These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

7 The parametric equations of a curve are

$$x = 2\theta - \sin 2\theta, \quad y = 2 - \cos 2\theta.$$

(i) Show that $\frac{dy}{dx} = \cot \theta$. [5]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$. [3]

(iii) For the part of the curve where $0 < \theta < 2\pi$, find the coordinates of the points where the tangent is parallel to the x-axis. [3]

Q2.

6 The parametric equations of a curve are

$$x = 2t + \ln t, \qquad y = t + \frac{4}{t},$$

where t takes all positive values.

(i) Show that $\frac{dy}{dx} = \frac{t^2 - 4}{t(2t+1)}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $t = 1$. [3]

(iii) The curve has one stationary point. Find the y-coordinate of this point, and determine whether this point is a maximum or a minimum. [4]

Q3.

5 (i) By differentiating $\frac{1}{\cos \theta}$, show that if $y = \sec \theta$ then $\frac{dy}{d\theta} = \sec \theta \tan \theta$. [3]

(ii) The parametric equations of a curve are

$$x = 1 + \tan \theta, \qquad y = \sec \theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Show that $\frac{dy}{dx} = \sin \theta$. [3]

(iii) Find the coordinates of the point on the curve at which the gradient of the curve is $\frac{1}{2}$. [3]

Q4.

3 The equation of a curve is $y = x + 2\cos x$. Find the x-coordinates of the stationary points of the curve for $0 \leqslant x \leqslant 2\pi$, and determine the nature of each of these stationary points. [7]

Q5.

5 The equation of a curve is $3x^2 + 2xy + y^2 = 6$. It is given that there are two points on the curve where the tangent is parallel to the x-axis.

(i) Show by differentiation that, at these points, $y = -3x$. [4]

(ii) Hence find the coordinates of the two points. [4]

Q6.

3 The parametric equations of a curve are

$$x = 3t + \ln(t-1), \quad y = t^2 + 1, \quad \text{for } t > 1.$$

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Find the coordinates of the only point on the curve at which the gradient of the curve is equal to 1. [4]

Q7.

6 It is given that the curve $y = (x-2)e^x$ has one stationary point.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q8.

7 The equation of a curve is

$$x^2 + y^2 - 4xy + 3 = 0.$$

(i) Show that $\dfrac{dy}{dx} = \dfrac{2y - x}{y - 2x}$. [4]

(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the x-axis. [5]

Q9.

4 The parametric equations of a curve are

$$x = 4\sin\theta, \quad y = 3 - 2\cos 2\theta,$$

where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Express $\dfrac{dy}{dx}$ in terms of θ, simplifying your answer as far as possible. [5]

Q10.

6 The equation of a curve is

$$x^2y + y^2 = 6x.$$

(i) Show that $\frac{dy}{dx} = \frac{6 - 2xy}{x^2 + 2y}$. [4]

(ii) Find the equation of the tangent to the curve at the point with coordinates (1, 2), giving your answer in the form $ax + by + c = 0$. [3]

Q11.

5 The equation of a curve is $y = x^3e^{-x}$.

(i) Show that the curve has a stationary point where $x = 3$. [3]

(ii) Find the equation of the tangent to the curve at the point where $x = 1$. [4]

Q12.

2 A curve has parametric equations

$$x = 3t + \sin 2t, \quad y = 4 + 2\cos 2t.$$

Find the exact gradient of the curve at the point for which $t = \frac{1}{6}\pi$. [4]

Q13.

5 Find the value of $\frac{dy}{dx}$ when $x = 4$ in each of the following cases:

(i) $y = x\ln(x - 3)$, [4]

(ii) $y = \frac{x - 1}{x + 1}$. [3]

Q14.

5 A curve has equation $x^2 + 2y^2 + 5x + 6y = 10$. Find the equation of the tangent to the curve at the point $(2, -1)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

Q15.

6 The curve $y = 4x^2 \ln x$ has one stationary point.

(i) Find the coordinates of this stationary point, giving your answers correct to 3 decimal places. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q16.

5 The parametric equations of a curve are

$$x = \ln(t+1), \quad y = e^{2t} + 2t.$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the normal to the curve at the point for which $t = 0$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

Q17.

5 The parametric equations of a curve are

$$x = e^{2t}, \quad y = 4te^t.$$

(i) Show that $\frac{dy}{dx} = \frac{2(t+1)}{e^t}$. [4]

(ii) Find the equation of the normal to the curve at the point where $t = 0$. [4]

Q18.

5 The equation of a curve is

$$x^2 - 2x^2y + 3y = 9.$$

(i) Show that $\frac{dy}{dx} = \frac{2x - 4xy}{2x^2 - 3}$. [4]

(ii) Find the equation of the normal to the curve at the point where $x = 2$, giving your answer in the form $ax + by + c = 0$. [4]

Q19.

7 The equation of a curve is

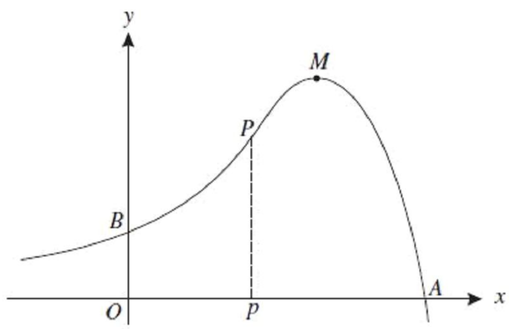
$$2x^2 + 3y^2 - 2xy = 10.$$

(i) Show that $\frac{dy}{dx} = \frac{y - 2x}{3y - x}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

Q20.

6



The diagram shows the curve $y = (4 - x)e^x$ and its maximum point M. The curve cuts the x-axis at A and the y-axis at B.

(i) Write down the coordinates of A and B. [2]

(ii) Find the x-coordinate of M. [4]

(iii) The point P on the curve has x-coordinate p. The tangent to the curve at P passes through the origin O. Calculate the value of p. [5]

Q21.

5 The curve with equation $y = x^2 \ln x$, where $x > 0$, has one stationary point.

(i) Find the x-coordinate of this point, giving your answer in terms of e. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q22.

4 The equation of a curve is $x^3 + y^3 = 9xy$.

(i) Show that $\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}$. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 4), giving your answer in the form $ax + by = c$. [3]

Q23.

4 The equation of a curve is $y = 2x - \tan x$, where x is in radians. Find the coordinates of the stationary points of the curve for which $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. [5]

Q24.

6 Find the exact coordinates of the point on the curve $y = xe^{-\frac{1}{2}x}$ at which $\frac{d^2y}{dx^2} = 0$. [7]

Q25.

6 The curve with equation $y = x \ln x$ has one stationary point.

(i) Find the exact coordinates of this point, giving your answers in terms of e. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q26.

8 The equation of a curve is $y^2 + 2xy - x^2 = 2$.

(i) Find the coordinates of the two points on the curve where $x = 1$. [2]

(ii) Show by differentiation that at one of these points the tangent to the curve is parallel to the x-axis. Find the equation of the tangent to the curve at the other point, giving your answer in the form $ax + by + c = 0$. [7]

Q27.

4 The parametric equations of a curve are

$$x = 1 - e^{-t}, \quad y = e^{t} + e^{-t}.$$

(i) Show that $\frac{dy}{dx} = e^{2t} - 1$. [3]

(ii) Hence find the exact value of t at the point on the curve at which the gradient is 2. [2]

Q28.

4 The parametric equations of a curve are

$$x = 1 + \ln(t - 2), \quad y = t + \frac{9}{t}, \quad \text{for } t > 2.$$

(i) Show that $\frac{dy}{dx} = \frac{(t^2 - 9)(t - 2)}{t^2}$. [3]

(ii) Find the coordinates of the only point on the curve at which the gradient is equal to 0. [3]

Q29.

8 The equation of a curve is

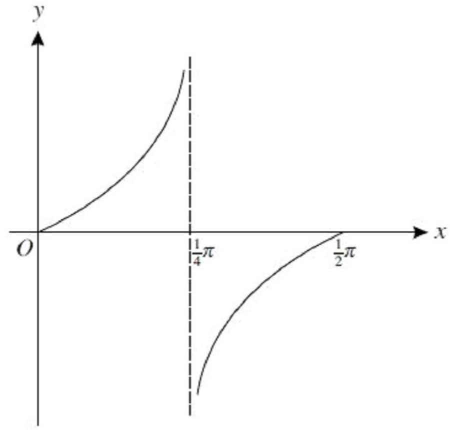
$$x^2 + 2xy - y^2 + 8 = 0.$$

(i) Show that the tangent to the curve at the point $(-2, 2)$ is parallel to the x-axis. [4]

(ii) Find the equation of the tangent to the curve at the other point on the curve for which $x = -2$, giving your answer in the form $y = mx + c$. [5]

Q30.

3



The diagram shows the part of the curve $y = \frac{1}{2}\tan 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. Find the x-coordinates of the points on this part of the curve at which the gradient is 4. [5]

Q31.

7 The parametric equations of a curve are

$$x = e^{3t}, \quad y = t^2e^t + 3.$$

(i) Show that $\frac{dy}{dx} = \frac{t(t+2)}{3e^{2t}}$. [4]

(ii) Show that the tangent to the curve at the point (1, 3) is parallel to the x-axis. [2]

(iii) Find the exact coordinates of the other point on the curve at which the tangent is parallel to the x-axis. [2]

Q32.

6 The parametric equations of a curve are

$$x = 1 + 2\sin^2\theta, \quad y = 4\tan\theta.$$

(i) Show that $\frac{dy}{dx} = \frac{1}{\sin\theta\cos^3\theta}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$, giving your answer in the form $y = mx + c$. [4]

Q33.

1 Find the gradient of the curve $y = \ln(5x + 1)$ at the point where $x = 4$. [3]

Q34.

8 The equation of a curve is $2x^2 - 3x - 3y + y^2 = 6$.

(i) Show that $\frac{dy}{dx} = \frac{4x - 3}{3 - 2y}$. [3]

(ii) Find the coordinates of the two points on the curve at which the gradient is -1. [6]

Q35.

4 The parametric equations of a curve are

$$x = \ln(1 - 2t), \quad y = \frac{2}{t}, \quad \text{for } t < 0.$$

(i) Show that $\frac{dy}{dx} = \frac{1 - 2t}{t^2}$. [3]

(ii) Find the exact coordinates of the only point on the curve at which the gradient is 3. [3]

Q36.

2 The curve with equation $y = \frac{\sin 2x}{e^{2x}}$ has one stationary point in the interval $0 \leqslant x \leqslant \frac{1}{2}\pi$. Find the exact x-coordinate of this point. [4]

Q37.

7 The equation of a curve is

$$3x^2 - 4xy + 2y^2 - 6 = 0.$$

(i) Show that $\frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$. [4]

(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the x-axis. [5]

Q38.

3 The equation of a curve is $y = \frac{1}{2}e^{2x} - 5e^x + 4x$. Find the exact x-coordinate of each of the stationary points of the curve and determine the nature of each stationary point. [6]

Q39.

5 The parametric equations of a curve are

$$x = \cos 2\theta - \cos\theta, \quad y = 4\sin^2\theta,$$

for $0 \leqslant \theta \leqslant \pi$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{8\cos\theta}{1 - 4\cos\theta}$. [4]

(ii) Find the coordinates of the point on the curve at which the gradient is -4. [4]

Q40.

2 The curve $y = \dfrac{e^{3x-1}}{2x}$ has one stationary point. Find the coordinates of this stationary point. [5]

Q41.

5 The parametric equations of a curve are

$$x = 1 + \sqrt{t}, \quad y = 3\ln t.$$

(i) Find the exact value of the gradient of the curve at the point P where $y = 6$. [5]

(ii) Show that the tangent to the curve at P passes through the point $(1, 0)$. [3]

Q42.

2 Find the gradient of each of the following curves at the point for which $x = 0$.

(i) $y = 3\sin x + \tan 2x$ [3]

(ii) $y = \dfrac{6}{1 + e^{2x}}$ [3]

Q43.

7 The equation of a curve is

$$2x^2 + 3xy + y^2 = 3.$$

(i) Find the equation of the tangent to the curve at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

(ii) Show that the curve has no stationary points. [4]

Q44.

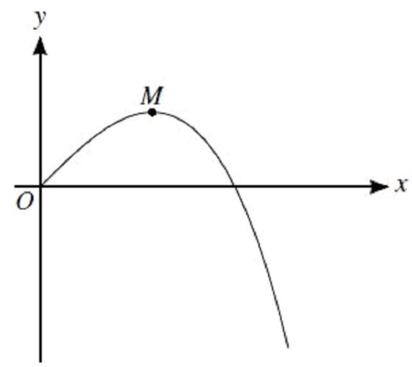
4 The parametric equations of a curve are

$$x = 2\ln(t + 1), \quad y = 4e^t.$$

Find the equation of the tangent to the curve at the point for which $t = 0$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

Q45.

8



The diagram shows the curve

$$y = \tan x \cos 2x, \text{ for } 0 \leqslant x < \tfrac{1}{2}\pi,$$

and its maximum point M.

(i) Show that $\dfrac{dy}{dx} = 4\cos^2 x - \sec^2 x - 2$. [5]

(ii) Hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [4]

Q46.

4 For each of the following curves, find the exact gradient at the point indicated:

(i) $y = 3\cos 2x - 5\sin x$ at $\left(\tfrac{1}{6}\pi, -1\right)$, [3]

(ii) $x^3 + 6xy + y^3 = 21$ at $(1, 2)$. [5]

Q47.

3 A curve has equation

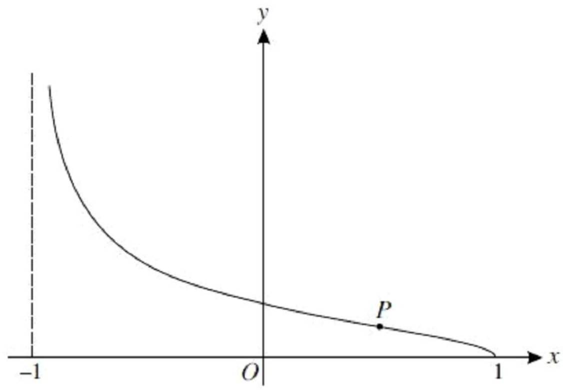
$$3\ln x + 6xy + y^2 = 16.$$

Find the equation of the normal to the curve at the point $(1, 2)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [7]

P3 (variant1 and 3)

Q1.

9



The diagram shows the curve $y = \sqrt{\left(\frac{1-x}{1+x}\right)}$.

(i) By first differentiating $\frac{1-x}{1+x}$, obtain an expression for $\frac{dy}{dx}$ in terms of x. Hence show that the gradient of the normal to the curve at the point (x, y) is $(1+x)\sqrt{(1-x^2)}$. [5]

(ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x-coordinate of P. [4]

Q2.

2 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \frac{\tan x}{x}$. [2]

Q3.

5 The curve with equation

$$6e^{2x} + ke^{y} + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

Q4.

2 The curve $y = \frac{\ln x}{x^3}$ has one stationary point. Find the x-coordinate of this point. [4]

Q5.

6 The equation of a curve is $3x^2 - 4xy + y^2 = 45$.

(i) Find the gradient of the curve at the point $(2, -3)$. [4]

(ii) Show that there are no points on the curve at which the gradient is 1. [3]

Q6.

3 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2\sin\theta.$$

Show that $\frac{dy}{dx} = \frac{2\cos\theta}{1 + 2\sin\theta}$. [5]

Q7.

4 The curve with equation $y = \frac{e^{2x}}{x^3}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

Q8.

5 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \frac{1 + x^2}{1 + e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

Q9.

4 A curve has equation $y = e^{-3x}\tan x$. Find the x-coordinates of the stationary points on the curve in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. Give your answers correct to 3 decimal places. [6]

Q10.

2 The parametric equations of a curve are

$$x = \frac{t}{2t+3}, \qquad y = e^{-2t}.$$

Find the gradient of the curve at the point for which $t = 0$. [5]

Q11.

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

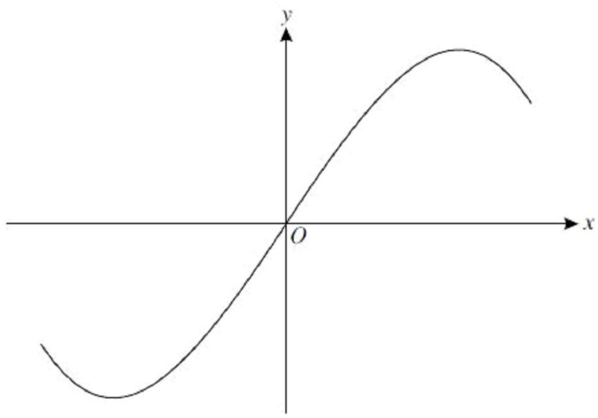
Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

Q12.

2 The equation of a curve is $y = \frac{e^{2x}}{1 + e^{2x}}$. Show that the gradient of the curve at the point for which $x = \ln 3$ is $\frac{9}{50}$. [4]

Q13.

8



The diagram shows the curve with parametric equations

$$x = \sin t + \cos t, \quad y = \sin^3 t + \cos^3 t,$$

for $\frac{1}{4}\pi < t < \frac{5}{4}\pi$.

(i) Show that $\frac{dy}{dx} = -3 \sin t \cos t$. [3]

(ii) Find the gradient of the curve at the origin. [2]

(iii) Find the values of t for which the gradient of the curve is 1, giving your answers correct to 2 significant figures. [4]

Q14.

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

Q15.

3 The parametric equations of a curve are

$$x = \frac{4t}{2t + 3}, \quad y = 2\ln(2t + 3).$$

(i) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

Q16.

1 The equation of a curve is $y = \frac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]

Q17.

4 The parametric equations of a curve are

$$x = e^{-t}\cos t, \quad y = e^{-t}\sin t.$$

Show that $\frac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

Q18.

4 A curve has equation $3e^{2x}y + e^{x}y^{3} = 14$. Find the gradient of the curve at the point (0, 2). [5]

Q19.

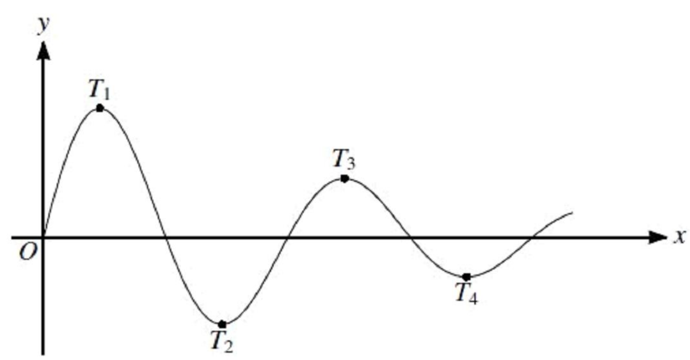
3 The parametric equations of a curve are

$$x = \ln(2t+3), \quad y = \frac{3t+2}{2t+3}.$$

Find the gradient of the curve at the point where it crosses the y-axis. [6]

Q20.

10



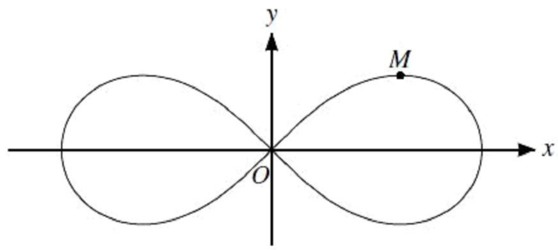
The diagram shows the curve $y = 10e^{-\frac{1}{2}x}\sin 4x$ for $x \geqslant 0$. The stationary points are labelled T_1, T_2, T_3, … as shown.

(i) Find the x-coordinates of T_1 and T_2, giving each x-coordinate correct to 3 decimal places. [6]

(ii) It is given that the x-coordinate of T_n is greater than 25. Find the least possible value of n. [4]

Q21.

6



The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M. Find the coordinates of M. [7]

Q22.

4 The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

Q23.

2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan \theta, \quad y = 2\cos^2 \theta \sin \theta.$$

Show that $\frac{dy}{dx} = 6\cos^5 \theta - 4\cos^3 \theta$. [5]