P3 (variant1 and 3)

Q1.

10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m intersect. [4]

(ii) Calculate the acute angle between the lines. [3]

(iii) Find the equation of the plane containing l and m, giving your answer in the form $ax + by + cz = d$. [5]

Q2.

10 The straight line l has equation $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $3x - y + 2z = 9$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find an equation for the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = d$. [5]

Q3.

3 Points A and B have coordinates $(-1, 2, 5)$ and $(2, -2, 11)$ respectively. The plane p passes through B and is perpendicular to AB.

(i) Find an equation of p, giving your answer in the form $ax + by + cz = d$. [3]

(ii) Find the acute angle between p and the y-axis. [4]

Q4.

10 With respect to the origin O, the lines l and m have vector equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

(i) Prove that l and m do not intersect. [4]

(ii) Calculate the acute angle between the directions of l and m. [3]

(iii) Find the equation of the plane which is parallel to l and contains m, giving your answer in the form $ax + by + cz = d$. [5]

Q5.

8 The point P has coordinates $(-1, 4, 11)$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(i) Find the perpendicular distance from P to l. [4]

(ii) Find the equation of the plane which contains P and l, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

Q6.

9 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

(i) Given that l and m intersect, show that

$$2a - b = 4.$$ [4]

(ii) Given also that l and m are perpendicular, find the values of a and b. [4]

(iii) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

Q7.

6 The points P and Q have position vectors, relative to the origin O, given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of PQ is the point A. The plane Π is perpendicular to the line PQ and passes through A.

(i) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(ii) The straight line through P parallel to the x-axis meets Π at the point B. Find the distance AB, correct to 3 significant figures. [5]

Q8.

10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p. [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

Q9.

6 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

The mid-point of AB is M. The point N lies on AC between A and C and is such that $AN = 2NC$.

(i) Find a vector equation of the line MN. [4]

(ii) It is given that MN intersects BC at the point P. Find the position vector of P. [4]

Q10.

7 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line AB and OP is perpendicular to AB.

(i) Find a vector equation for the line AB. [1]

(ii) Find the position vector of P. [4]

(iii) Find the equation of the plane which contains AB and which is perpendicular to the plane OAB, giving your answer in the form $ax + by + cz = d$. [4]

Q11.

6 The straight line l passes through the points with coordinates $(-5, 3, 6)$ and $(5, 8, 1)$. The plane p has equation $2x - y + 4z = 9$.

(i) Find the coordinates of the point of intersection of l and p. [4]

(ii) Find the acute angle between l and p. [4]

Q12.

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda \overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

(iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

Q13.

9 The line l has equation $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$, where a is a constant. The plane p has equation $2x - 2y + z = 10$.

(i) Given that l does not lie in p, show that l is parallel to p. [2]

(ii) Find the value of a for which l lies in p. [2]

(iii) It is now given that the distance between l and p is 6. Find the possible values of a. [5]

Q14.

10 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane m is parallel to $\overrightarrow{OC}$ and contains A and B.

(i) Find the equation of m, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the length of the perpendicular from C to the line through A and B. [5]

Q15.

8 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$$

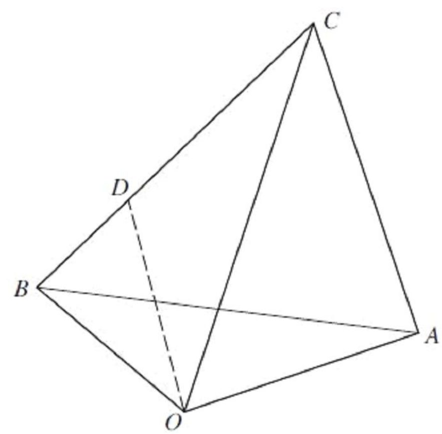
where p is a constant. It is given that the lines intersect.

(i) Find the value of p and determine the coordinates of the point of intersection. [5]

(ii) Find the equation of the plane containing the two lines, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

Q16.

9



The diagram shows three points A, B and C whose position vectors with respect to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. The point D lies on BC, between B and C, and is such that $CD = 2DB$.

(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the position vector of D. [1]

(iii) Show that the length of the perpendicular from A to OD is $\frac{1}{3}\sqrt{(65)}$. [4]

Q17.

6 Two planes have equations $3x - y + 2z = 9$ and $x + y - 4z = -1$.

(i) Find the acute angle between the planes. [3]

(ii) Find a vector equation of the line of intersection of the planes. [6]

Q18.

7 The straight line l has equation $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$. The plane p passes through the point $(4, -1, 2)$ and is perpendicular to l.

(i) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Find the perpendicular distance from the origin to p. [3]

(iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q. [3]

Q19.

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Q20.

10 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

Q21.

7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]