**Q1.**

| | | |
|---|---|---|
| 3 (i) | State or imply indefinite integral of $e^{2x}$ is $\frac{1}{2}e^{2x}$, or equivalent | B1 |
| | Substitute correct limits correctly | M1 |
| | Obtain answer $R = \frac{1}{2} e^{2p} - \frac{1}{2}$, or equivalent | A1 |
| | | **[3]** |
| (ii) | Substitute $R = 5$ and use logarithmic method to obtain an equation in $2p$ | M1* |
| | Solve for $p$ | M1 (dep*) |
| | Obtain answer $p = 1.2$ (1.1989 ...) | A1 |
| | | **[3]** |

**Q2.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Make relevant use of the cos(A + B) formula | M1* | |
| | | Make relevant use of cos2A and sin2A formulae | M1* | |
| | | Obtain a correct expression in terms of cosA and sinA | A1 | |
| | | Use $\sin^2 A = 1 - \cos^2 A$ to obtain an expression in terms of cosA | M1(dep*) | |
| | | Obtain given answer correctly | A1 | **5** |
| | (ii) | Replace integrand by $\frac{1}{4}\cos 3x + \frac{3}{4}\cos x$, or equivalent | B1 | |
| | | Integrate, obtaining $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$, or equivalent | B1 + B1√ | |
| | | Use limits correctly | M1 | |
| | | Obtain given anser | A1 | **5** |

**Q3.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Make relevant use of the sin(*A* + *B*) formula | B1 | |
| | | Make relevant use of sin2*A* and cos2*A* formulae | M1 | |
| | | Obtain a correct expression in terms of sin *x* and cos *x* | A1 | |
| | | Use $\cos^2 x = 1 - \sin^2 x$ to obtain an expression in terms of sin *x* | M1(dep*) | |
| | | Obtain given answer correctly | A1 | **5** |
| | (ii) | Replace integrand by $\frac{3}{4}\sin x - \frac{1}{4}\sin 3x$, or equivalent | B1 | |
| | | Integrate, obtaining $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$, or equivalent | B1√ + B1√ | |
| | | Use limits correctly | M1 | |
| | | Obtain given answer correctly | A1 | **5** |

**Q4.**

| | | | | |
|---|---|---|---|---|
| 6 | (i) | State correct expression $\frac{1}{2}+\frac{1}{2}\cos 2x$, or equivalent | B1 | [1] |
| | (ii) | Integrate an expression of the form $a+b\cos 2x$, where $ab \neq 0$, correctly | M1 | |
| | | State correct integral $\frac{1}{2}x+\frac{1}{4}\sin 2x$, or equivalent | A1 | |
| | | Use correct limits correctly | M1 | |
| | | Obtain given answer correctly | A1 | [4] |
| | (iii) | Use identity $\sin^2 x = 1-\cos^2 x$ and attempt indefinite integration | M1 | |
| | | Obtain integral $x-\left(\frac{1}{2}x-\frac{1}{4}\sin 2x\right)$, or equivalent | A1 | |
| | | Use limits and obtain answer $\frac{1}{6}\pi-\frac{\sqrt{3}}{8}$ | A1 | [3] |
| | | [Solutions that use the result of part **(ii)**, score M1A1 for integrating 1 and A1 for the final answer.] | | |

## Q5.

| | | | |
|---|---|---|---|
| 3 | Obtain integral $\frac{1}{2}\sin 2x - \cos x$ | B1 + B1 | |
| | Substitute limits correctly in an integral of the form $a \sin 2x + b \cos x$ | M1 | |
| | Use correct exact values, e.g. of $\cos\left(\frac{1}{6}\pi\right)$ | M1 | |
| | Obtain answer $1-\frac{1}{4}\sqrt{3}$, or equivalent | A1 | [5] |

## Q6.

| | | | | |
|---|---|---|---|---|
| 3 | (i) | Show or imply correct ordinates 1, 0.5, 0.414213 ... | B1 | |
| | | Use correct formula, or equivalent, with $h = 1$ and three ordinates | M1 | |
| | | Obtain answer 1.21 with no errors seen | A1 | [3] |
| | (ii) | Justify the statement that the rule gives an over-estimate | B1 | [1] |

## Q7.

| | | | |
|---|---|---|---|
| 2 | Obtain integral $\ln(x+2)$ | B1 | |
| | Substitute correct limits correctly | M1 | |
| | Use law for the logarithm of a product, a quotient or a power | M1 | |
| | Obtain given answer following full and correct working | A1 | [4] |

## Q8.

| | | | | |
|---|---|---|---|---|
| 2 | (i) | State or imply correct ordinates 0.27067..., 0.20521..., 0.14936... | B1 | |
| | | Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates | M1 | |
| | | Obtain answer 0.21 with no errors seen | A1 | [3] |
| | (ii) | Justify statement that the trapezium rule gives an over-estimate | B1 | [1] |

## Q9.

| | | | | |
|---|---|---|---|---|
| 4 | (a) | Obtain integral $a \sin 2x$ with $a = \pm\left(1, 2 \text{ or } \frac{1}{2}\right)$ | M1 | |
| | | Use limits and obtain $\frac{1}{2}$ (AG) | A1 | [2] |
| | (b) | Use $\tan^2 x = \sec^2 x - 1$ and attempt to integrate both terms | M1 | |
| | | Obtain $3\tan x - 3x$ | A1 | |
| | | Attempt to substitute limits, using exact values | M1 | |
| | | Obtain answer $2\sqrt{3} - \frac{\pi}{2}$ | A1 | [4] |

## Q10.

| | | | | |
|---|---|---|---|---|
| 6 | (a) | Rewrite integrand as $12e^{x} + 4e^{3x}$ | B1 | |
| | | Integrate to obtain $12e^{x}$ … | B1 | |
| | | Integrate to obtain … $+\frac{4}{3}e^{3x}$ | B1 | |
| | | Include … $+ c$ | B1 | [4] |
| | (b) | Use identity $\tan^2\theta = \sec^2\theta - 1$ | B1 | |
| | | Integrate to obtain $2\tan\theta + \theta$ or equivalent | B1 | |
| | | Use limits correctly for integral of form $a\tan\theta + b\theta$ | M1 | |
| | | Confirm given answer $\frac{1}{2}(8 + \pi)$ | A1 | [4] |

## Q11.

| | | | | |
|---|---|---|---|---|
| 2 | (i) | Show or imply correct ordinates 1, $\sqrt{2}$ or 1.414, 3 | B1 | |
| | | Use correct formula, or equivalent, with $h = 1$ | M1 | |
| | | Obtain 3.41 | A1 | [3] |
| | (ii) | Obtain 6 – 3.41 and hence 2.59, following their answer to **(i)** provided less than 6 | B1√ | |
| | | Refer, in some form, to two line segments replacing curve and conclude with clear justification of given result that answer is an under-estimate. | B1 | [2] |

## Q12.

| | | | | |
|---|---|---|---|---|
| 4 | (a) | Obtain integral form of $k\cos\frac{1}{2}x$ | M1 | |
| | | Obtain correct $-2\cos\frac{1}{2}x$ | A1 | |
| | | Use limits correctly to obtain 1 | A1 | [3] |
| | (b) | Rewrite integrand as $e^{-x} + 1$ | B1 | |
| | | Integrate to obtain $-e^{-x}$ … | B1 | |
| | | Integrate to obtain … $+ x + c$ | B1 | [3] |

**Q13.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Expand to obtain 4 $\sin^2 x$ + 4 $\sin x \cos x + \cos^2 x$ | B1 | |
| | | Use 2 $\sin x \cos x = \sin 2x$ | B1 | |
| | | Attempt to express $\sin^2 x$ or $\cos^2 x$ (or both) in terms of $\cos 2x$ | M1 | |
| | | Obtain correct $\frac{1}{2}k(1-\cos 2x)$ for their $k \sin^2 x$ or equivalent | A1√ | |
| | | Confirm given answer $\frac{5}{2} + 2\sin 2x - \frac{3}{2}\cos 2x$ | A1 | [5] |
| | (ii) | Integrate to obtain form $px + q \cos 2x + r \sin 2x$ | M1 | |
| | | Obtain $\frac{5}{2}x - \cos 2x - \frac{3}{4}\sin 2x$ | A1 | |
| | | Substitute limits in integral of form $px + q \cos 2x + r \sin 2x$ and attempt simplification | DM1 | |
| | | Obtain $\frac{5}{8}\pi + \frac{1}{4}$ or exact equivalent | A1 | [4] |

**Q14.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Replace $\tan^2 x$ by $\sec^2 x - 1$ | B1 | |
| | | Express $\cos^2 x$ in the form $\pm\frac{1}{2} \pm \frac{1}{2}\cos 2x$ | M1 | |
| | | Obtain given answer $\sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$ correctly | A1 | |
| | | Attempt integration of expression | M1 | |
| | | Obtain $\tan x + \frac{1}{4}\sin 2x - \frac{1}{2}x$ | A1 | |
| | | Use limits correctly for integral involving at least $\tan x$ and $\sin 2x$ | M1 | |
| | | Obtain $\frac{5}{4} - \frac{1}{8}\pi$ or exact equivalent | A1 | [7] |
| | (ii) | State or imply volume is $\int \pi(\tan x + \cos x)^2 \, dx$ | B1 | |
| | | Attempt expansion and simplification | M1 | |
| | | Integrate to obtain one term of form $k \cos x$ | M1 | |
| | | Obtain $\pi(\frac{5}{4} - \frac{1}{8}\pi) + \pi(2-\sqrt{2})$ or equivalent | A1 | [4] |

**Q15.**

| | | | | |
|---|---|---|---|---|
| 3 | (i) | Either | | |
| | | Use $\sin 2x = 2\sin x \cos x$ to convert integrand to $k \sin^2 2x$ | M1 | |
| | | Use $\cos 4x = 1 - 2\sin^2 2x$ | M1 | |
| | | State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent | A1 | |
| | | Or | | |
| | | Use $\cos^2 x = \frac{1-\cos 2x}{2}$ and/or $x = \frac{1-\cos 2x}{2}$ to obtain an equation in $\cos 2x$ only | M1 | |
| | | Use $\cos^2 2x = \frac{1+\cos 4x}{2}$ | M1 | |
| | | State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent | A1 | [3] |
| | (ii) | State correct integral $\frac{3}{2}x - \frac{3}{8}\sin 4x$, or equivalent | B1 | |
| | | Attempt to substitute limits, using exact values | M1 | |
| | | Obtain given answer correctly | A1 | [3] |

## Q16.

| | | | |
|---|---|---|---|
| 1 | Integrate and obtain term of the form $k \ln(7 - 2x)$ | M1 | |
| | State $y = -2\ln(7 - 2x)(+c)$ | A1 | |
| | Evaluate $c$ | DM1 | |
| | Obtain answer $y = -2\ln(7 - 2x) + 2$ | A1√ | [4] |

## Q17.

| | | | | | |
|---|---|---|---|---|---|
| 6 | (a) | Obtain indefinite integral $-\frac{1}{2}\cos 2x + \sin x$ | | B1 + B1 | |
| | | Use limits with attempted integral | | M1 | |
| | | Obtain answer 2 correctly with no errors | | A1 | 4 |
| | (b) | (i) | Identify $R$ with correct definite integral and attempt to integrate | M1 | |
| | | | Obtain indefinite integral $\ln(x + 1)$ | B1 | |
| | | | Obtain answer $R = \ln(p + 1) - \ln 2$ | A1 | 3 |
| | | (ii) | Use exponential method to solve an equation of the form $\ln x = k$ | M1 | |
| | | | Obtain answer $p = 13.8$ | A1 | 2 |

## Q18.

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Obtain derivative of the form $k \sec^2 2x$, where $k = 2$ or $k = 1$ | M1 | |
| | | Obtain correct derivative $2\sec^2 2x$ | A1 | 2 |
| | (ii) | State or imply the indefinite integral is $\frac{1}{k}\tan 2x$, where $k = 2$ or $k = 1$ | M1* | |
| | | Substitute limits correctly | M1(dep*) | |
| | | Obtain given answer $\frac{1}{2}\sqrt{3}$ | A1 | |
| | | Use $\tan^2 2x = \sec^2 2x - 1$ and attempt to integrate both terms, or equivalent | M1 | |
| | | Substitute limits in indefinite integral of the form $\frac{1}{k}\tan 2x - x$, where $k = 2$ or $k = 1$ | M1 | |
| | | Obtain answer $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$, or equivalent | A1 | 6 |
| | (iii) | State that the integrand is equivalent to $\frac{1}{2}\sec^2 2x$ | B1 | |
| | | Obtain answer $\frac{1}{4}\sqrt{3}$ | B1 | 2 |

**Q19.**

| | | | |
|---|---|---|---|
| 1 | State indefinite integral of the form $k\ \text{In}(2x + 1)$, where $k = \frac{1}{2}$, 1 or 2 | M1 | |
| | State correct integral $\frac{1}{2}\text{In}(2x + 1)$ | A1 | |
| | Use limits correctly, allow use of limits $x = 4$ and $x = 1$ in an incorrect form | M1 | |
| | Obtain given answer | A1 | [4] |

**Q20.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Expand and use sin $2A$ formula | M1 | |
| | | Use cos $2A$ formula at least once | M1 | |
| | | Obtain any correct expression in terms of cos $2x$ and sin $2x$ only – can be implied | A1 | |
| | | Obtain given answer correctly | A1 | [4] |
| | (ii) | State indefinite integral $5x - 2\sin 2x - \frac{3}{2}\cos 2x$ | B2 | |
| | | [Award B1 if one error in one term] | | |
| | | Substitute limits correctly – must be correct limits | M1 | |
| | | Obtain answer $\frac{1}{4}(5\pi - 2)$, or exact simplified equivalent | A1 | [4] |

**Q21.**

| | | | |
|---|---|---|---|
| 5 | Integrate and state term $\ln x$ | B1 | |
| | Obtain term of the form $k\ln(2x + 1)$ | M1 | |
| | State correct term $-2\ln(2x + 1)$ | A1 | |
| | Substitute limits correctly | M1 | |
| | Use law for the logarithm of a product, quotient or power | M1 | |
| | Obtain given answer correctly | A1 | [6] |

**Q22.**

| | | | | |
|---|---|---|---|---|
| 5 | (i) | Use double angle formulae and obtain $a + b\cos 4x$ | M1 | |
| | | Obtain answer $\frac{1}{2} + \frac{1}{2}\cos 4x$, or equivalent | A1 | [2] |
| | (ii) | Integrate and obtain $\frac{1}{2}x + \frac{1}{8}\sin 4x$ | A1√ + A1√ | |
| | | Substitute limits correctly | M1 | |
| | | Obtain answer $\frac{1}{16}\pi + \frac{1}{8}$, or exact equivalent | A1 | [4] |

## Q23.

| | | | | |
|---|---|---|---|---|
| 3 | (i) | Show or imply correct ordinates 1, 1.15470…, 2 | B1 | |
| | | Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and three ordinates | M1 | |
| | | Obtain answer 1.39 with no errors seen | A1 | [3] |
| | (ii) | Make recognisable sketch of $y = \sec x$ for $0 \le x \le \frac{1}{3}\pi$ | B1 | |
| | | Using a correct graph, explain that the rule gives an over-estimate | B1 | [2] |

## Q24.

| | | | | |
|---|---|---|---|---|
| 8 | (a) | Integrate and obtain term $k\cos 2x$, where $k = \pm\frac{1}{2}$ or $\pm 1$ | M1 | |
| | | Obtain term $-\frac{1}{2}\cos 2x$ | A1 | |
| | | Obtain term $\tan x$ | B1 | |
| | | Substitute correct limits correctly | M1 | |
| | | Obtain exact answer $\dfrac{3}{4} + \sqrt{3}$ | A1 | [5] |
| | (b) | Integrate and obtain $\frac{1}{2}\ln x + \ln(x+1)$ or $\frac{1}{2}\ln 2x + \ln(x+1)$ | B1 + B1 | |
| | | Substitute correct limits correctly | M1 | |
| | | Obtain given answer following full and correct working | A1 | [4] |

## Q25.

| | | | |
|---|---|---|---|
| 3 | Integrate and obtain $\dfrac{1}{2}e^{2x}$ term | B1 | |
| | Obtain $2e^{x}$ term | B1 | |
| | Obtain $x$ | B1 | |
| | Use limits correctly, allow use of limits $x = 1$ and $x = 0$ into an incorrect form | M1 | |
| | Obtain given answer | A1 | [5] |
| | S. R. Feeding limits into original integrand, 0/5 | | |

## Q26.

| | | | | |
|---|---|---|---|---|
| **4** | **(a)** | Obtain integral of the form $ke^{1-2x}$ with any non-zero $k$ | M1 | |
| | | Correct integral | A1 | [2] |
| | **(b)** | Attempt to use double angle formula to expand $\cos(3x + 3x)$ | M1 | |
| | | State correct expression $\frac{1}{2} - \frac{1}{2}\cos 6x$ or equivalent | A1 | |
| | | Integrate an expression of the form $a + b\cos 6x$, where $ab \neq 0$, correctly | M1 | |
| | | State correct integral $\frac{1}{2}x - \frac{1}{12}\sin 6x$, or equivalent | A1 | [4] |

## Q27.

| | | | |
|---|---|---|---|
| **2** | Integrate and obtain term of the form $k\ln(4x + 1)$ | M1 | |
| | State correct term $\frac{1}{2}\ln(4x+1)$ | A1 | |
| | Substitute limits correctly | M1 | |
| | Use law for the logarithm of a quotient or a power | M1 | |
| | Obtain given answer correctly | A1 | [5] |

## Q28.

| | | | | |
|---|---|---|---|---|
| **8** | **(i)** | Make relevant use of the $\cos(A + B)$ formula | M1* | |
| | | Make relevant use of the $\cos 2A$ and $\sin 2A$ formulae | M1* | |
| | | Obtain a correct expression in terms of $\cos x$ and $\sin x$ | A1 | |
| | | Use $\sin^2 x = 1 - \cos^2 x$ to obtain an expression in terms of $\cos x$ | M1(dep*) | |
| | | Obtain given answer correctly | A1 | [5] |
| | **(ii)** | Replace integrand by $\frac{1}{2}\cos 3x + \frac{1}{2}\cos x$, or equivalent | B1 | |
| | | Integrate, obtaining $\frac{1}{6}\sin 3x + \frac{1}{2}\sin x$, or equivalent | B1 + B1√ | |
| | | Use limits correctly | M1 | |
| | | Obtain given answer | A1 | [5] |

## Q29.

| | | | |
|---|---|---|---|
| **4** | State at least one correct integral | B1 | |
| | Use limits correctly to obtain an equation in $e^{2k}$, $e^{4k}$ | M1 | |
| | Carry out recognizable solution method for quadratic in $e^{2k}$ | M1 | |
| | Obtain $e^{2k} = 1$ and $e^{2k} = 3$ | A1 | |
| | Use logarithmic method to solve an equation of the form $e^{\lambda a} = b$, where $b > 0$ | M1 | |
| | Obtain answer $k = \frac{1}{2}\ln 3$ | A1 | [6] |

## Q30.

| | | | | |
|---|---|---|---|---|
| 4 | (i) | State correct expression $\frac{1}{2}+\frac{1}{2}\cos 2x$, or equivalent | B1 | [1] |
| | (ii) | Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly | M1 | |
| | | State correct integral $\frac{1}{2}x+\frac{1}{4}\sin 2x$, or equivalent | A1 | |
| | | Obtain correct integral (for $\sin 2x$ term) of $-\frac{1}{2}\cos 2x$ | B1 | |
| | | Attempt to substitute limits, using exact values | M1 | |
| | | Obtain given answer correctly | A1 | [5] |

## Q31.

| | | | | |
|---|---|---|---|---|
| 6 | (a) | State or imply correct ordinates 0.125, 0.08743…, 0.21511… | B1 | |
| | | Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates | M1 | |
| | | Obtain answer 0.11 with no errors seen | A1 | [3] |
| | (b) | Attempt to expand brackets and divide by $e^{2x}$ | M1 | |
| | | Integrate a term of form $ke^{-x}$ or $ke^{-2x}$ correctly | A1√ | |
| | | Obtain 2 correct terms | A1 | |
| | | Fully correct integral $x + 4e^{-x} - 2e^{-2x} + c$ | A1 | [4] |

## Q32.

| | | | | |
|---|---|---|---|---|
| 6 | (a) | Obtain integral $ke^{-\frac{1}{2}x}$ with any non-zero $k$ | M1 | |
| | | Correct integral | A1 | [2] |
| | (b) | State indefinite integral of the form $k\ln(3x-1)$, where $k = 2$, 6 or 3 | M1 | |
| | | State correct integral $2\ln(3x-1)$ | A1 | |
| | | Substitute limits correctly (must be a function involving a logarithm) | M1 | |
| | | Use law for the logarithm of a power or a quotient | M1 | |
| | | Obtain given answer correctly | A1 | [5] |

## Q33.

| | | | | | |
|---|---|---|---|---|---|
| 6 | (a) | (i) | Attempt to divide by $e^{2x}$ and attempt to integrate 2 terms | M1 | |
| | | | Integrate a term of form $ke^{-2x}$ correctly | A1√ | |
| | | | Fully correct integral $x - 3e^{-2x}\ (+c)$ | A1 | [3] |
| | | (ii) | State correct expression $\frac{1}{2}\cos 2x+\frac{1}{2}$ or equivalent | B1 | |
| | | | Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly | M1 | |
| | | | State correct integral $\frac{3\sin 2x}{4}+\frac{3x}{2}(+c)$ | A1 | [3] |
| | (b) | | State or imply correct ordinates 5.46143…, 4.78941…, 4.32808… | B1 | |
| | | | Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates | M1 | |
| | | | Obtain answer 4.84 with no errors seen | A1 | [3] |

## Q34.

| | | | | |
|---|---|---|---|---|
| 1 | (i) | State indefinite integral of the form $k \ln (4x - 1)$, where $k = 2, 4$, or ½ | M1 | |
| | | State correct integral ½ $\ln (4x - 1)$ | A1 | [2] |
| | (ii) | Substitute limits correctly | M1 | |
| | | Use law for the logarithm of a power or a quotient | M1 | |
| | | Obtain ln 3 correctly | A1 | [3] |

## Q35.

| | | | | | |
|---|---|---|---|---|---|
| 6 | (a) | | Expand brackets and use $\sin^2 x + \cos^2 x = 1$ | M1 | |
| | | | Obtain $1 - \sin 2x$ | A1 | |
| | | | Integrate and obtain term of form $\pm k \cos 2x$, where $k$ = ½, 1 or 2 | M1 | |
| | | | State correct integral $x + \dfrac{\cos 2x}{2} (+c)$ | A1 | [4] |
| | (b) | (i) | State or imply correct ordinates 1.4142…, 1.0823…, 1 | B1 | |
| | | | Use correct formula, or equivalent, correctly with $h = \dfrac{\pi}{8}$ and three ordinates | M1 | |
| | | | Obtain answer 0.899 with no errors seen | A1 | [3] |
| | | (ii) | Make a recognisable sketch of $y = \operatorname{cosec} x$ for $0 < x \leqslant \frac{1}{2}\pi$ | B1 | |
| | | | Justify statement that the trapezium rule gives an over-estimate | B1 | [2] |

## Q36.

| | | | |
|---|---|---|---|
| 5 (i) | Express left-hand side as a single fraction | M1 | |
| | Use $\sin 2\theta = 2\sin\theta\cos\theta$ at some point | B1 | |
| | Complete proof with no errors seen (AG) | A1 | [3] |





| | | | |
|---|---|---|---|
| (ii) (a) | State $\dfrac{2}{\sin\frac{1}{4}\pi}$ or equivalent | B1 | |
| | Obtain $2\sqrt{2}$ or exact equivalent (dependent on first B1) | B1 | [2] |
| (b) | State or imply $k\sin 2\theta$ for any $k$ | B1 | |
| | Integrate to obtain $-\frac{3}{2}\cos 2\theta$ | B1 | |
| | Substitute both limits correctly to obtain 3 | B1 | [3] |

**Q37.**

| | | | |
|---|---|---|---|
| 6 (a) | Integrate to obtain form $k\ln(2x-7)$ | M1 | |
| | Obtain correct $3\ln(2x-7)$ | A1 | |
| | Substitute limits correctly (dependent on first M1) | DM1 | |
| | Use law for logarithm of a quotient or power (dependent on first M1) | DM1 | |
| | Confirm $\ln 125$ following correct work and sufficient detail (AG) | A1 | [5] |
| (b) | Evaluate $y$ at (1), 5, 9, 13, 17 | M1 | |
| | Use correct formula, or equivalent, with $h=4$ and five $y$-values | M1 | |
| | Obtain 13.5 | A1 | [3] |

**Q38.**

| | | | |
|---|---|---|---|
| 3 (a) | Integrate to obtain form $k\sin(\frac{1}{3}x+2)$ where $k \neq 4$ | M1 | |
| | Obtain $12\sin(\frac{1}{3}x+2)$ $(+c)$ | A1 | [2] |
| (b) | State or imply correct $y$-values 2, $\sqrt{20}$, $\sqrt{68}$, $\sqrt{148}$ | B1 | |
| | Use correct formula, or equivalent, with $h=4$ and four $y$-values | M1 | |
| | Obtain 79.2 | A1 | [3] |

**Q39.**

| | | | | |
|---|---|---|---|---|
| 1 | | State or imply correct $y$–values 6, 4, 0, 8, 24 | B1 | |
| | | Use correct formula, or equivalent, with $h = 1$ and five $y$–values | M1 | |
| | | Obtain 27 | A1 | [3] |

## Q40.

| | | | | |
|---|---|---|---|---|
| 3 | (a) | Express integrand in the form $p\cos\theta + 2$ | M1 | |
| | | State correct $2\cos\theta + 2$ | A1 | |
| | | Integrate to obtain $2\sin\theta + 2\theta$ $(+c)$ | A1 | [3] |
| | (b) | Integrate to obtain form $k\ln(2x+3)$ | M1 | |
| | | Obtain correct $\frac{1}{2}\ln(2x+3)$ | A1 | |
| | | Apply limits correctly | DM1 | |
| | | Obtain $\frac{1}{2}\ln 15$ | A1 | [4] |

## Q41.

| | | | | |
|---|---|---|---|---|
| 2 | (i) | Integrate to obtain form $pe^{-x} + qe^{-3x}$ where $p \neq 1, q \neq 6$ | M1 | |
| | | Obtain $-e^{-x} - 2e^{-3x}$ (allow unsimplified) | A1 | |
| | | Apply both limits to $pe^{-x} + qe^{-3x}$ (allow $p = 1, q = 6$) | M1 | |
| | | Obtain $3 - e^{-a} - 2e^{-3a}$ | A1 | [4] |
| | (ii) | State 3 following a result of the form $k + pe^{-x} + qe^{-3x}$ | B1√ | [1] |

## P3 (variant1 and 3)

## Q1.

| | | | | |
|---|---|---|---|---|
| 4 | (i) | State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$ | B1 | |
| | | Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent | M1 | |
| | | Simplify and obtain the given identity correctly | A1 | [3] |
| | (ii) | Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$ | B1 | |
| | | Substitute limits correctly in an integral of the form $a\sin 2x + b\sin 4x$ | M1 | |
| | | Obtain given answer following full, correct and exact working | A1 | [3] |

## Q2.

| | | | | |
|---|---|---|---|---|
| 8 | (i) | State or imply the form $\frac{A}{x+1}+\frac{B}{x+3}$ and use a relevant method to find $A$ or $B$ | M1 | |
| | | Obtain $A = 1$, $B = -1$ | A1 | [2] |
| | (ii) | Square the result of part (i) and substitute the fractions of part (i) | M1 | |
| | | Obtain the given answer correctly | A1 | [2] |
| | (iii) | Integrate and obtain $-\frac{1}{x+1}-\ln(x+1)+\ln(x+3)-\frac{1}{x+3}$ | B3 | |
| | | Substitute limits correctly in an integral containing at least two terms of the correct form | M1 | |
| | | Obtain given answer following full and exact working | A1 | [5] |

**Q3.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Use correct $\cos(A + B)$ formula to express $\cos 3\theta$ in terms of trig functions of $2\theta$ and $\theta$ | M1 | |
| | | Use correct trig formulae and Pythagoras to express $\cos 3\theta$ in terms of $\cos\theta$ | M1 | |
| | | Obtain a correct expression in terms of $\cos\theta$ in any form | A1 | |
| | | Obtain the given identity correctly | A1 | [4] |
| | | [SR: Give M1 for using correct formulae to express RHS in terms of $\cos\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of either only $\cos 3\theta$ and $\cos\theta$, or only $\cos 2\theta$, $\sin 2\theta$, $\cos\theta$, and $\sin\theta$, and A1 for obtaining the given identity correctly.] | | |
| | (ii) | Use identity and integrate, obtaining terms $\frac{1}{4}(\frac{1}{3}\sin 3\theta)$ and $\frac{1}{4}(3\sin\theta)$, or equivalent | B1 + B1 | |
| | | Use limits correctly in an integral of the form $k\sin 3\theta + l\sin\theta$ | M1 | |
| | | Obtain answer $\frac{2}{3}-\frac{3}{8}\sqrt{3}$, or any exact equivalent | A1 | [4] |

**Q4.**

| | | | | |
|---|---|---|---|---|
| 7 | (i) | State or imply $dx = 2t\,dt$ or equivalent | B1 | |
| | | Express the integral in terms of $x$ and $dx$ | M1 | |
| | | Obtain given answer $\int_1^5 (2x-2)\ln x\,dx$, including change of limits **AG** | A1 | [3] |
| | (ii) | Attempt integration by parts obtaining $(ax^2 + bx)\ln x \pm \int (ax^2+bx)\frac{1}{x}dx$ or equivalent | M1 | |
| | | Obtain $(x^2 - 2x)\ln x - \int (x^2-2x)\frac{1}{x}dx$ or equivalent | A1 | |
| | | Obtain $(x^2 - 2x)\ln x - \frac{1}{2}x^2 + 2x$ | A1 | |
| | | Use limits correctly having integrated twice | M1 | |
| | | Obtain $15\ln 5 - 4$ or exact equivalent | A1 | [5] |
| | | [Equivalent for M1 is $(2x - 2)(ax\ln x + bx) - \int (ax\ln x + bx)\,2dx$] | | |

**Q5.**

| | | | | |
|---|---|---|---|---|
| **10** | **(i)** | Separate variables correctly and integrate of at least one side | M1 | |
| | | Carry out an attempt to find $A$ and $B$ such that $\frac{1}{N(1800-N)} \equiv \frac{A}{N} + \frac{B}{1800-N}$, or equivalent | M1 | |
| | | Obtain $\frac{2}{N} + \frac{2}{1800-N}$ or equivalent | A1 | |
| | | Integrates to produce two terms involving natural logarithms | M1 | |
| | | Obtain $2\ln N - 2\ln(1800 - N) = t$ or equivalent | A1 | |
| | | Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a \ln N$, $b \ln(1800)$ and $ct$ | M1 | |
| | | Obtain $2\ln N - 2\ln(1800 - N) = t - 2\ln 5$ or equivalent | A1 | |
| | | Use laws of logarithms to remove logarithms | M1 | |
| | | Obtain $N = \frac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent | A1 | [9] |
| | **(ii)** | State or imply that $N$ approaches 1800 | B1 | [1] |

## Q6.

| | | | |
|---|---|---|---|
| **3** | Attempt integration by parts and reach $k(1-x)e^{-\frac{1}{2}x} \pm k\int e^{-\frac{1}{2}x}\,dx$, or equivalent | M1 | |
| | Obtain $-2(1-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$, or equivalent | A1 | |
| | Integrate and obtain $-2(1-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$, or equivalent | A1 | |
| | Use limits $x = 0$ and $x = 1$, having integrated twice | M1 | |
| | Obtain the given answer correctly | A1 | [5] |

## Q7.

| | | | | |
|---|---|---|---|---|
| **9** | **(i)** | State or imply $\frac{dx}{dt} = k(10-x)(20-x)$ and show $k = 0.01$ | B1 | [1] |
| | **(ii)** | Separate variables correctly and attempt integration of at least one side | M1 | |
| | | Carry out an attempt to find $A$ and $B$ such that $\frac{1}{(10-x)(20-x)} \equiv \frac{A}{10-x} + \frac{B}{20-x}$, or equivalent | M1 | |
| | | Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent | A1 | |
| | | Integrate and obtain $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x)$, or equivalent | A1√ | |
| | | Integrate and obtain term $0.01t$, or equivalent | A1 | |
| | | Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a\ln(10-x)$, $b\ln(20-x)$ and $ct$ | M1 | |
| | | Obtain answer in any form, e.g. $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x) = 0.01t + \frac{1}{10}\ln 2$ | A1√ | |
| | | Use laws of logarithms to correctly remove logarithms | M1 | |
| | | Rearrange and obtain $x = 20(\exp(0.1t) - 1)/(2\exp(0.1t) - 1)$, or equivalent | A1 | [9] |
| | **(iii)** | State that $x$ approaches 10 | B1 | [1] |

## Q8.

| | | | |
|---|---|---|---|
| 7 | Separate variables correctly and attempt integration on at least one side | M1 | |
| | Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side | A1 | |
| | Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x}\,dx$ ) | M1 | |
| | Obtain or imply $2xe^{3x} + \int 2e^{3x}\,dx$ or equivalent | A1 | |
| | Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$ | A1 | |
| | Substitute $x = 0$, $y = 2$ in an expression containing terms $Ay^3$, $Bxe^{3x}$, $Ce^{3x}$, where $ABC \neq 0$, and find the value of $c$ | M1 | |
| | Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent | A1 | |
| | Substitute $x = 0.5$ to obtain $y = 2.44$ | A1 | [8] |

## Q9.

| | | | |
|---|---|---|---|
| 5 | Separate variables correctly | B1 | |
| | Integrate and obtain term $\ln x$ | B1 | |
| | Integrate and obtain term $\frac{1}{2}\ln(y^2 + 4)$ | B1 | |
| | Evaluate a constant or use limits $y = 0$, $x = 1$ in a solution containing $a\ln x$ and $b\ln(y^2 + 4)$ | M1 | |
| | Obtain correct solution in any form, e.g. $\frac{1}{2}\ln(y^2 + 4) = \ln x + \frac{1}{2}\ln 4$ | A1 | |
| | Rearrange as $y^2 = 4(x^2 - 1)$, or equivalent | A1 | [6] |

## Q10.

| | | | |
|---|---|---|---|
| 4 | Separate variables correctly | B1 | |
| | Obtain term $k\ln(4 - x^2)$, or terms $k_1\ln(2 - x) + k_2\ln(2 + x)$ | B1 | |
| | Obtain term $-2\ln(4 - x^2)$, or $-2\ln(2 - x) - 2\ln(2 + x)$, or equivalent | B1 | |
| | Obtain term $t$, or equivalent | B1 | |
| | Evaluate a constant or use limits $x = 1$, $t = 0$ in a solution containing terms $a\ln(4 - x^2)$ and $bt$ or terms $c\ln(2 - x)$, $d\ln(2 + x)$ and $bt$ | M1 | |
| | Obtain correct solution in any form, e.g. $-2\ln(4 - x^2) = t - 2\ln 3$ | A1 | |
| | Rearrange and obtain $x^2 = 4 - 3\exp(-\frac{1}{2}t)$, or equivalent (allow use of $2\ln 3 = 2.20$) | A1 | [7] |

## Q11.

| | | | |
|---|---|---|---|
| **9** | State or imply form $A + \frac{B}{2x+1} + \frac{C}{x+2}$ | B1 | |
| | State or obtain $A = 2$ | B1 | |
| | Use correct method for finding $B$ or $C$ | M1 | |
| | Obtain $B = 1$ | A1 | |
| | Obtain $C = -3$ | A1 | |
| | Obtain $2x + \frac{1}{2}\ln(2x+1) - 3\ln(x+2)$ [Deduct B1√ for each error or omission] | B3√ | |
| | Substitute limits in expression containing $a\ln(2x+1) + b\ln(x+2)$ | M1 | |
| | Show full and exact working to confirm that $8 + \frac{1}{2}\ln 9 - 3\ln 6 + 3\ln 2$, or an equivalent expression, simplifies to given result $8 - \ln 9$ | A1 | [10] |

[SR: If $A$ omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B0√B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{M}{2x+1} + \frac{Nx}{x+2}$ or $\frac{Px}{2x+1} + \frac{Q}{x+2}$, give B0B0M1A0A0 in (**i**); B1√B1√B1√, if recover correct form, M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1$, $D = 2$, $E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -3$.]

[SR: For a solution starting with $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$, give M1A1 for one of $C = -3$, $F = 4$, $G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = 1$.]

## Q12.

| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | Substitute for $x$, separate variables correctly and attempt integration of both sides | M1 | |
| | | Obtain term $\ln y$, or equivalent | A1 | |
| | | Obtain term $e^{-3t}$, or equivalent | A1 | |
| | | Evaluate a constant, or use $t = 0$, $y = 70$ as limits in a solution containing terms $a\ln y$ and $be^{-3t}$ | M1 | |
| | | Obtain correct solution in any form, e.g. $\ln y - \ln 70 = e^{-3t} - 1$ | A1 | |
| | | Rearrange and obtain $y = 70\exp(e^{-3t} - 1)$, or equivalent | A1 | [6] |
| | **(ii)** | Using answer to part (**i**), either express $p$ in terms of $t$ or use $e^{-3t} \to 0$ to find the limiting value of $y$ | M1 | |
| | | Obtain answer $\frac{100}{e}$ from correct exact work | A1 | [2] |

## Q13.

| | | | | |
|---|---|---|---|---|
| **8** | **(i)** | State or imply the form $A+\frac{B}{x+1}+\frac{C}{2x-3}$ | B1 | |
| | | State or obtain $A = 2$ | B1 | |
| | | Use a correct method for finding a constant | M1 | |
| | | Obtain $B = -2$ | A1 | |
| | | Obtain $C = -1$ | A1 | [5] |
| | **(ii)** | Obtain integral $2x - 2\ln(x+1) - \frac{1}{2}\ln(2x-3)$ | B3√ | |
| | | (Deduct B1√ for each error or omission. The f.t. is on $A$, $B$, $C$.) | | |
| | | Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$ | M1 | |
| | | Obtain the given answer following full and exact working | A1 | [5] |
| | | [SR: If $A$ omitted from the form of fractions, give B0B0M1A0A0 in **(i)**; B1√B1√M1A0 in **(ii)**.] | | |
| | | [SR: For a solution starting with $\frac{B}{x+1}+\frac{Dx+E}{2x-3}$, give M1A1 for one of $B = -2$, $D = 4$, $E = -7$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -1$.] | | |
| | | [SR: For a solution starting with $\frac{Fx+G}{x+1}+\frac{C}{2x-3}$ or with $\frac{Fx}{x+1}+\frac{C}{2x-3}$, give M1A1 for one of $C = -1$, $F = 2$, $G = 0$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = -2$.] | | |

**Q14.**

| | | | | |
|---|---|---|---|---|
| **8** | **(a)** | Carry out integration by parts and reach $ax^2\ln x + b\int\frac{1}{2}x^2\,dx$ | M1* | |
| | | Obtain $2x^2\ln x - \int\frac{1}{x}.2x^2\,dx$ | A1 | |
| | | Obtain $2x^2\ln x - x^2$ | A1 | |
| | | Use limits, having integrated twice | M1 (dep*) | |
| | | Confirm given result $56\ln 2 - 12$ | A1 | [5] |





| | | | |
|---|---|---|---|
| **(b)** | State or imply $\frac{du}{dx} = 4\cos 4x$ | B1 | |
| | Carry out complete substitution except limits | M1 | |
| | Obtain $\int(\frac{1}{4}-\frac{1}{4}u^2)\,du$ or equivalent | A1 | |
| | Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants $k_1, k_2$ | M1 | |
| | Use appropriate limits to obtain $\frac{11}{96}$ | A1 | [5] |

**Q15.**

| | | | | |
|---|---|---|---|---|
| **10** | **(i)** | State $\frac{dV}{dt} = 80 - kV$ | B1 | |
| | | Correctly separate variables and attempt integration of one side | M1 | |
| | | Obtain $a\ln(80 - kV) = t$ or equivalent | M1* | |
| | | Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent | A1 | |
| | | Use $t = 0$ and $V = 0$ to find constant of integration or as limits | M1 (dep*) | |
| | | Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent | A1 | |
| | | Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly | A1 | [7] |
| | **(ii)** | Use iterative formula correctly at least once | M1 | |
| | | Obtain final answer 0.14 | A1 | |
| | | Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145) | A1 | [3] |
| | **(iii)** | State a value between 530 and 540 $cm^3$ inclusive | B1 | |
| | | State or imply that volume approaches 569 $cm^3$ (allowing any value between 567 and 571 inclusive) | B1 | [2] |

# Q16.

| | | | | |
|---|---|---|---|---|
| **4** | **(i)** | State $R = 2$ | B1 | |
| | | Use trig formula to find $\alpha$ | M1 | |
| | | Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen | A1 | [3] |
| | **(ii)** | Substitute denominator of integrand and state integral $k\tan(x - \alpha)$ | M1* | |
| | | State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$ | A1√ | |
| | | Substitute limits | M1 (dep*) | |
| | | Obtain the given answer correctly | A1 | [4] |

# Q17.

| | | | | |
|---|---|---|---|---|
| **8** | **(i)** | Separate variables correctly and integrate at least one side | M1 | |
| | | Obtain term ln $t$, or equivalent | B1 | |
| | | Obtain term of the form $a\ \ln(k - x^3)$ | M1 | |
| | | Obtain term $-\frac{2}{3}\ln(k - x^3)$, or equivalent | A1 | |
| | | *EITHER*: Evaluate a constant or use limits $t = 1, x = 1$ in a solution containing $a$ ln $t$ and $b\ \ln(k - x^3)$ | M1* | |
| | | Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k - x^3) + \frac{2}{3}\ln(k - 1)$ | A1 | |
| | | Use limits $t = 4$, $x = 2$, and solve for $k$ | M1(dep*) | |
| | | Obtain $k = 9$ | A1 | |
| | | *OR*: Using limits $t = 1, x = 1$ and $t = 4, x = 2$ in a solution containing $a$ ln $t$ and $b$ ln $(k - x^3)$ obtain an equation in $k$ | M1* | |
| | | Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k - 8) + \frac{2}{3}\ln(k - 1)$ | A1 | |
| | | Solve for $k$ | M1(dep*) | |
| | | Obtain $k = 9$ | A1 | |
| | | Substitute $k = 9$ and obtain $x = (9 - 8t^{-\frac{3}{2}})^{\frac{1}{3}}$ | A1 | [9] |
| | **(ii)** | State that $x$ approaches $9^{\frac{1}{3}}$, or equivalent | B1√ | [1] |

## Q18.

| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | *EITHER*: Use double angle formulae correctly to express LHS in terms of trig functions of $2\theta$ | M1 | |
| | | Use trig formulae correctly to express LHS in terms of sin $\theta$, converting at least two terms | M1 | |
| | | Obtain expression in any correct form in terms of sin $\theta$ | A1 | |
| | | Obtain given answer correctly | A1 | |
| | | *OR*: Use double angle formulae correctly to express RHS in terms of trig functions of $2\theta$ | M1 | |
| | | Use trig formulae correctly to express RHS in terms of cos $4\theta$ and cos $2\theta$ | M1 | |
| | | Obtain expression in any correct form in terms of cos $4\theta$ and cos $2\theta$ | A1 | |
| | | Obtain given answer correctly | A1 | [4] |
| | **(ii)** | State indefinite integral $\frac{1}{4}\sin 4\theta - \frac{4}{2}\sin 2\theta + 3\theta$, or equivalent | B2 | |
| | | (award B1 if there is just one incorrect term) | | |
| | | Use limits correctly, having attempted to use the identity | M1 | |
| | | Obtain answer $\frac{1}{32}(2\pi - \sqrt{3})$, or any simplified exact equivalent | A1 | [4] |

## Q19.

| | | | | |
|---|---|---|---|---|
| **10** | **(i)** | State or imply $\frac{dA}{dt} = kV$ | M1* | |
| | | Obtain equation in $r$ and $\frac{dr}{dt}$, e.g. $8\pi r\frac{dr}{dt} = k\frac{4}{3}\pi r^3$ | A1 | |
| | | Use $\frac{dr}{dt} = 2$, $r = 5$ to evaluate $k$ | M1(dep*) | |
| | | Obtain given answer | A1 | [4] |
| | **(ii)** | Separate variables correctly and integrate both sides | M1 | |
| | | Obtain terms $-\frac{1}{r}$ and $0.08t$, or equivalent | A1 + A1 | |
| | | Evaluate a constant or use limits $t = 0$, $r = 5$ with a solution containing terms of the form $\frac{a}{r}$ and $bt$ | M1 | |
| | | Obtain solution $r = \frac{5}{(1-0.4t)}$, or equivalent | A1 | [5] |
| | **(iii)** | State the set of values $0 \leqslant t < 2.5$, or equivalent<br>[Allow $t < 2.5$ and $0 < t < 2.5$ to earn B1.] | B1 | [1] |

## Q20.

| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | State or imply $dx = 2\cos\theta\, d\theta$, or $\frac{dx}{d\theta} = 2\cos\theta$, or equivalent | B1 | |
| | | Substitute for $x$ and $dx$ throughout the integral | M1 | |
| | | Obtain the given answer correctly, having changed limits and shown sufficient working | A1 | [3] |
| | **(ii)** | Replace integrand by $2 - 2\cos 2\theta$, or equivalent | B1 | |
| | | Obtain integral $2\theta - \sin 2\theta$, or equivalent | B1√ | |
| | | Substitute limits correctly in an integral of the form $a\theta \pm b\sin 2\theta$, where $ab$ þ 0 | M1 | |
| | | Obtain answer $\frac{1}{3}\pi - \frac{\sqrt{3}}{2}$ or exact equivalent | A1 | [4] |
| | | [The f.t. is on integrands of the form $a + c\cos 2\theta$, where $ac$ þ 0.] | | |

## Q21.

| | | | | |
|---|---|---|---|---|
| 10 | (i) | State or imply $\frac{dx}{dt} = k(20 - x)$ | B1 | |
| | | Show that $k = 0.05$ | B1 | [2] |
| | (ii) | Separate variables correctly and integrate both sides | B1 | |
| | | Obtain term $-\ln(20 - x)$, or equivalent | B1 | |
| | | Obtain term $\frac{1}{20}t$, or equivalent | B1 | |
| | | Evaluate a constant or use limits $t = 0$, $x = 0$ in a solution containing terms $a \ln(20 - x)$ and $bt$ | M1* | |
| | | Obtain correct answer in any form, e.g. $\ln 20 - \ln(20 - x) = \frac{1}{20}t$ | A1 | [5] |
| | (iii) | Substitute $t = 10$ and calculate $x$ | M1(dep*) | |
| | | Obtain answer $x = 7.9$ | A1 | [2] |
| | (iv) | State that $x$ approaches 20 | B1 | [1] |

## Q22.

| | | | |
|---|---|---|---|
| 5 | State or imply form $\frac{A}{2x+1} + \frac{B}{x+2}$ | B1 | |
| | Use relevant method to find $A$ or $B$ | M1 | |
| | Obtain $\frac{4}{2x+1} - \frac{1}{x+2}$ | A1 | |
| | Integrate and obtain $2\ln(2x+1) - \ln(x+2)$ (ft on their $A$, $B$) | B1√B1√ | |
| | Apply limits to integral containing terms $a\ln(2x+1)$ and $b\ln(x+2)$ and apply a law of logarithms correctly. | M1 | |
| | Obtain given answer $\ln 50$ correctly | A1 | [7] |

## Q23.

| | | | | |
|---|---|---|---|---|
| 9 | (i) | State $\frac{dA}{dt} = k\sqrt{2A-5}$ | B1 | [1] |
| | (ii) | Separate variables correctly and attempt integration of each side | M1 | |
| | | Obtain $(2A-5)^{\frac{1}{2}} = \ldots$ or equivalent | A1 | |
| | | Obtain $= kt$ or equivalent | A1 | |
| | | Use $t = 0$ and $A = 7$ to find value of arbitrary constant | M1 | |
| | | Obtain $C = 3$ or equivalent | A1 | |
| | | Use $t = 10$ and $A = 27$ to find $k$ | M1 | |
| | | Obtain $k = 0.4$ or equivalent | A1 | |
| | | Substitute $t = 20$ and values for $C$ and $k$ to find value of $A$ | M1 | |
| | | Obtain 63 | cwo A1 | [9] |

## Q24.

| | | | |
|---|---|---|---|
| 4 | Separate variables and attempt integration of at least one side | M1 | |
| | Obtain term $\ln(x+1)$ | A1 | |
| | Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ | M1 | |
| | Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ | A1 | |
| | Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$ | M1 | |
| | Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2}\ln \sin 2\theta - \frac{1}{2}\ln\frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$) | A1√ | |
| | Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent | A1 | [7] |

## Q25.

| | | | | |
|---|---|---|---|---|
| 8 | (i) | Use any relevant method to determine a constant | M1 | |
| | | Obtain one of the values $A = 3$, $B = 4$, $C = 0$ | A1 | |
| | | Obtain a second value | A1 | |
| | | Obtain the third value | A1 | [4] |
| | (ii) | Integrate and obtain term $-3\ \ln(2-x)$ | B1√ | |
| | | Integrate and obtain term $k \ln(4+x^2)$ | M1 | |
| | | Obtain term $2\ \ln(4+x^2)$ | A1√ | |
| | | Substitute correct limits correctly in a complete integral of the form $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$ | M1 | |
| | | Obtain given answer following full and correct working | A1 | [5] |

## Q26.

| | | | | |
|---|---|---|---|---|
| 4 | (i) | Separate variables and attempt integration on both sides | M1* | |
| | | Obtain $2N^{0.5}$ on left-hand side or equivalent | A1 | |
| | | Obtain $-60e^{-0.02t}$ on right-hand side or equivalent | A1 | |
| | | Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{-0.02t}$ | DM1* | |
| | | Obtain $2N^{0.5} = -60e^{-0.02t} + 80$ or equivalent | A1 | |
| | | Conclude with $N = (40 - 30e^{-0.02t})^2$ or equivalent | A1 | [6] |
| | (ii) | State number approaches 1600 or equivalent, following expression of form $(c + de^{-0.02t})^n$ | B1√ | [1] |

## Q27.

| | | | | | |
|---|---|---|---|---|---|
| **10** | **(i)** | State or imply $\frac{du}{dx} = \sec^2 x$ | B1 | |
| | | Express integrand in terms of $u$ and $du$ | M1 | |
| | | Integrate to obtain $\frac{u^{n+1}}{n+1}$ or equivalent | A1 | |
| | | Substitute correct limits correctly to confirm given result $\frac{1}{n+1}$ | A1 | [4] |
| | **(ii)** | **(a)** Use $\sec^2 x = 1 + \tan^2 x$ twice | M1 | |
| | | Obtain integrand $\tan^4 x + \tan^2 x$ | A1 | |
| | | Apply result from part **(i)** to obtain $\frac{1}{3}$ | A1 | [3] |
| | | **Or**<br>Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)** | M1 | |
| | | Obtain $\int u^2 du$ | A1 | |
| | | Apply limits correctly and obtain $\frac{1}{3}$ | A1 | |
| | | **(b)** Arrange, perhaps implied, integrand to<br>$t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$ | B1 | |
| | | Attempt application of result from part **(i)** at least twice | M1 | |
| | | Obtain $\frac{1}{8} + \frac{4}{6} + \frac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent | A1 | [3] |

## Q28.

| | | | |
|---|---|---|---|
| **6** | Separate variables correctly and attempt integration of one side | B1 | |
| | Obtain term $\ln x$ | B1 | |
| | State or imply and use a relevant method to find $A$ or $B$ | M1 | |
| | Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$ | | |
| | Integrate and obtain $-\frac{1}{2}\ln(1-y) + \frac{1}{2}\ln(1+y)$, or equivalent<br>[If the integral is directly stated as $k_1 \ln$ or $k_2 \ln$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$] | A1 √ | |
| | Evaluate a constant, or use limits $x = 2$, $y = 0$ in a solution containing terms $a \ln x$, $b \ln(1-y)$ and $c \ln(1+y)$, where $abc \neq 0$<br>[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k \ln(1-y^2)$] | M1 | |
| | Obtain solution in any correct form, e.g. $\frac{1}{2}\ln \ = \ln x - \ln 2$ | A1 | |
| | Rearrange and obtain $y =$ , or equivalent, free of logarithms | A1 | [8] |

## Q29.

| | | | |
|---|---|---|---|
| **4** | Separate variables correctly and integrate one side | M1 | |
| | Obtain $\ln y = ...$ or equivalent | A1 | |
| | Obtain $= 3\ln(x^2+4)$ or equivalent | A1 | |
| | Evaluate a constant or use $x = 0$, $y = 32$ as limits in a solution containing terms $a \ln y$ and $b \ln\left(x^2+4\right)$ | M1 | |
| | Obtain $\ln y = 3\ln(x^2+4) + \ln 32 - 3\ln 4$ or equivalent | A1 | |
| | Obtain $y = \frac{1}{2}\left(x^2+4\right)$ or equivalent | A1 | [6] |

**Q30.**

| | | | | |
|---|---|---|---|---|
| **7** | **(i)** | State or imply $du = 2\cos 2x\ dx$ or equivalent | B1 | |
| | | Express integrand in terms of $u$ and $du$ | M1 | |
| | | Obtain $\int \frac{1}{2}u^3\left(1-u^2\right) du$ or equivalent | A1 | |
| | | Integration to obtain an integral of the form $k_1u^4 + k_2u^6, k_1, k_2 \neq 0$ | M1 | |
| | | Use limits 0 and 1 or (if reverting to $x$) 0 and $\frac{1}{4}\pi$ correctly | DM1 | |
| | | Obtain $\frac{1}{24}$, or equivalent | A1 | [6] |
| | **(ii)** | Use 40 and upper limit from part **(i)** in appropriate calculation | M1 | |
| | | Obtain $k = 10$ with no errors seen | A1 | [2] |

**Q31.**

| | | | | |
|---|---|---|---|---|
| 3 | *EITHER:* | Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$ | M1* | |
| | | Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent | A1 | |
| | | Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent | A1 | |
| | | Substitute limits $x = 1$ and $x = 4$, having integrated twice | M1(dep*) | |
| | | Obtain answer $4(\ln 4 - 1)$, or exact equivalent | A1 | |
| | *OR1:* | Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u} - m\int e^{\frac{1}{2}u}\,du$ | M1* | |
| | | Obtain $2ue^{\frac{1}{2}u} - 2\int e^{\frac{1}{2}u}\,du$, or equivalent | A1 | |
| | | Integrate again and obtain $2ue^{\frac{1}{2}u} - 4e^{\frac{1}{2}u}$, or equivalent | A1 | |
| | | Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice | M1(dep*) | |
| | | Obtain answer $4\ln 4 - 4$, or exact equivalent | A1 | |
| | *OR2:* | Using $u = \sqrt{x}$, or equivalent, integrate and obtain $ku\ln u - m\int u.\frac{1}{u}\,du$ | M1* | |
| | | Obtain $4u\ln u - 4\int 1\,du$, or equivalent | A1 | |
| | | Integrate again and obtain $4u\ln u - 4u$, or equivalent | A1 | |
| | | Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int \ln u\,du$ as $u\ln u \pm u$ | M1(dep*) | |
| | | Obtain answer $8\ln 2 - 4$, or exact equivalent | A1 | |
| | *OR3:* | Integrate by parts and reach $I = \frac{x\ln x \pm x}{\sqrt{x}} + k\int \frac{x\ln x \pm x}{x\sqrt{x}}\,dx$ | M1* | |
| | | Obtain $I = \frac{x\ln x - x}{\sqrt{x}} + \frac{1}{2}I - \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx$ | A1 | |
| | | Integrate and obtain $I = 2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent | A1 | |
| | | Substitute limits $x = 1$ and $x = 4$, having integrated twice | M1(dep*) | |
| | | Obtain answer $4\ln 4 - 4$, or exact equivalent | A1 | [5] |

## Q32.

| | | | | |
|---|---|---|---|---|
| 5 | (i) | Use Pythagoras | M1 | |
| | | Use the $\sin 2A$ formula | M1 | |
| | | Obtain the given result | A1 | [3] |
| | (ii) | Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$ | M1* | |
| | | Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$ | A1 | |
| | | Substitute limits correctly | M1(dep)* | |
| | | Obtain the given answer correctly having shown appropriate working | A1 | [4] |

## Q33.

| | | | | |
|---|---|---|---|---|
| 10 | (i) | State or imply $V = \pi h^3$ | B1 | |
| | | State or imply $\frac{dV}{dt} = -k\sqrt{h}$ | B1 | |
| | | Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$, or equivalent | M1 | |
| | | Obtain the given equation | A1 | [4] |
| | | [The M1 is only available if $\frac{dV}{dh}$ is in terms of $h$ and has been obtained by a correct method.] | | |
| | | [Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.] | | |

| | | | |
|---|---|---|---|
| (ii) | Separate variables and integrate at least one side | M1 | |
| | Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent | A1 | |
| | Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ | M1 | |
| | Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ | M1 | |
| | Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$ | A1 | |
| (ii) | Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent | A1 | [6] |
| (iii) | Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$ | B1 | [1] |

# Q34.

| | | | |
|---|---|---|---|
| 2 | Carry out complete substitution including the use of $\frac{du}{dx} = 3$ | M1 | |
| | Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$ | A1 | |
| | Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$ | M1 | |
| | Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and $+ c$ | A1 | [4] |

# Q35.

| | | |
|---|---|---|
| **10** | Use $2\cos^2 x = 1 + \cos 2x$ or equivalent | B1 |
| | Separate variables and integrate at least one side | M1 |
| | Obtain $\ln\left(y^3+1\right) = \ldots$ or equivalent | A1 |
| | Obtain $\ldots = 2x + \sin 2x$ or equivalent | A1 |
| | Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3+1\right)$, $bx$ or $c \sin 2x$ | M1* |
| | Obtain $\ln\left(y^3+1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant | A1 |
| | Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as $x$-coordinate at stationary point | B1 |
| | Use correct process to find $y$-coordinate for at least one $x$-coordinate | M1(d*M) |
| | Obtain 5.9 | A1 |
| | Obtain 48.1 | A1 [10] |

## Q36.

| | | | |
|---|---|---|---|
| **2** | State $\frac{du}{dx} = 3\sec^2 x$ or equivalent | B1 | |
| | Express integral in terms of $u$ and $du$ (accept unsimplified and without limits) | M1 | |
| | Obtain $\int \frac{1}{3} u^{\frac{1}{2}}\,du$ | A1 | |
| | Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$ | M1 | |
| | Obtain $\frac{14}{9}$ | A1 | [5] |

## Q37.

| | | | |
|---|---|---|---|
| **4** | Separate variables correctly and recognisable attempt at integration of at least one side | M1 | |
| | Obtain $\ln y$, or equivalent | B1 | |
| | Obtain $k\ln\left(2+e^{3x}\right)$ | B1 | |
| | Use $y(0) = 36$ to find constant in $y = A\left(2+e^{3x}\right)^k$ or $\ln y = k\ln\left(2+e^{3x}\right) + c$ or equivalent | M1* | |
| | Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2+e^{3x}\right)^k\right)$ | *M1 | |
| | Obtain $y = 4\left(2+e^{3x}\right)^2$ | A1 | [6] |

## Q38.

| | | | |
|---|---|---|---|
| **5** | Separate variables correctly and attempt integration of at least one side | B1 | |
| | Obtain term in the form $a\sqrt{(2x+1)}$ | M1 | |
| | Express $1/(\cos^2\theta)$ as $\sec^2\theta$ | B1 | |
| | Obtain term of the form $k\tan\theta$ | M1 | |
| | Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k \tan\theta$, $ak \neq 0$ | M1 | |
| | Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2}\tan\theta + \frac{1}{2}$ | A1 | |
| | Rearrange and obtain $x = \frac{1}{8}(\tan\theta + 1)^2 - \frac{1}{2}$, or equivalent | A1 | **7** |

## Q39.

| | | | | |
|---|---|---|---|---|
| **8** | **(i)** | Use a correct method for finding a constant | M1 | |
| | | Obtain one of $A = 3$, $B = 3$, $C = 0$ | A1 | |
| | | Obtain a second value | A1 | |
| | | Obtain a third value | A1 | **4** |
| | **(ii)** | Integrate and obtain term $-3\ln(2-x)$ | B1√ | |
| | | Integrate and obtain term of the form $k\ln(2+x^2)$ | M1 | |
| | | Obtain term $\frac{3}{2}\ln(2+x^2)$ | A1√ | |
| | | Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$ | M1 | |
| | | Obtain given answer after full and correct working | A1 | **5** |

## Q40.

| | | | | |
|---|---|---|---|---|
| **2** | **(i)** | State or imply ordinates 2, 1.1547…, 1, 1.1547… | B1 | |
| | | Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates | M1 | |
| | | Obtain answer 1.95 | A1 | **[3]** |
| | **(ii)** | Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval | B1 | |
| | | Justify a statement that the estimate will be an overestimate | B1 | **[2]** |

## Q41.

| | | | | |
|---|---|---|---|---|
| 7 | (i) | Separate variables correctly and attempt to integrate at least one side | B1 | |
| | | Obtain term $\ln R$ | B1 | |
| | | Obtain $\ln x - 0.57x$ | B1 | |
| | | Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$ | M1 | |
| | | Obtain correct solution in any form | A1 | |
| | | Obtain a correct expression for $R$, e.g. $R = xe^{(3.80-0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285-0.57x)}$ | A1 | [6] |
| | (ii) | Equate $\frac{dR}{dx}$ to zero and solve for $x$ | M1 | |
| | | State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75 | A1 | |
| | | Obtain $R = 28.8$ (allow 28.9) | A1 | [3] |

## Q42.

| | | | | |
|---|---|---|---|---|
| 6 | (i) | State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000… | B1 | |
| | | Use correct formula or equivalent with $h = 0.1$ and four $y$ values | M1 | |
| | | Obtain 0.255 with no errors seen | A1 | [3] |
| | (ii) | Obtain or imply $a = -6$ | B1 | |
| | | Obtain $x^4$ term including correct attempt at coefficient | M1 | |
| | | Obtain or imply $b = 27$ | A1 | |
| | | Either Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of $a$ and $b$ | B1√ | |
| | | Obtain 0.259 | B1 | |
| | | Or Use correct trapezium rule with at least 3 ordinates | M1 | |
| | | Obtain 0.259 (from 4) | A1 | [5] |

## Q43.

| | | | | |
|---|---|---|---|---|
| 8 | (i) | Sensibly separate variables and attempt integration of at least one side | M1 | |
| | | Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent | A1 | |
| | | Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\,dx$ | M1 | |
| | | Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\,dx$ or equivalent | A1 | |
| | | Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent | A1 | |
| | | Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent | A1 | [6] |
| | (ii) | Use $x = 0$ and $y = 100$ to find constant | M*1 | |
| | | Substitute 25 and calculate value of $y$ | DM*1 | |
| | | Obtain 203 | A1 | [3] |

## Q44.

| | | | |
|---|---|---|---|
| **10** | State or imply $\frac{du}{dx} = e^x$ | B1 | |
| | Substitute throughout for $x$ and d$x$ | M1 | |
| | Obtain $\int \frac{u}{u^2 + 3u + 2} du$ or equivalent (ignoring limits so far) | A1 | |
| | State or imply partial fractions of form $\frac{A}{u+2} + \frac{B}{u+1}$, following their integrand | B1 | |
| | Carry out a correct process to find at least one constant for their integrand | M1 | |
| | Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$ | A1 | |
| | Integrate to obtain $a\ln(u+2) + b\ln(u+1)$ | M1 | |
| | Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their $A$ and $B$ | A1√ | |
| | Apply appropriate limits and use at least one logarithm property correctly | M1 | |
| | Obtain given answer $\ln\frac{8}{5}$ legitimately | A1 | [10] |
| | **SR** for integrand $\frac{u^2}{u(u+1)(u+2)}$ | | |
| | State or imply partial fractions of form $\frac{A}{u} + \frac{B}{u+1} + \frac{C}{u+2}$ | (B1) | |
| | Carry out a correct process to find at least one constant | (M1) | |
| | Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$ | (A1) | |
| | …complete as above. | | |