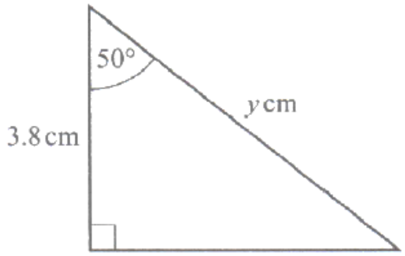
Trigonometry – 2021 IGCSE 0580

1. Nov/2021/Paper_13/No.24b

(b)



Show that the value of y is 5.9, correct to 2 significant figures.

Cosine

$\cos 50^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$y \times \cos 50^\circ = \frac{3.8\text{ cm}}{\cancel{y}} \cancel{\times y}$

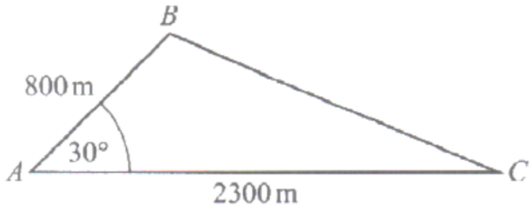
$\frac{y \cancel{\cos 50}}{\cancel{\cos 50}} = \frac{3.8\text{ cm}}{\cos 50^\circ}$

$y = 5.91$

$\approx 5.9\text{ cm}$

[3]

2. Nov/2021/Paper_21/No.18



The diagram shows some land in the shape of a triangle ABC.
Houses are built on this land.
Each house requires $400\ m^2$ of land.

Find the greatest number of houses that can be built on this land.

Area of triangle $= \frac{1}{2} ab \sin C$

$A = \frac{1}{2} \times 800 \times 2300 \sin 30°$

$A = 460{,}000 m^2$

1 house requires $400 m^2$
? $\Rightarrow 460{,}000 m^2$

$\frac{460{,}000}{400} = 1150$

1150 houses .... [3]

3. Nov/2021/Paper_21/No.20

Solve $3(2 + \cos x) = 5$ for $0° \leqslant x \leqslant 360°$.

$3(2 + \cos x) = 5$

$6 + 3\cos x = 5$

$3\cos x = 5 - 6$

$\frac{3\cos x}{3} = \frac{-1}{3}$

$\cos x = -\frac{1}{3}$

$\alpha = \cos^{-1}(\frac{1}{3})$

$\alpha = 70.5°$

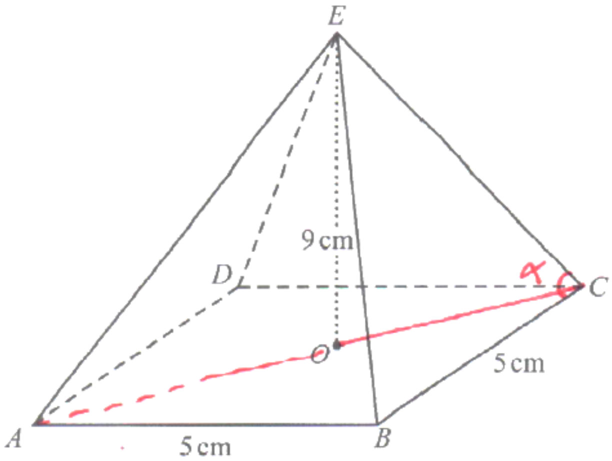
$x_1 = 180° - 70.5$
$= 109.5°$

$x_2 = 180° + 70.5°$

$x_2 = 250.5°$

109.5°, 250.5° .... [3]

4. Nov/2021/Paper_21/No.21



The diagram shows a pyramid *ABCDE*.
The pyramid has a square horizontal base *ABCD* with side 5 cm.
The vertex *E* is vertically above the centre *O* of the base.
The height *OE* of the pyramid is 9 cm.

Calculate the angle that *EC* makes with the base *ABCD*.

$\tan\alpha = \frac{OE}{OC}$

$OE = 9\text{ cm}$

$OC = \frac{1}{2} AC$

$= \frac{1}{2}\left(\sqrt{5^2+5^2}\right)$

$= \frac{1}{2}\left(\sqrt{50}\right)$

$= \frac{1}{2}(7.07106)$

$= 3.5355$

$OC = 3.54\text{ cm}$

$\tan\alpha = \frac{9\text{cm}}{3.54}$

$\tan^{-1}\alpha = 68.553$

$= 68.6^\circ$

$\angle EC$ with the plane $ABCD$

$= 68.6^\circ$

68.6° [4]

Question 22 is printed on the next page.

5. Nov/2021/Paper_22/No.22

2 Solve the equation $7\sin x + 2 = 0$ for $0° \leqslant x \leqslant 360°$.

$7\sin x + 2 = 0$

$\sin x = -\frac{2}{7}$

$\alpha = \sin^{-1}\left(\frac{2}{7}\right)$

$\alpha = 16.6°$

$x_1 = 180° + 16.6$

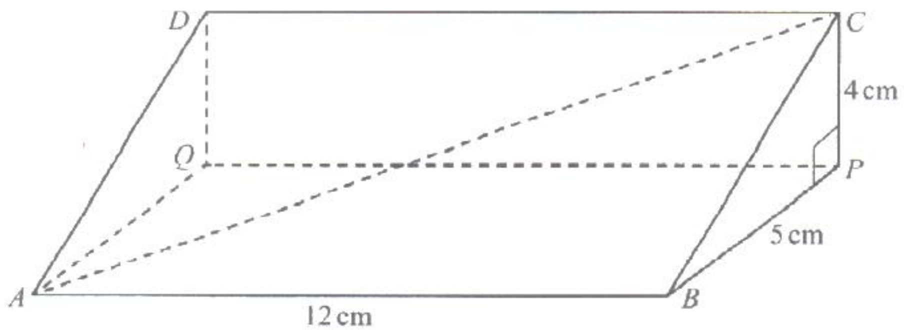
$= 196.6°$

$x_2 = 360° - 16.6$

$= 343.4°$

$196.6°,\ 343.4°$ .... [3]

6. Nov/2021/Paper_23/No.23



NOT TO SCALE

The diagram shows a triangular prism.
Angle $BPC = 90°$.

(a) Calculate AC.

$AP^2 = 12^2 + 5^2$

$AP^2 = 144 + 25$

$AP^2 = \sqrt{169}$

$AP = \underline{\underline{13\text{ cm}}}$

$AC^2 = AP^2 + CP^2$

$AC^2 = 13^2 + 4^2$

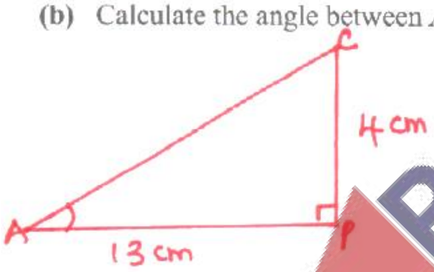
$AC^2 = 169 + 16$

$AC^2 = \sqrt{185}$

$AC = \underline{\underline{13.60}}$

$AC =$ ....13.60.... cm [3]

(b) Calculate the angle between AC and the base $ABPQ$.



$\tan \angle CAP = 4/13$

$\tan^{-1} = 0.3076$

$\simeq \underline{\underline{17.1°}}$

....17.1°.... [3]

7. Nov/2021/Paper_23/No.24

$\tan x = \sqrt{3}$ and $0° \leqslant x \leqslant 360°$.

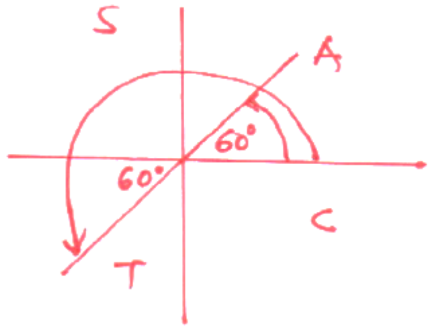
Find all the possible values of x.

$\tan x = \sqrt{3}$

$x = \tan^{-1}\sqrt{3}$

$x_1 = = 60°$

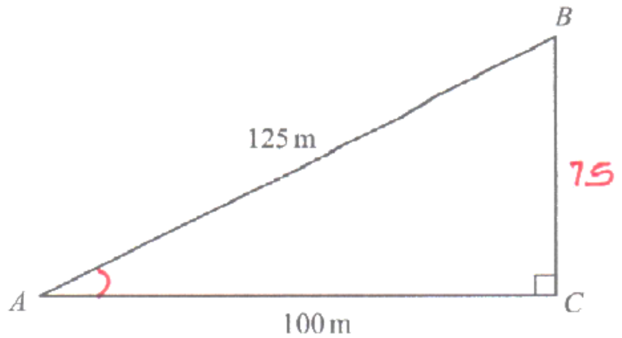
$x_2 = 180 + 60° = 240°$



$60°,\ 240°$ .... [2]

8. Nov/2021/Paper_31/No.8a(I, ii)

(a)



NOT TO SCALE

The diagram shows a right-angled triangle, ABC.

(i) Show that $BC = 75$ m. using Phythagoras theorem.

$BC^2 = 125^2 - 100^2$

$BC^2 = 15625 - 10,000$

$BC^2 = \sqrt{5625}$

$BC = \underline{\underline{75cm}}$

[2]

(ii) Calculate angle BAC.

Using trigonometric ratio's

$\cos \angle BAC = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$\angle BAC = \dfrac{100}{125}$

$\angle BAC^{-1} = 0.8$

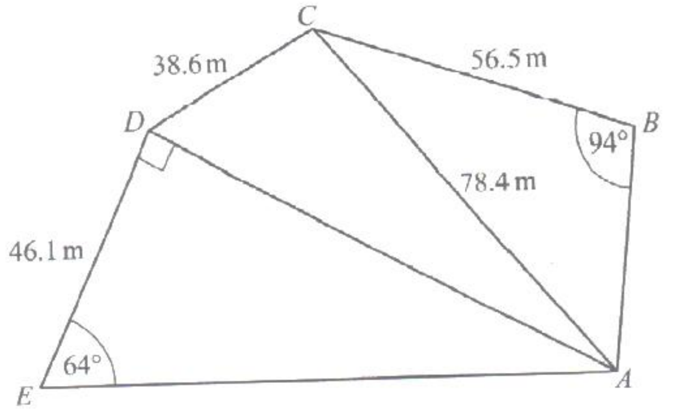
$= 36.8698$

$\simeq \underline{\underline{36.9^\circ}}$

Angle BAC = ... 36.9° .... [2]

9. Nov/2021/Paper_42/No.3

(a)



NOT TO SCALE

ABCDE is a pentagon.

(i) Calculate *AD* and show that it rounds to 94.5 m, correct to 1 decimal place.

$\tan 64° = \dfrac{AD}{46.1}$

$AD = 46.1 \tan 64° = 94.519$

$\simeq 94.5$

[2]

(ii) Calculate angle *BAC*.

using Sine rule.

$\dfrac{\sin BAC}{56.5m} = \dfrac{\sin 94°}{78.4m}$

$\angle BAC = \dfrac{56.5m \sin 94}{78.4m}$

$\angle BAC = \sin^{-1}(0.718907)$

$\angle BAC = 45.964$

$\simeq 46.0°$

Angle *BAC* = 46.0° [3]

(iii) Calculate the largest angle in triangle *CAD*.

The largest angle is opposite the longest side of the triangle

using Cosine rule.

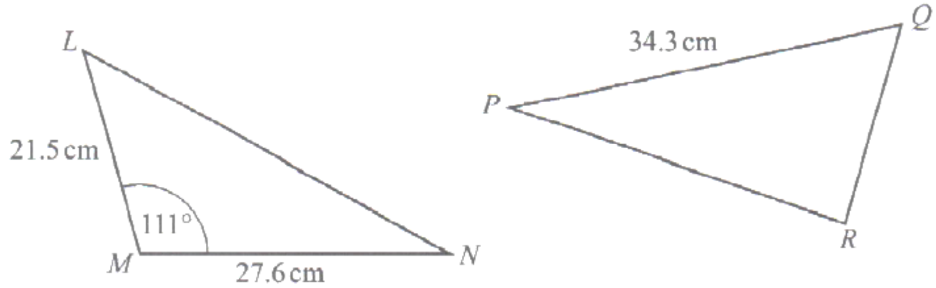
$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \cos \angle ACD$

$\angle ACD = \dfrac{94.5^2 - 2(78.4^2 + 38.6^2)}{(-2(78.4)\,38.6)}$

$\angle ACD = 102.3°$

102.3° [4]

(b)



NOT TO SCALE

Triangle PQR has the same area as triangle LMN.

Calculate the shortest distance from R to the line PQ.

Area of triangle PQR = Area of triangle LMN

$\frac{1}{2} \times 34.3\text{ cm} \times y = \frac{1}{2} \times 21.5 \times 27.6 \sin 111°$

$17.15y = 296.7 \sin 11$

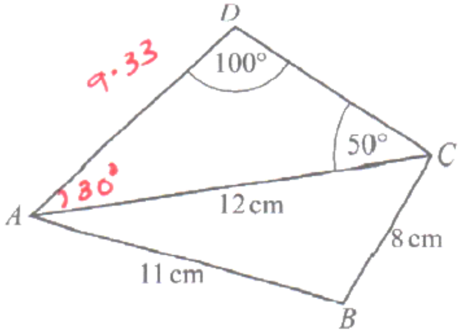
$\frac{\cancel{17.15}y}{\cancel{17.15}} = \frac{276.993}{17.15}$

$y = 16.15$

$\approx \underline{16.2\text{ cm}}$

.... cm [3]

10. Nov/2021/Paper_43/No.6



NOT TO SCALE

(a) Calculate AD.

using sine rule.

$$\frac{AD}{\sin 50^\circ} = \frac{12\text{ cm}}{\sin 100}$$

$$AD = \frac{12 \sin 50}{\sin 100} = 9.33$$

AD = 9.33 cm [3]

(b) Calculate angle BAC and show that it rounds to 40.42°, correct to 2 decimal places.

using cosine rule;

$$\angle BAC = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{12^2 + 11^2 - 8^2}{2 \times 12 \times 11}$$

$$= \frac{144 + 121 - 64}{264}$$

$$\angle BAC = \frac{265 - 64}{264}$$

$$= \frac{201}{264}$$

$$= 0.76136$$

$$\cos^{-1} = 40.415^\circ = \underline{40.42^\circ}$$

$$\approx \underline{40.42}$$

[4]

(c) Calculate the area of the quadrilateral $ABCD$.

Area of Triangle ADC + Area of triangle ABC

$$\left(\tfrac{1}{2} \times 9.33 \times 12 \sin 30^\circ\right) + \tfrac{1}{2} \times 12 \times 11 \sin 40.42$$

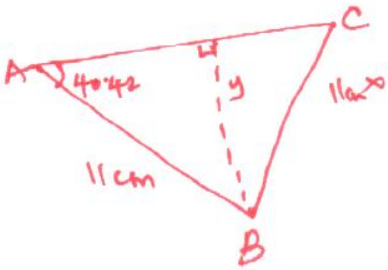
$$27.99 + 42.793$$

$$= 70.783$$

$$= \underline{70.8}$$

70.8 cm² [3]

(d) Calculate the shortest distance from B to AC.



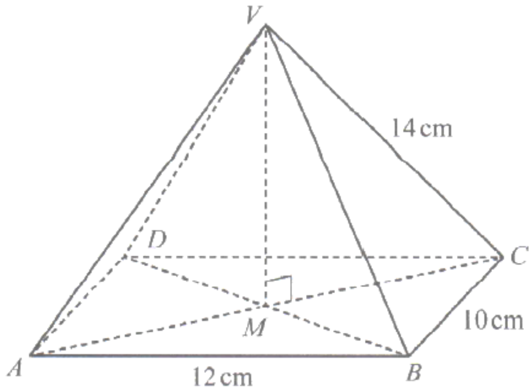
$$11\text{cm} \times \sin 40.42 = \frac{y}{11\text{cm}} \times 11\text{cm}$$

$$y = 11 \sin 40.42$$

$$y = \underline{7.13\text{cm}}$$

7.13 cm [3]

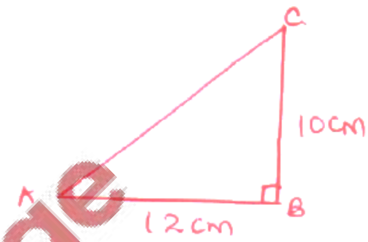
11. March/2021/Paper_22/No.23



NOT TO SCALE

The diagram shows a pyramid *VABCD* with a rectangular base.
V is vertically above *M*, the intersection of the diagonals *AC* and *BD*.
$AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm.

Calculate the angle that *VC* makes with the base *ABCD*.



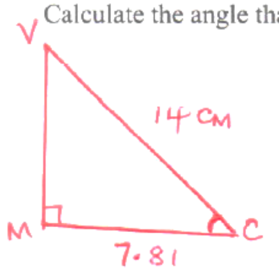
$AC^2 = 12^2 + 10^2$

$AC^2 = 144 + 100$

$AC^2 = 244$

$AC = \sqrt{244}$

$= \underline{15.62\text{ cm}}$



$\angle VCM =$

$\cos VCM = \dfrac{7.81}{14}$

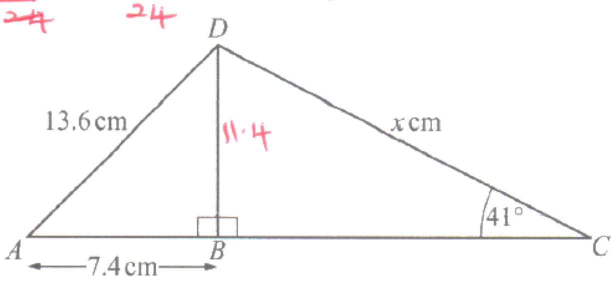
$\cos^{-1} VCM = 56.092$

$= \underline{56.1^\circ}$

.......56.1°....... [4]

12. March/2021/Paper_32/No.9e

(e)



NOT TO SCALE

Calculate the value of x.

$BD^2 = AD^2 - AB^2$

$BD^2 = 13.6^2 - 7.4^2$

$BD^2 = 184.96 - 54.76$

$BD^2 = 130.2$

$BD = \sqrt{130.2}$

$= 11.41$

$\simeq 11.4$ cm

$x \times \sin 41^\circ = \frac{11.4}{\cancel{x}} \times \cancel{x}$

$\frac{x \cancel{\sin 41}}{\cancel{\sin 41}} = \frac{11.4}{\sin 41}$

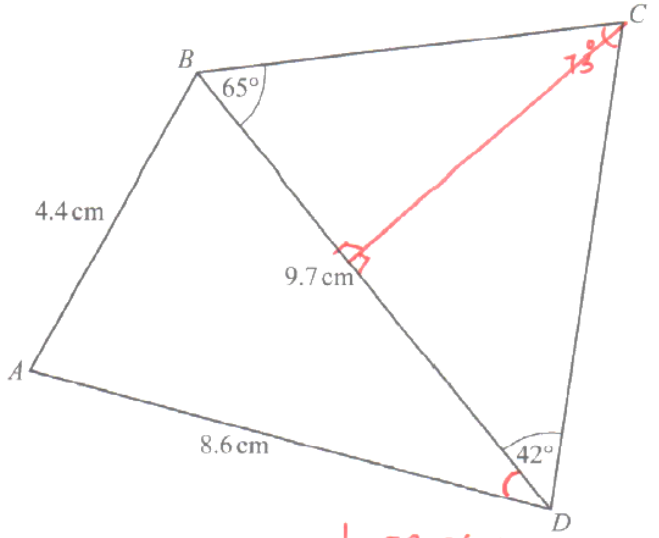
$x = 17.376$

$\simeq 17.38$ cm

$x =$ 17.38 cm [5]

13. March/2021/Paper_42/No.5

5



(a) Calculate angle ADB.

$\cos D = \dfrac{b^2 + a^2 - d^2}{2 \times a \times b}$

$= \dfrac{8.6^2 + 9.7^2 - 4.4^2}{2 \times 9.7 \times 8.6}$

$\dfrac{73.96 + 94.09 - 19.36}{166.84}$

$= \dfrac{148.69}{166.84}$

$\cos^{-1} 0.891213138$

$= 26.97°$

≈ 27

Angle ADB = ... 27° .... [3]

(b) Calculate DC.

$\angle BCD = 180° - (65° + 42°)$

$= 73°$

$\dfrac{9.7}{\sin 73°} = \dfrac{DC}{\sin 65°}$

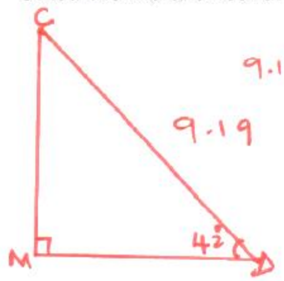
$DC = \dfrac{9.7 \sin 65°}{\sin 73°}$

9.1928

≈ 9.19

DC = ... 9.19 .... cm [4]

(c) Calculate the shortest distance from C to BD.



$9.19 \times \sin 42° = \dfrac{CM}{9.19} \times 9.19$

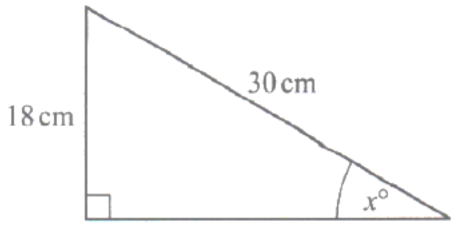
$CM = 9.19 \sin 42°$

$= 6.1493$

≈ 6.15

.... 6.15 .... cm [3]

14. June/2021/Paper_13/No.24



NOT TO SCALE

The diagram shows a right-angled triangle.

Show that the value of x is 36.9, correct to 1 decimal place.

$$\sin x = \frac{18}{30}$$

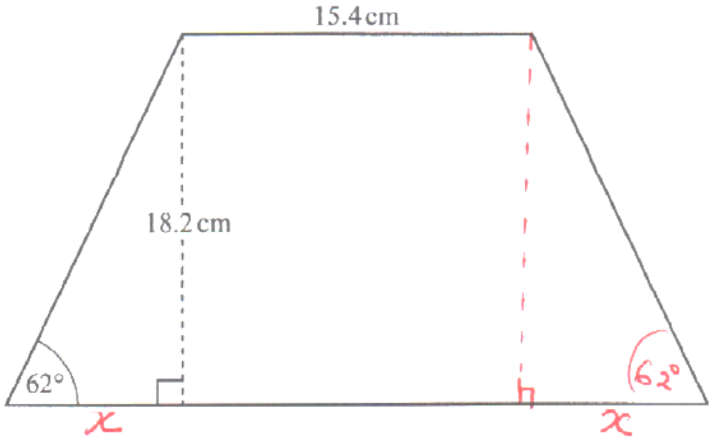
$$x = \sin^{-1}\left(\frac{18}{30}\right)$$

$$x = 36.869$$

$$x = 36.87$$

[2]

15. June/2021/Paper_22/No.14



NOT TO SCALE

The diagram shows a trapezium.
The trapezium has one line of symmetry.

Work out the area of the trapezium.

$\tan 62 = \frac{18.2}{x}$

$x \tan 62 = 18.2$

$x = \frac{18.2}{\tan 62} = 9.68$

$2x = \underline{\underline{19.35}}$

$= 19.35 + 15.4 + 15.4$

$34.75 + 15.4 = 50.15$

$\frac{1}{2}(50.15) \times 18.2 = 456.365$

$= \underline{456.4}$

............456.4............ cm^2 [4]

16. June/2021/Paper_22/No.23

Find all the solutions of $4\sin x = 3$ for $0° \leqslant x \leqslant 360°$.

$\sin x = 3/4$

$x = \sin^{-1}(3/4)$

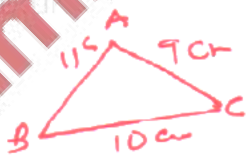
$x = \underline{\underline{48.6°}}$

$180 - 48.6° = \underline{\underline{131.4°}}$

48.6, 131.4° .... [2]

17. June/2021/Paper_23/No.23

A triangle has sides of length 11 cm, 10 cm and 9 cm.



Calculate the largest angle in the triangle.

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{10^2 + 9^2 - 11^2}{2 \times 10 \times 9}$

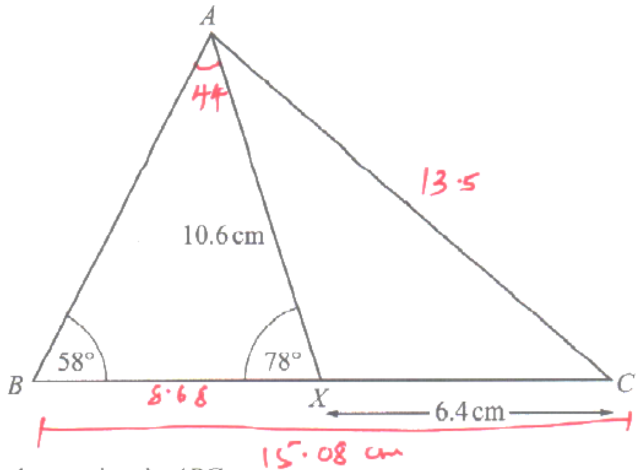
$\cos C = \dfrac{100 + 81 - 121}{180}$

$\cos C = \dfrac{60}{180}$

$\cos^{-1} C = 0.33\dot{3} = \underline{\underline{70.528°}}$

70.528° .... [4]

18. June/2021/Paper_41/No.5



NOT TO SCALE

Angles in straight line add up to 180°

$\angle AXC = 180° - 78°$

$= 102°$

The diagram shows triangle ABC.
X is a point on BC.
$AX = 10.6$ cm, $XC = 6.4$ cm, angle $ABC = 58°$ and angle $AXB = 78°$.

(a) Calculate AC.

Use Cosine rule

$AC^2 = AX^2 + XC^2 - 2(AX)(XC)\cos 102°$

$AC^2 = 10.6^2 + 6.4^2 - 2 \times 10.6 \times 6.4 \cos 102°$

$AC^2 = 112.36 + 40.96 - 135.68 \cos 102°$

$AC^2 = 153.32 - (-28.20945)$

$AC^2 = \sqrt{181.529}$

$AC = 13.473$

≈ 13.5 cm

$AC =$ ......13.7...... cm [4]

(b) Calculate BX.

$180 - (56 + 78)$
$= 44°$

$\dfrac{BX}{\sin 44} = \dfrac{10.6}{\sin 58}$

$BX = \dfrac{10.6 \sin 44}{\sin 58}$

$BX = 8.682$

$= 8.68$

$BX =$ ......8.68...... cm [4]

(c) Calculate the area of triangle ABC.

Area of Triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 13.5 \times 15.08 \sin 50.2°$

$= 6.75 \times 15.08 \sin 50.2$

$= 78.20$ cm^2

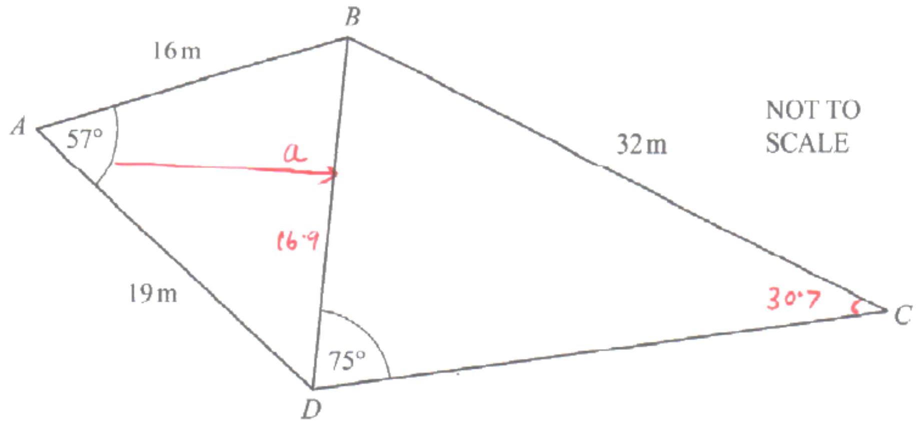
$\angle ACB = \dfrac{10.6}{\sin ACB} = \dfrac{13.5}{\sin 102°}$

$= \dfrac{10.6 \sin 102}{13.5}$

$= 50.2°$

......78.20...... cm^2 [3]

19. June/2021/Paper_42/No.6



The diagram shows a quadrilateral *ABCD* made from two triangles, *ABD* and *BCD*.

(a) Show that $BD = 16.9$ m, correct to 1 decimal place.

cosine rule.

$a^2 = b^2 + d^2 - 2bd \cos A$

$a^2 = 19^2 + 16^2 - 2 \times 19 \times 16 \cos 57$

$a^2 = 361 + 256 - 608 \cos 57°$

$a^2 = 617 - 608 \cos 57°$

$a^2 = 617 - 331.14053$

$a^2 = \sqrt{258.859} = 16.907$

$= 16.9$

[3]

(b) Calculate angle *CBD*.

Sine rule.

$\frac{\sin 75°}{32} = \frac{\sin c}{16.9}$

$\sin c = \frac{16.9 \sin 75}{32}$

$\sin^{-1} c = 30.67$

$= 30.7$

Since angles in a triangle add up to $180°$.

$\angle CBD = 180° - (75 + 30.7)$

$= 74.3°$

Angle *CBD* = $74.3°$ [4]

(c) Find the area of the quadrilateral *ABCD*.

Area of triangle $ABD = \frac{1}{2} \times ab \sin C$

$= \frac{1}{2} \times 16 \times 19 \sin 57$

$= 127.478 \text{ cm}^2$

Area of triangle $BCD = \frac{1}{2} \times 16.9 \times 32 \sin 74.3$

$= 260.31 \text{ m}^2$

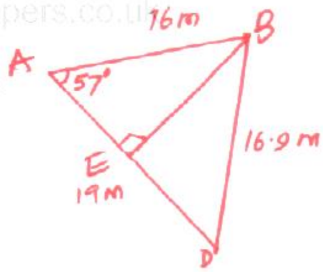
Total area of ABCD

$= 127.478 + 260.31$

$= 387.788$

$\approx 388 \text{ m}^2$

388 m² [3]



(d) Find the shortest distance from B to AD.

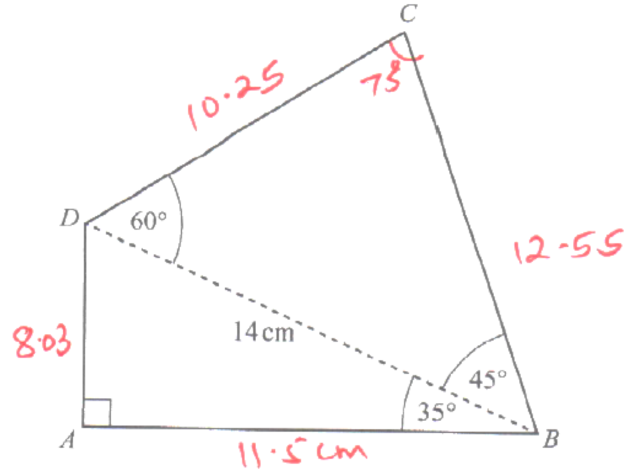
$$16 \times \frac{BE}{16} = \sin 57^\circ \times 16$$

$$BE = 16 \sin 57$$

$$= \underline{13 \cdot 42}$$

13·42 .... m [3]

20. June/2021/Paper_43/No.9



NOT TO SCALE

Calculate the perimeter of the quadrilateral *ABCD*.

Length of AD =
Using trigonometric ratios AD =

$14\text{cm} \times \sin 35^\circ = \frac{AD}{\cancel{14\text{cm}}} \times \cancel{14\text{cm}}$

$AD = 14 \sin 35$

$AD = \underline{8.03}$

$AB = 14 \times \cos 35^\circ = \frac{AB}{\cancel{14\text{cm}}} \times \cancel{14}$

$AB = 11.468$

$= \underline{11.5\text{cm}}$

Since Angles in a triangle Sum up to 180°.

$\angle DCB = 180^\circ - (60 + 45^\circ)$

$= 180^\circ - 105^\circ$

$= \underline{75^\circ}$

Using Sine rule length of DC =

$\frac{14\text{cm}}{\sin 75^\circ} = \frac{CD}{\sin 45^\circ}$ (Cross Multiply).

$\frac{14 \sin 45}{\sin 75} = \frac{CD\ \cancel{\sin 75}}{\cancel{\sin 75}}$

$CD = \underline{10.25\text{cm}}$

$\frac{\sin 60^\circ}{BC} = \frac{\sin 45^\circ}{10.25}$

$BC = \frac{10.25 \sin 60}{\sin 45^\circ}$

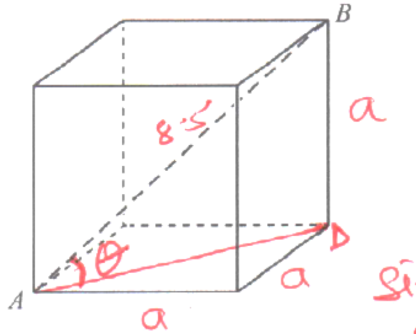
$BC = \underline{12.55\text{cm}}$

42·33 .... cm [7]

Perimeter $= 8.03 + 11.5 + 10.25 + 12.55$

$= \underline{42.33\text{ CM}}$

(b)



Since it is a cube all sides are equal.

The diagram shows a cube.
The length of the diagonal AB is 8.5 cm.

(i) Calculate the length of an edge of the cube.

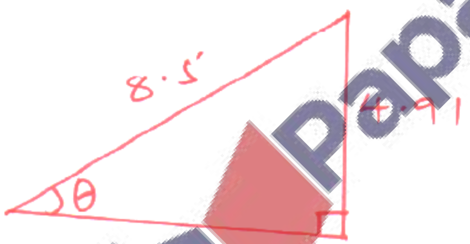
$$A.^2 = a^2 + a^2$$
$$= 2a^2$$

$$AD^2 + BD^2 = AB^2$$
$$2a^2 + a^2 = 8.5^2$$
$$3a^2 = 8.5^2$$

$$\frac{3a^2}{3} = \frac{72.25}{3}$$
$$a^2 = 24.083$$
$$a = \sqrt{24.083} = 4.9074$$
$$4.91 \text{ cm}$$

............4.91............ cm [3]

(ii) Calculate the angle between AB and the base of the cube.



$$\sin\theta = \frac{4.91}{8.5}$$

$$\sin^{-1}\theta = 0.5776$$
$$\simeq 35.28$$
$$= 35.3^{\circ}$$

............35.3°............ [3]