Cambridge Assessment International Education

# Cambridge International AS & A Level

**THINKING SKILLS** **9694/31**

Paper 3 Problem Analysis and Solution **May/June 2024**

**2 hours**

*8683003745*

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

**INSTRUCTIONS**

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
  Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
  In some questions, if you do not show your working, full marks will not be awarded.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

**1** Frank runs a funfair which has 7 small rides and 6 big rides. There are two options for paying for entry to the fair and for rides: Wristband and Tokens.

- Wristband: $12 gives entry to the fair and an unlimited number of rides.
- Tokens: Entry to the fair is free, tokens cost $1.20 each or 10 tokens cost $10. Each go on a small ride requires 1 token and each go on a big ride requires 2 tokens.

A wristband can only be used by the person who buys it; tokens can be used by anyone.

Jim went to the fair and chose to use the Tokens option. By the time he left the fair he had had 4 goes on small rides and 6 goes on big rides.

**(a)** What is the smallest amount of money that Jim could have saved if he had used the Wristband option instead? [2]

Kenny and two of his friends are going to the fair. They will all go on the same rides together, and they might go on some of the rides more than once. They will have as many goes on rides as possible such that it will be cheaper for them to use the Tokens option.

**(b) (i)** What is the greatest possible number of goes on rides that each of the friends will be able to have? [2]

**(ii)** What is the least possible number of goes on rides that each of the friends will be able to have? [2]

Frank's funfair is open from 14:00 to 22:00 each day. Each small ride lasts 5 minutes and each big ride lasts 10 minutes, including time for getting on and off the ride. The queuing time for small rides is 10 minutes and the queuing time for big rides is 20 minutes. It takes 2 minutes to walk between rides.

Pete is going to the fair tomorrow. He will arrive at the first ride at 16:00 and must get off the final ride no later than 21:30. He will go on an equal number of small and big rides and will fit in as many rides as possible.

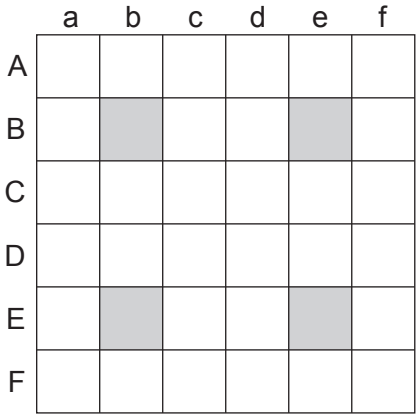
**(c)** What is the earliest time at which Pete might get off his final ride? [3]

Pete works out that if he arrives at the first ride 11 minutes earlier than planned, he will be able to fit in another two rides.

**(d)** Is Pete correct? Explain your answer. [1]

**[Turn over for Question 2]**

**2** *Trayles* is a game for two players. The equipment for playing the game consists of 28 discs, which are placed into a bag at the start of the game, and the 6 × 6 grid of squares shown below.




The rows and columns have been labelled to identify positions on the grid. For instance, the four shaded squares are identified as Bb, Be, Eb and Ee.

Each of the 28 discs has a number from 1 to 7 on one face and each of the seven numbers appears on four discs.

The game proceeds as follows:

- Both players take one disc from the bag and show their number to each other.
- The first player must place their disc on one of the shaded squares and then take another disc from the bag and show its number to the other player.
- Play alternates and at each turn a player must place their disc on an unoccupied square that is adjacent to (immediately above, below, to the left or to the right of) at least one other disc already on the grid, or on a shaded square that is unoccupied.
- A player's turn is not completed until another disc has been taken from the bag and its number shown to the other player, except after each player's final turn when there are no more discs in the bag.

**(a)** How many squares are available to the second player when deciding where to place their first disc? [1]

The players score points each time they complete a line of three **different** numbers in consecutive squares, either horizontally or vertically, but not diagonally. This line is called a *trayle*.

- When a trayle is completed that does not include a disc on a shaded square, the player who completes it scores the sum of the three numbers.
- When a trayle is completed that does include a disc on a shaded square, the player who completes it scores double the number on the shaded square plus the sum of the other two numbers.

**(b)** What is the most that can be scored for a single trayle? [1]

When the score for a trayle has been recorded, the three discs are turned over so the numbers are no longer visible. The discs remain on the grid, but they cannot be part of another trayle.

Sometimes the placing of a disc could form more than one trayle. When this occurs, the player must choose which trayle to score points from and turn over the relevant three discs.

**(c) (i)** What is the maximum number of trayles that points can be scored from in a game of *Trayles*? [1]

**(ii)** What is the maximum number of points that could be scored in total in a full game of *Trayles*? [3]

Wilfred and Lara are playing a game of *Trayles*. This is the current appearance of the grid.

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| A | | ◯ | | | | |
| B | ⑥ | ◯ | | ◯ | ◯ | ◯ |
| C | | ◯ | | ③ | ① | |
| D | ⑤ | ④ | | ④ | ⑦ | |
| E | ⑥ | ④ | | ④ | | |
| F | | ◯ | ◯ | ◯ | | |

It is Wilfred's turn and his disc is a 3. He is currently 2 points behind Lara. Lara's disc is a 6.

**(d) (i)** What is the highest number of points that Wilfred could score on this turn? State the square on which he would place his disc. [1]

**(ii)** Explain how Lara could regain the lead if Wilfred did place his disc on this square. [1]

Wilfred sees that, by placing his disc on a different square, he could score fewer points but ensure that Lara cannot regain the lead on her turn.

**(e) (i)** What is the second-highest number of points that Wilfred could score on this turn? State all the squares on which he could place his disc to obtain this score. [2]

**(ii)** Identify the three squares that Wilfred's trayle will occupy, and state Lara's only possible score when she places her 6, as a result of Wilfred's choice. [2]

Earlier, the first trayle of this game had been completed when Lara placed a 2 on square Ab, earning her 11 points.

**(f)** Deduce the number on the overturned disc on square Bb and the number on the overturned disc on square Cb. [3]

**3** A singing competition takes place over six rounds. In each round the performances of all of the singers are ranked and points are awarded according to the following rules:

- The number of points scored by the highest-ranked singer is twice the total number of singers in that round.
- The number of points scored for each other position is two less than the position above.
- If two singers are ranked equally, the points for those two positions are shared between them. Similarly, if three singers are ranked equally, the points for those three positions are shared between them, and so on.

There are eight singers in round 1; so the highest-ranked singer is awarded 16 points and the next-ranked singer is awarded 14. If two singers were tied for the first and second positions, they would each be awarded 15 points.

Any number of singers in the round could also receive a bonus. The bonus is always 2 points.

At the end of each round, the singer with the lowest total score is eliminated and does not take part in any of the later rounds. If two or more singers have the same lowest total score then one of them is chosen to be eliminated at random.

The scores for each of the first five rounds are shown in the table below.

| *Name* | *Score for each round* | | | | | | *Total Score* |
|---|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *6* | |
| Akmal | 11 | 16 | 4 | 7 | 6 | | 44 |
| Beau | 4 | 11 | 10 | 5 | 4 | | 34 |
| Charlie | 12 | 2 | 6 | | | | 20 |
| Daryl | 14 | 8 | 6 | 2 | | | 30 |
| Eliza | 16 | 13 | 8 | 10 | 6 | | 53 |
| Feroza | 9 | 10 | 14 | 10 | 8 | | 51 |
| George | 8 | 4 | | | | | 12 |
| Hank | 4 | | | | | | 4 |

**(a)** How many of the singers were awarded 2 bonus points in round 1? [2]

At the end of round 4, Daryl was chosen at random to be eliminated from the two singers who shared the same total score.

**(b)** Who was the other singer with the same lowest total score? [1]

**(c)** Which singers were awarded 2 bonus points in round 2? Explain your answer. [3]

**(d)** There are four different ways in which the points awarded in round 3 could have been achieved.

Identify the set of singers receiving bonus points in each of these four cases. [4]

**4** Jez runs his own company, carrying out repairs and routine services on laptops and tablets. His business is very popular, and he always has plenty of work of each type waiting to be done.

Jobs always take a whole number of hours. A laptop repair takes at least 1 hour and at most 6 hours to complete. A tablet repair takes at least 1 hour and at most 3 hours to complete. Routine services always take 1 hour to complete.

For repairing laptops, Jez charges a basic fee of $100 for the first hour, then $50 for each subsequent hour. For repairing tablets, he charges a basic fee of $90 for the first hour, then $50 for each subsequent hour. For a routine service of a laptop or a tablet, he charges a fixed fee of $60.

Before he begins a job, Jez is not able to predict how long a repair will take, so he always assumes that it could take the maximum time. He selects his next job at random from those he is certain that he will be able to complete on the same day.

Jez works an 8-hour day.

**(a)** What is the least amount of money that Jez might take in one day? [2]

**(b)** What is the greatest amount of money that Jez could take in one day? [2]

Jez pays himself a wage of $40 per hour and the other costs of running the business are $100 per working day.

Jez decides that instead of working five 8-hour days in a week, he will work four 10-hour days in a week. He says that the least profit that he might make in one week will be increased by this change in his working pattern.

**(c)** Is Jez correct? [3]

Jez now decides to work only three 10-hour days, but he employs an apprentice, Becky, to help him. Becky works the same hours as Jez and is paid $25 per hour. She is able to carry out routine services on laptops and tablets, but not to do repairs.

Customers are given a 20% reduction if Becky carries out the work on their laptop or tablet. Jez continues to pay himself $40 an hour. The other costs of running the business increase by 50%.

**(d)** What is the least profit that Jez might now make in one day? [2]

After a few months, Becky tells Jez that she would like to reduce her hours. If Jez allows this, he wants to be certain that he would make a profit of at least $200 each day. Becky would only be able to work at times when Jez was also working.

**(e)** For how many hours a day would Becky need to be employed? [3]

Jez agrees with Becky that, instead of reducing her hours, he will send her on a training course so that she will be able to repair tablets as well as carry out services. Following this, he will be able to pay her more than $25 an hour. However, she will need to continue to work the same three 10-hour days as him.

**(f)** What is the most that Jez could pay Becky per hour so that he can still make a profit of at least $200 each day? [3]

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