UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Level

Additional Materials: Answer Booklet/Paper Electronic Calculator

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Start each question on a new answer sheet.
Calculators should be used where appropriate.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 Study the information below and answer the questions. Show your working.
The results in a sailing regatta are decided by adding up the positions each crew achieves in each race. It is not possible to finish at exactly the same time as another crew in a race, so each race position is always awarded to only one crew.

Your rank in the regatta is determined by the number of crews ahead of you. The crew with the lowest total score in the regatta is ranked 1st overall, the second-lowest is ranked 2nd etc. If two or more crews tie for a particular rank, then subsequent rankings are adjusted accordingly. For example, if two crews are both ranked 6th in the regatta then the next rank awarded is 8 th (since there are seven crews ahead). The rankings are recalculated after each race. There are 10 crews competing in the regatta, and they all finish every race.
(a) Show that it is possible to finish 3rd in each of the first three races, and yet be ranked 4th so far, by listing possible positions for the crews ahead of you.
(b) What is the best ranking you could have after two races, if you finished 6th in both of them?
(c) If you finish 4th in every race in a five-race regatta, what is the worst final ranking you can have? Suggest positions for the crews who beat you.
(d) What is the lowest position you could finish in the first three races, if you finished in the same position each time (e.g. 2nd, 2nd, 2nd), and still be able to be ranked 1st before the fourth race? Justify your answer.

2 Study the information below and answer the questions. Show your working.
The crime statistics in the village used to be simply related to whether the one burglar (known as Carradine) was in prison or not. When he was in prison the crime rate was zero; when he was out it was about one burglary per week.

The published crime figures are given as a four-month running total. For example, the figure for May shows the total number of crimes reported in February, March, April, and May.

|  | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2010 | 10 | 9 | 8 | 8 | 4 | 1 | 0 | 5 | 10 | 14 | 17 | 15 |
| 2011 | 14 | 10 | 7 | 4 | 0 | 0 | 5 | 7 | 13 | 17 | 16 | 14 |
| 2012 | 8 | 4 | 0 | 0 | 0 | 6 | 10 | 14 | 18 | 17 | 17 | 18 |

(a) (i) Which entry in the table enables us to deduce that there were no crimes reported in April 2010?
(ii) How many crimes were reported in 2011?
(iii) How many crimes were reported in September 2010?
(iv) How many crimes were reported in November 2009?

Prison terms are never less than eight weeks long.
(b) Assuming that all crimes have been reported immediately, approximately what length of time did Carradine spend in prison last time?

A scheme for online mapping of crime was introduced on 1 January 2012, with the intention of using it to hold local police accountable. The unintended consequence was that $25 \%$ of people said that they would now not report a crime as it would lower the profile of their area, and so decrease the value of their house when they wanted to sell it.

Carradine's behaviour has not changed, and yet the reported crime figures have not decreased.
(c) Give two plausible but distinct reasons to explain this inconsistency.

Carradine claims that he commits one burglary on the same night every week when he is not in prison, and those are the only crimes that he commits. The detective knows that there may in reality be a delay of up to two weeks before a crime is reported, and is able to work out that Carradine cannot be telling the truth.
(d) How can the detective use the data in the table to support his claim that Carradine cannot be telling the truth?

3 Study the information below and answer the questions. Show your working.
A cartwheel in a film often appears to be turning backwards or standing still even though the cart is moving forwards.

Consider a simple cartwheel with 4 spokes, turning one full revolution (clockwise) per second. This will appear not to be moving if the camera takes 1 frame every $1 / 4$ second (or 4 frames per second), or 1 frame every $1 / 2$ second ( 2 frames per second), or 1 frame every second, since each spoke will land precisely where one of the other spokes was in the previous frame.

In the example below, the wheel will appear to turn backwards because, after 1 frame, a spoke is just behind (i.e. anticlockwise from) where a spoke was previously.


Frame 1


Frame 2


Frame 3


Frame 4

In this series of 4 frames, the cartwheel has turned just less than $90^{\circ}$ clockwise each time, and yet would appear to be turning anticlockwise. A similar effect is created if the wheel turns just less than $180^{\circ}$, or just less than any multiple of $90^{\circ}$.

For the purposes of the whole of this question, you should assume that:

- a 4-spoke cartwheel will appear to be turning forwards whenever the next frame shows the spokes up to $45^{\circ}$ beyond (i.e. clockwise from) where the spokes were last;
- a 4-spoke cartwheel will appear to be turning backwards whenever the next frame shows the spokes up to $45^{\circ}$ behind (i.e. anticlockwise from) where the spokes were last;
- at $45^{\circ}$ the cartwheel's direction of motion will be ambiguous;
- cameras can only take whole numbers of frames per second.

For parts (a) to (d), the cartwheel is turning at 1 revolution per second.
(a) Will the cartwheel appear to be going backwards or forwards if the camera takes 36 frames per second? Justify your answer.
(b) At what number of frames per second will a 4-spoke cartwheel's direction of motion be ambiguous?
(c) What is the minimum number of frames per second to make the cartwheel's motion appear backwards? Justify your answer.

The method of determining when a cartwheel's motion appears to be forwards or backwards described above involves halving the angle between the spokes. This halving method can be applied however many spokes a cartwheel has.
(d) A particular cartwheel has 12 spokes. My camera takes 25 frames per second.

What is the slowest that this cartwheel can be turning (measured in revolutions per second) for its direction of motion to be ambiguous when filmed by my camera?
(e) Two differently-sized cartwheels, both with 12 spokes, appeared to be turning in different directions when filmed by my camera ( 25 frames per second).

Suggest a possible speed of rotation (measured in revolutions per second) for each of the two wheels. Justify your answer.
(f) A cartwheel was filmed with an old camera which took 14 frames per second. The cartwheel appeared to go backwards when revolving at 1 revolution per second.

How many spokes could the cartwheel have? List all the possibilities. (You may assume that no cartwheel has more than 25 spokes.)

4 Study the information below and answer the questions. Show your working.
Four underground caverns close to the village of Letiby are popular with tourists. They are known as John, Paul, George and Ringo, and guided tours take place daily, as given below.

| Cavern | Tours begin at | Tour takes |
| :---: | :---: | :---: |
| John | $09: 30$ and every hour to 17:30 | 90 mins |
| Paul | $09: 40$ and every hour to $17: 40$ | 75 mins |
| George | $09: 50$ and every hour to $17: 50$ | 80 mins |
| Ringo | $10: 15$ | $11: 45$ |

Tickets are available at the Letiby Tourist Information Centre, which is where all the tours begin and end.

| Tour |  |
| :--- | :--- |
| Prices |  |
| John | $\$ 12.50$ |
| Paul | $\$ 10.80$ |
| George | $\$ 11.60$ |
| Ringo | $\$ 14.50$ |

Every ticket for a tour that begins before 12:00 is also a voucher for $20 \%$ off one tour that begins after 14:00 on the same day. Only one voucher may be used per person per tour.

There are 32 tours altogether every day, shared between 8 guides. Each morning the guides are assigned, at random, a number between 1 and 8 inclusive. Number 1 leads the 1st, 9th, 17th and 25th tours to depart from the Information Centre, number 2 leads the 2nd, 10th, 18th and 26th tours, and so on.

| Yesterday's Rota |  |
| :---: | :---: |
| 1 | Martha |
| 2 | Bill |
| 3 | Jude |
| 4 | Michelle |
| 5 | Jojo |
| 6 | Sadie |
| 7 | Maxwell |
| 8 | Lucy |

(a) How many tours each day begin between 12:00 and 14:00?
(b) At what time yesterday did Jojo
(i) begin his second tour?
(ii) get back to the Information Centre at the end of his last tour?

Yesterday, Penny arrived at the Information Centre at 09:41 and just missed the 09:40 Paul tour. She joined the next available tour, which was the 09:50 George tour, led by Jude. Subsequently, every time she returned to the Information Centre she bought a ticket for the next tour to depart for a cavern that she had not already visited, until she had been into all four.
(c) (i) At what times did Penny's second, third and fourth tours begin, and who led each of them?
(ii) How much did the four tours cost Penny altogether?
(d) Had she arrived in time to join the 09:40 Paul tour, how much
(i) earlier or later would she have completed her fourth tour?
(ii) more or less would she have paid overall?

Eleanor plans to visit all four caverns tomorrow. She intends to arrive in Letiby before 09:15.
(e) Suggest a timetable for her that will allow her to go on the four tours for the lowest possible total cost.

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