

Chapter 1 Algebra

May/June 2002

- 2 Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]
- 3 The polynomial $x^4 + 4x^2 + x + a$ is denoted by $p(x)$. It is given that $(x^2 + x + 2)$ is a factor of $p(x)$.
Find the value of a and the other quadratic factor of $p(x)$. [4]
- 6 Let $f(x) = \frac{4x}{(3x+1)(x+1)^2}$.
- (i) Express $f(x)$ in partial fractions. [5]
- (ii) Hence show that $\int_0^1 f(x) dx = 1 - \ln 2$. [5]

Oct/Nov 2002

- 1 Solve the inequality $|9 - 2x| < 1$. [3]
- 6 Let $f(x) = \frac{6 + 7x}{(2 - x)(1 + x^2)}$.
- (i) Express $f(x)$ in partial fractions. [4]
- (ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,
$$f(x) = 3 + 5x - \frac{1}{2}x^2 - \frac{15}{4}x^3$$
 [5]

May/June 2003

- 3 Solve the inequality $|x - 2| < 3 - 2x$. [4]
- 4 The polynomial $x^4 - 2x^3 - 2x^2 + a$ is denoted by $f(x)$. It is given that $f(x)$ is divisible by $x^2 - 4x + 4$.
- (i) Find the value of a . [3]
- (ii) When a has this value, show that $f(x)$ is never negative. [4]
- 6 Let $f(x) = \frac{9x^2 + 4}{(2x+1)(x-2)^2}$.
- (i) Express $f(x)$ in partial fractions. [5]
- (ii) Show that, when x is sufficiently small for x^3 and higher powers to be neglected,
$$f(x) = 1 - x + 5x^2$$
 [4]

Oct/Nov 2003

1 Solve the inequality $|2^x - 8| < 5$. [4]

2 Expand $(2 + x^2)^{-2}$ in ascending powers of x , up to and including the term in x^4 , simplifying the coefficients. [4]

8 Let $f(x) = \frac{x^3 - x - 2}{(x - 1)(x^2 + 1)}$.

(i) Express $f(x)$ in the form

$$A + \frac{B}{x - 1} + \frac{Cx + D}{x^2 + 1},$$

where A, B, C and D are constants. [5]

(ii) Hence show that $\int_2^3 f(x) dx = 1$. [4]

May/June 2004

2 Solve the inequality $|2x + 1| < |x|$. [4]

9 Let $f(x) = \frac{x^2 + 7x - 6}{(x - 1)(x - 2)(x + 1)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = -3 + 2x - \frac{3}{2}x^2 + \frac{11}{4}x^3. [5]$$

Oct/Nov 2004

1 Expand $\frac{1}{(2 + x)^3}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

3 The polynomial $2x^3 + ax^2 - 4$ is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$.

(i) Find the value of a . [2]

When a has this value,

(ii) factorise $p(x)$, [2]

(iii) solve the inequality $p(x) > 0$, justifying your answer. [2]

- 8 An appropriate form for expressing $\frac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

- (a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\frac{4x}{(x+4)(x^2+3)},$ [1]

(ii) $\frac{2x+1}{(x-2)(x+2)^2}.$ [2]

(b) Show that $\int_3^4 \frac{3x}{(x+1)(x-2)} dx = \ln 5.$ [6]

May/June 2005

- 1 Expand $(1+4x)^{-\frac{1}{2}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

- 5 The polynomial $x^4 + 5x + a$ is denoted by $p(x)$. It is given that $x^2 - x + 3$ is a factor of $p(x)$.

(i) Find the value of a and factorise $p(x)$ completely. [6]

(ii) Hence state the number of real roots of the equation $p(x) = 0$, justifying your answer. [2]

Oct/Nov 2005

- 1 Given that a is a positive constant, solve the inequality

$$|x - 3a| > |x - a|. \quad [4]$$

9 (i) Express $\frac{3x^2 + x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{3x^2 + x}{(x+2)(x^2+1)}$ in ascending powers of x , up to and including the term in x^3 . [5]

May/June 2006

2 Solve the inequality $2x > |x - 1|.$ [4]

9 (i) Express $\frac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\frac{10}{(2-x)(1+x^2)}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [5]

Oct/Nov 2006

1 Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer. [4]

5 (i) Simplify $(\sqrt{1+x} + \sqrt{1-x})(\sqrt{1+x} - \sqrt{1-x})$, showing your working, and deduce that

$$\frac{1}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}. \quad [2]$$

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{1+x} + \sqrt{1-x}}$$

in ascending powers of x , up to and including the term in x^2 . [4]

8 Let $f(x) = \frac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^2 f(x) dx = 2 + \ln \frac{5}{3}$. [5]

May/June 2007

1 Expand $(2+3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

2 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(x+2)$ is a factor of $p(x)$.

(i) Find the value of a . [2]

(ii) When a has this value, find the quadratic factor of $p(x)$. [2]

Oct/Nov 2007

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $x^2 + x + 2$ is a factor of $p(x)$. Find the value of a and the other quadratic factor of $p(x)$. [4]

9 (i) Express $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x , up to and including the term in x^2 . [5]

May/June 2008

1 Solve the inequality $|x-2| > 3|2x+1|$. [4]

7 Let $f(x) \equiv \frac{x^2+3x+3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^3 f(x) dx = 3 - \frac{1}{2} \ln 2$. [4]

Oct/Nov 2008

2 Expand $(1+x)\sqrt{1-2x}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

5 The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a . [3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer. [3]

May/June 2009

5 When $(1+2x)(1+ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x , the coefficient of the term in x is zero.

(i) Find the value of a . [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

8 (i) Express $\frac{100}{x^2(10-x)}$ in partial fractions. [4]

(ii) Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{100}x^2(10-x),$$

obtaining an expression for t in terms of x . [6]

Oct/Nov 2009/31

1 Solve the inequality $2-3x < |x-3|$. [4]

8 (i) Express $\frac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x+3}{(x+1)^2(3x+2)}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [5]

Oct/Nov 2009/32

5 The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $p(x)$. The result of differentiating $p(x)$ with respect to x is denoted by $p'(x)$. It is given that $(x+2)$ is a factor of $p(x)$ and of $p'(x)$.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

8 (i) Express $\frac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1+x}{(1-x)(2+x^2)}$ in ascending powers of x , up to and including the term in x^2 . [5]

May/June 2010/31

1 Solve the inequality $|x+3a| > 2|x-2a|$, where a is a positive constant. [4]

8 (i) Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part (i), show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}. \quad [2]$$

(iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2} dx = \frac{7}{12} - \ln \frac{3}{2}$. [5]

May/June 2010/32

5 The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x+2)$ the remainder is 9.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

- 10 (i) Find the values of the constants A , B , C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}. \quad [5]$$

- (ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x - 1)} dx = \frac{3}{2} + \frac{1}{2} \ln\left(\frac{16}{27}\right). \quad [5]$$

May/June 2010/33

- 1 Solve the inequality $|x - 3| > 2|x + 1|$. [4]

- 9 (i) Express $\frac{4 + 5x - x^2}{(1 - 2x)(2 + x)^2}$ in partial fractions. [5]

- (ii) Hence obtain the expansion of $\frac{4 + 5x - x^2}{(1 - 2x)(2 + x)^2}$ in ascending powers of x , up to and including the term in x^2 . [5]