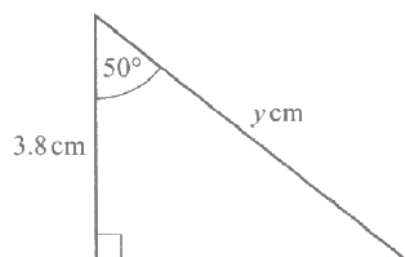


1. Nov/2021/Paper_13/No.24b

(b)



NOT TO SCALE

Show that the value of y is 5.9, correct to 2 significant figures.

Cosine

$$\cos 50^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

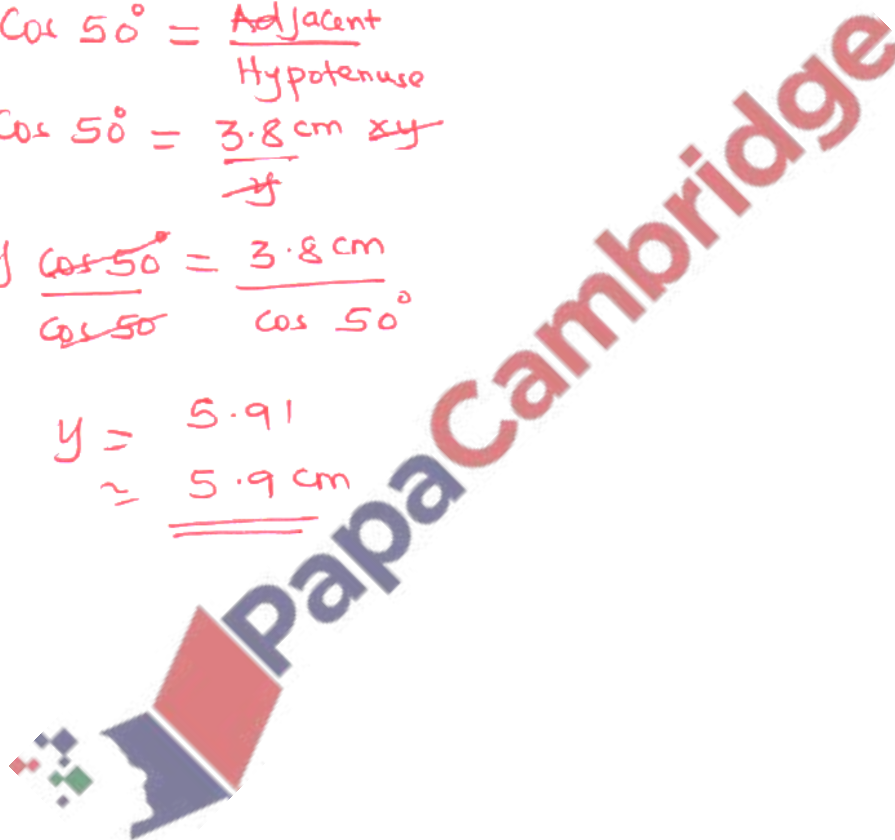
$$y \times \cos 50^\circ = \frac{3.8 \text{ cm}}{y}$$

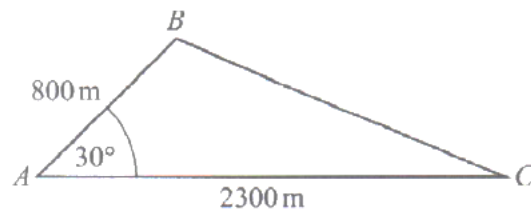
$$y \frac{\cos 50^\circ}{\cos 50^\circ} = \frac{3.8 \text{ cm}}{\cos 50^\circ}$$

$$y = \frac{3.8}{\cos 50^\circ}$$

$$y \approx \underline{\underline{5.9 \text{ cm}}}$$

[3]



NOT TO
SCALE

The diagram shows some land in the shape of a triangle ABC .
Houses are built on this land.
Each house requires 400 m^2 of land.

Find the greatest number of houses that can be built on this land.

$$\text{Area of triangle} = \frac{1}{2} ab \sin c$$

$$A = \frac{1}{2} \times 800 \times 2300 \sin 30^\circ$$

$$A = 460,000 \text{ m}^2$$

$$\begin{array}{l} \text{1 house} \\ \text{house} \end{array} \text{ requires } 400 \text{ m}^2 \\ \text{?} \quad \quad \quad = 460,000 \text{ m}^2$$

$$\frac{460,000}{400} = \underline{\underline{1150}}$$

..... 1150 houses [3]

Solve $3(2 + \cos x) = 5$ for $0^\circ \leq x \leq 360^\circ$.

$$3(2 + \cos x) = 5$$

$$6 + 3 \cos x = 5$$

$$3 \cos x = 5 - 6$$

$$\frac{3 \cos x}{3} = \frac{-1}{3}$$

$$\cos x = -\frac{1}{3}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = \underline{\underline{70.5^\circ}}$$

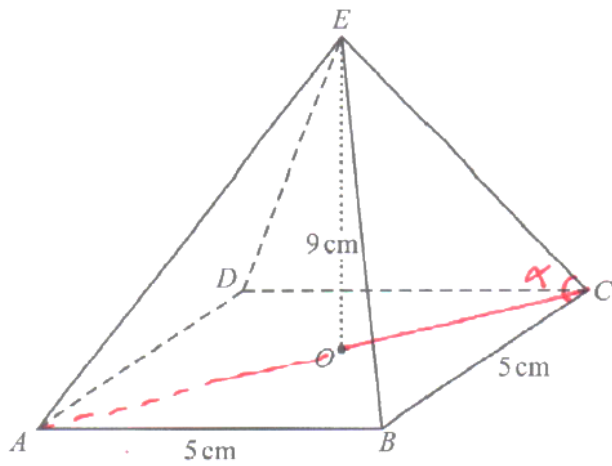
$$x_1 = 180^\circ - 70.5^\circ$$

$$= \underline{\underline{109.5^\circ}}$$

$$x_2 = 180^\circ + 70.5^\circ$$

$$x_2 = \underline{\underline{250.5^\circ}}$$

..... 109.5°, 250.5° [3]



NOT TO SCALE

The diagram shows a pyramid $ABCDE$.
 The pyramid has a square horizontal base $ABCD$ with side 5 cm .
 The vertex E is vertically above the centre O of the base.
 The height OE of the pyramid is 9 cm .

Calculate the angle that EC makes with the base $ABCD$.

$$\tan \alpha = \frac{OE}{OC}$$

$$OE = 9\text{ cm}$$

$$OC = \frac{1}{2} AC$$

$$= \frac{1}{2} (\sqrt{5^2 + 5^2})$$

$$= \frac{1}{2} (\sqrt{50})$$

$$= \frac{1}{2} (7.07106)$$

$$= 3.5355$$

$$OC = 3.54\text{ cm}$$

$$\tan \alpha = \frac{9\text{ cm}}{3.54}$$

$$\tan^{-1} = 68.553$$

$$= \underline{\underline{68.6^\circ}}$$

$$\angle EC \text{ with Plane } ABCD$$

$$= \underline{\underline{68.6^\circ}}$$

..... 68.6 [4]

Question 22 is printed on the next page.

5. Nov/2021/Paper_22/No.22

2 Solve the equation $7 \sin x + 2 = 0$ for $0^\circ \leq x \leq 360^\circ$.

$$7 \sin x + 2 = 0$$

$$\sin x = -\frac{2}{7}$$

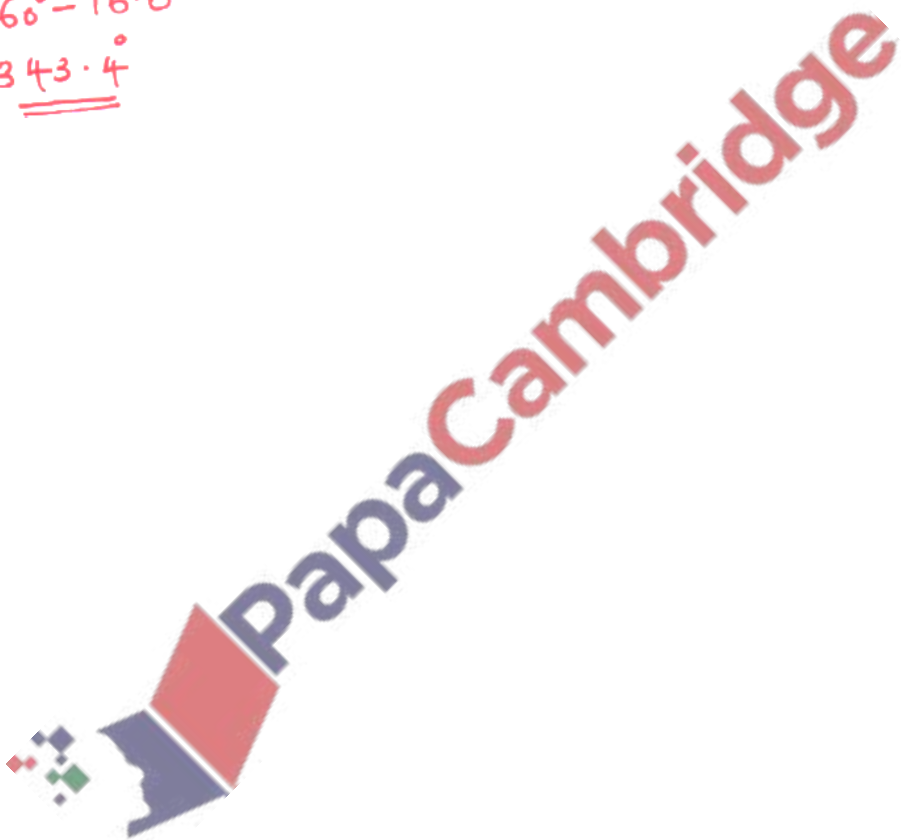
$$\alpha = \sin^{-1}\left(\frac{2}{7}\right)$$

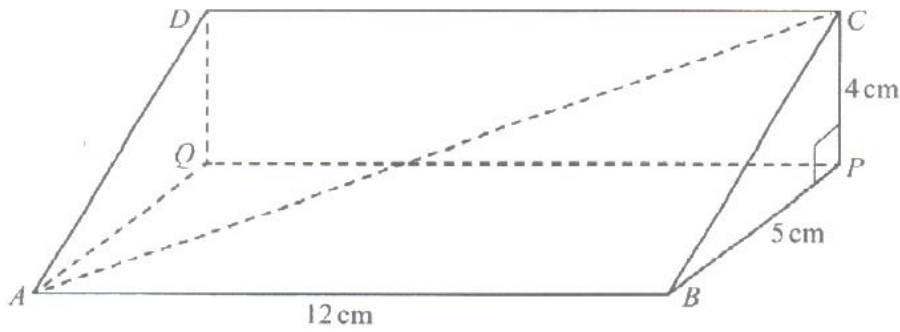
$$\alpha = \underline{\underline{16.6^\circ}}$$

$$\begin{aligned} x_1 &= 180^\circ + 16.6 \\ &= \underline{\underline{196.6^\circ}} \end{aligned}$$

$$\begin{aligned} x_2 &= 360^\circ - 16.6 \\ &= \underline{\underline{343.4^\circ}} \end{aligned}$$

$$\underline{\underline{196.6^\circ, 343.4^\circ}} \quad [3]$$





NOT TO SCALE

The diagram shows a triangular prism.
Angle $BPC = 90^\circ$.

(a) Calculate AC .

$$AP^2 = 12^2 + 5^2$$

$$AP^2 = 144 + 25$$

$$AP^2 = \sqrt{169}$$

$$AP = \underline{13 \text{ cm}}$$

$$AC^2 = AP^2 + CP^2$$

$$AC^2 = 13^2 + 4^2$$

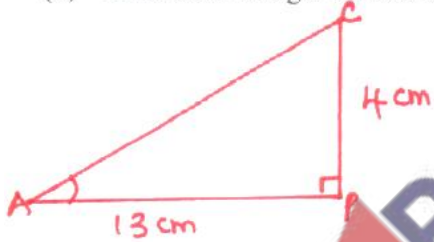
$$AC^2 = 169 + 16$$

$$AC^2 = \sqrt{185}$$

$$AC = \underline{13.60}$$

$AC = \underline{13.60} \dots \dots \dots \text{ cm [3]}$

(b) Calculate the angle between AC and the base $ABPQ$.



$$\tan \angle CAP = \frac{4}{13}$$

$$\tan^{-1} = 0.3076$$

$$\approx \underline{17.1^\circ}$$

$\underline{17.1^\circ} \dots \dots \dots [3]$

7. Nov/2021/Paper_23/No.24

$$\tan x = \sqrt{3} \text{ and } 0^\circ \leq x \leq 360^\circ.$$

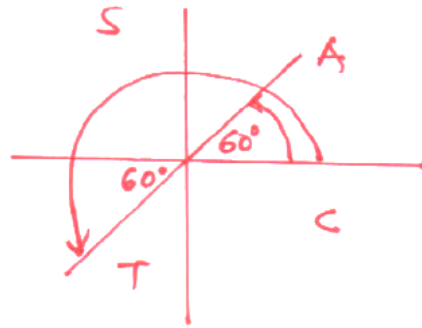
Find all the possible values of x .

$$\tan x = \sqrt{3}$$

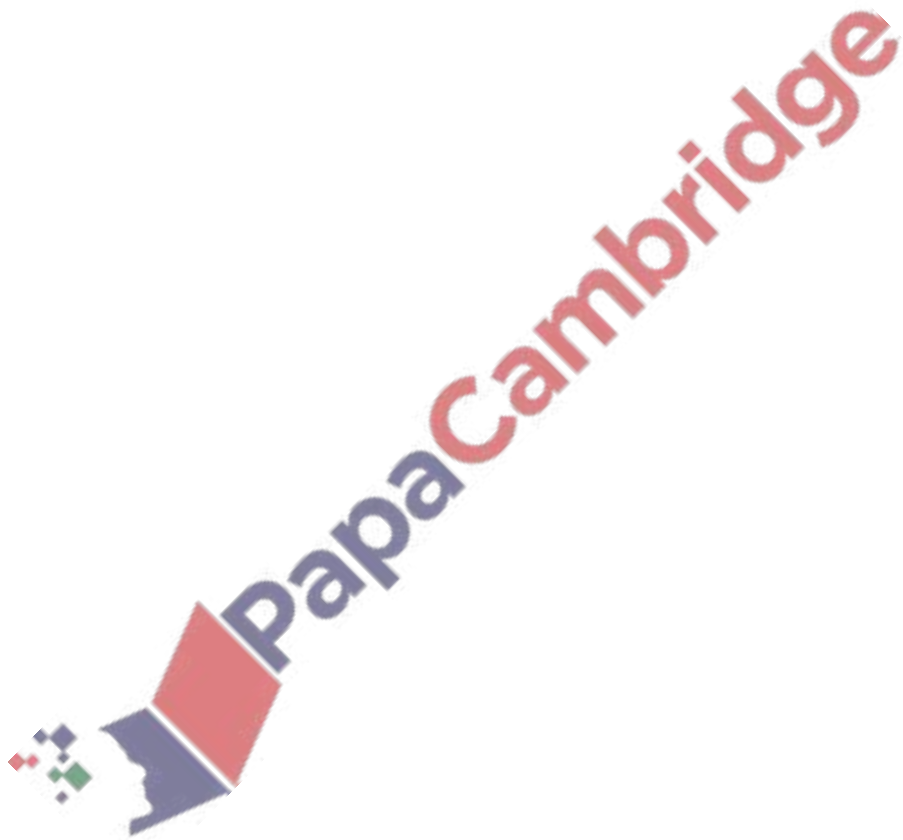
$$x = \tan^{-1} \sqrt{3}$$

$$x_1 = 60^\circ$$

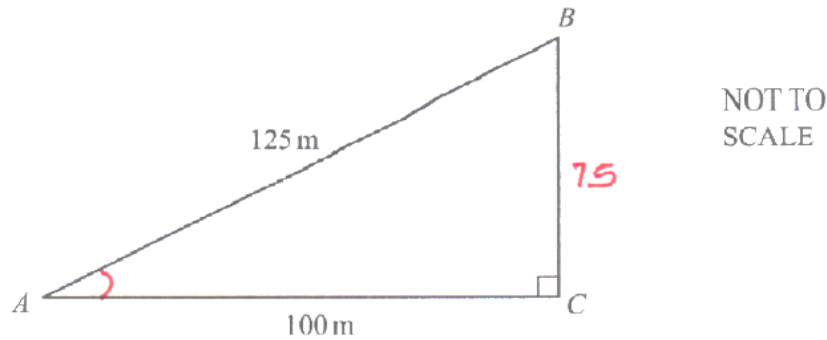
$$x_2 = 180 + 60 = 240^\circ$$



60°, 240° [2]



(a)



The diagram shows a right-angled triangle, ABC .

(i) Show that $BC = 75$ m.

using Pythagoras theorem.

$$BC^2 = 125^2 - 100^2$$

$$BC^2 = 15625 - 10,000$$

$$BC^2 = \sqrt{5625}$$

$$BC = \underline{\underline{75\text{cm}}}$$

[2]

(ii) Calculate angle BAC .

Using trigonometric ratios

$$\cos \angle BAC = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\angle BAC = \frac{100}{125}$$

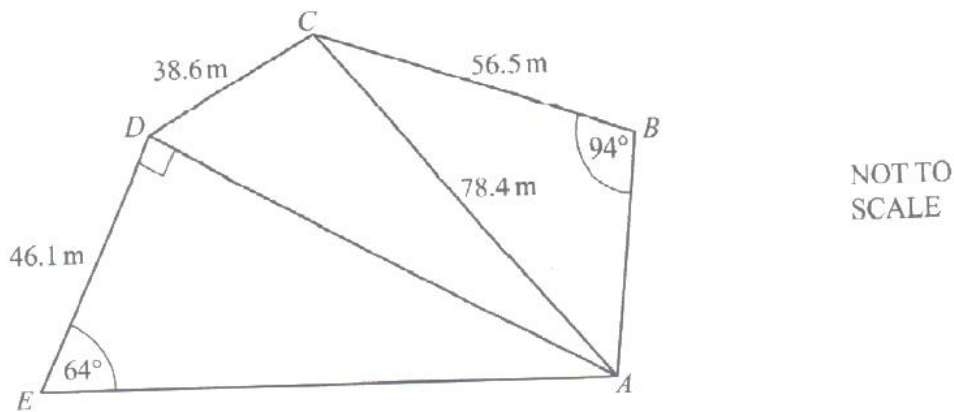
$$\angle BAC = 0.8$$

$$= 36.8698$$

$$\approx \underline{\underline{36.9^\circ}}$$

$$\text{Angle } BAC = \dots\dots\dots 36.9^\circ \dots\dots\dots [2]$$

(a)



$ABCDE$ is a pentagon.

- (i) Calculate AD and show that it rounds to 94.5 m, correct to 1 decimal place.

$$\tan 64^\circ = \frac{AD}{46.1}$$

$$AD = 46.1 \tan 64^\circ = 94.519 \approx \underline{94.5} \quad [2]$$

- (ii) Calculate angle BAC .

Using Sine rule.

$$\frac{\sin BAC}{56.5\text{m}} = \frac{\sin 94^\circ}{78.4\text{m}}$$

$$\angle BAC = 45.964 \approx \underline{46.0}$$

$$\angle BAC = \frac{56.5\text{m} \sin 94^\circ}{78.4\text{m}}$$

$$\angle BAC = \sin^{-1}(0.718907)$$

$$\text{Angle } BAC = \underline{46.0} \quad [3]$$

- (iii) Calculate the largest angle in triangle CAD .

The largest angle is opposite the longest side of the triangle using Cosine rule.

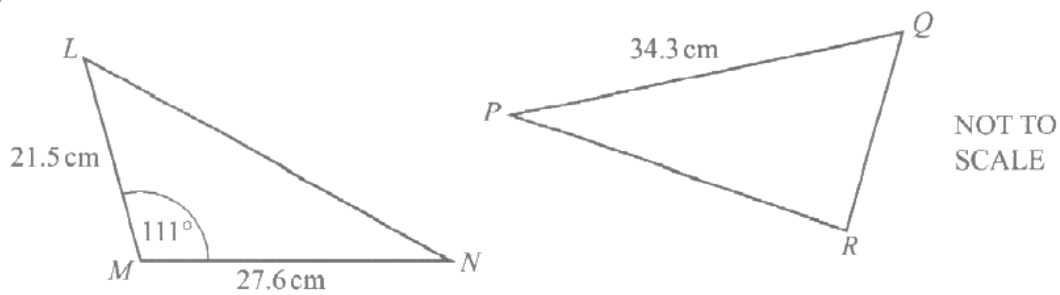
$$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \cos \angle ACD$$

$$\angle ACD = \frac{94.5^2 - 2(78.4^2 + 38.6^2)}{-2(78.4)(38.6)}$$

$$\angle ACD = \underline{102.3}$$

$$\underline{102.3} \quad [4]$$

(b)



Triangle PQR has the same area as triangle LMN .

Calculate the shortest distance from R to the line PQ .

Area of triangle $PQR = \text{Area of triangle } LMN$

$$\frac{1}{2} \times 34.3 \text{ cm} \times y = \frac{1}{2} \times 21.5 \times 27.6 \sin 111^\circ$$

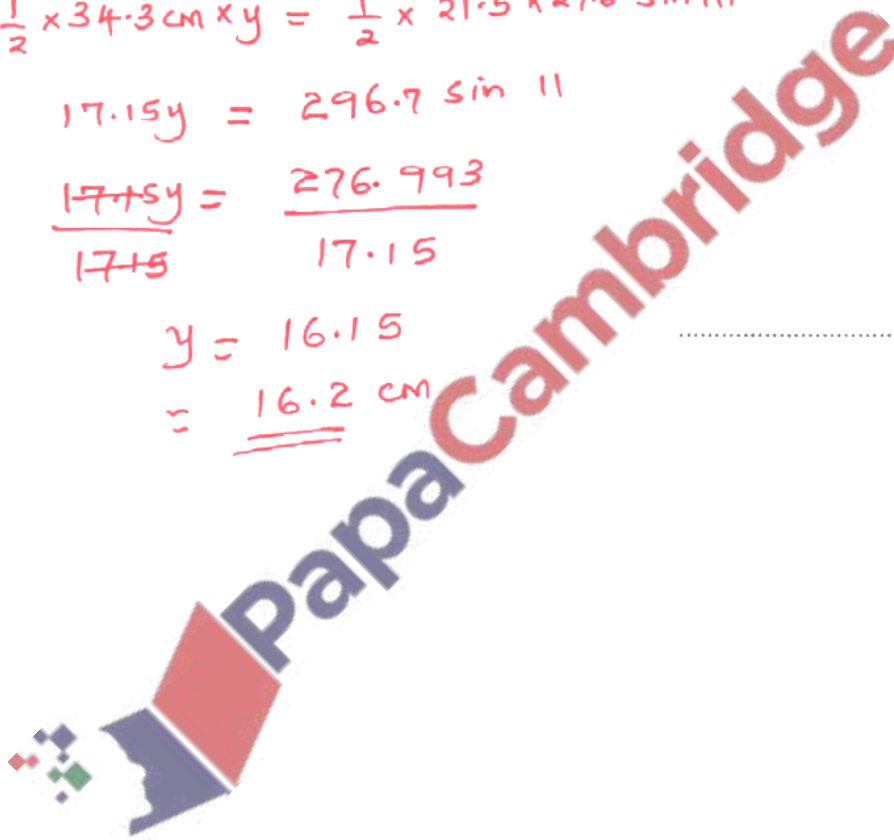
$$17.15y = 296.7 \sin 11$$

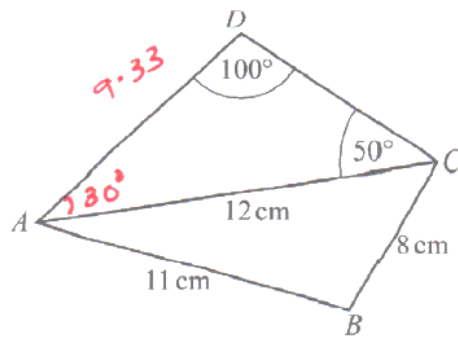
$$\frac{17.15y}{17.15} = \frac{276.993}{17.15}$$

$$y = 16.15$$

$$= \underline{\underline{16.2 \text{ cm}}}$$

..... cm [3]





NOT TO SCALE

(a) Calculate AD.

Using sine rule.

$$\frac{AD}{\sin 50^\circ} = \frac{12 \text{ cm}}{\sin 100^\circ}$$

$$AD = \frac{12 \sin 50}{\sin 100} = 9.33$$

AD = 9.33 cm [3]

(b) Calculate angle BAC and show that it rounds to 40.42°, correct to 2 decimal places.

using cosine rule;

$$\begin{aligned} \angle BAC &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{12^2 + 11^2 - 8^2}{2 \times 12 \times 11} \\ &= \frac{144 + 121 - 64}{264} \end{aligned}$$

$$\begin{aligned} \angle BAC &= \frac{265 - 64}{264} \\ &= \frac{201}{264} \\ &= 0.76136 \end{aligned}$$

$$\begin{aligned} \cos^{-1} &= 40.415^\circ = \underline{\underline{40.42^\circ}} \quad [4] \\ &\approx \underline{\underline{40.42}} \end{aligned}$$

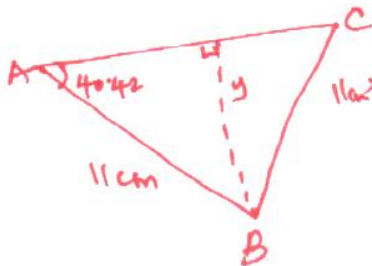
(c) Calculate the area of the quadrilateral ABCD.

Area of Triangle ADC + Area of triangle ABC

$$\begin{aligned} &\left(\frac{1}{2} \times 9.33 \times 12 \sin 30^\circ \right) + \frac{1}{2} \times 12 \times 11 \sin 40.42^\circ \\ &= 27.99 + 42.793 \\ &= 70.783 \\ &= \underline{\underline{70.8}} \end{aligned}$$

70.8 cm² [3]

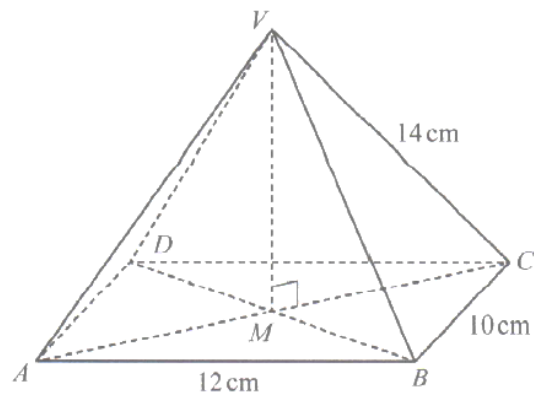
(d) Calculate the shortest distance from B to AC.



$$11 \sin 40.42^\circ = \frac{y}{11 \text{ cm}}$$

$$\begin{aligned} y &= 11 \sin 40.42^\circ \\ y &= \underline{\underline{7.13 \text{ cm}}} \end{aligned}$$

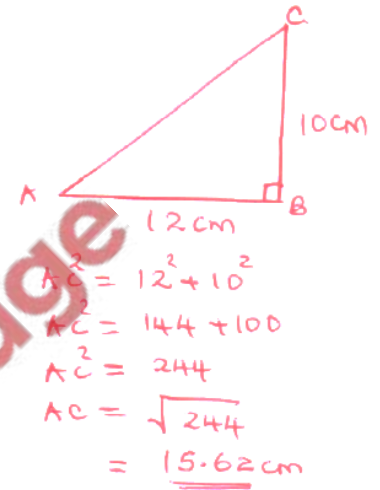
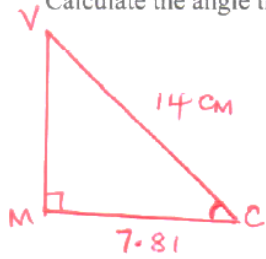
7.13 cm [3]



NOT TO SCALE

The diagram shows a pyramid $VABCD$ with a rectangular base. V is vertically above M , the intersection of the diagonals AC and BD . $AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm.

Calculate the angle that VC makes with the base $ABCD$.

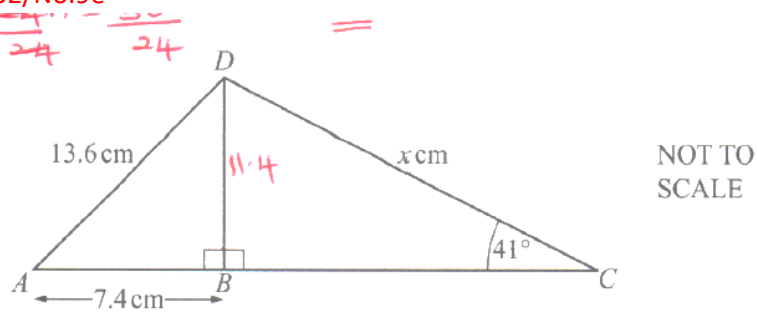


$\angle VCM =$
 $\cos \angle VCM = \frac{7.81}{14}$
 $\cos^{-1} = 56.092$
 $= \underline{\underline{56.1^\circ}}$

..... 56.1° [4]



(e)

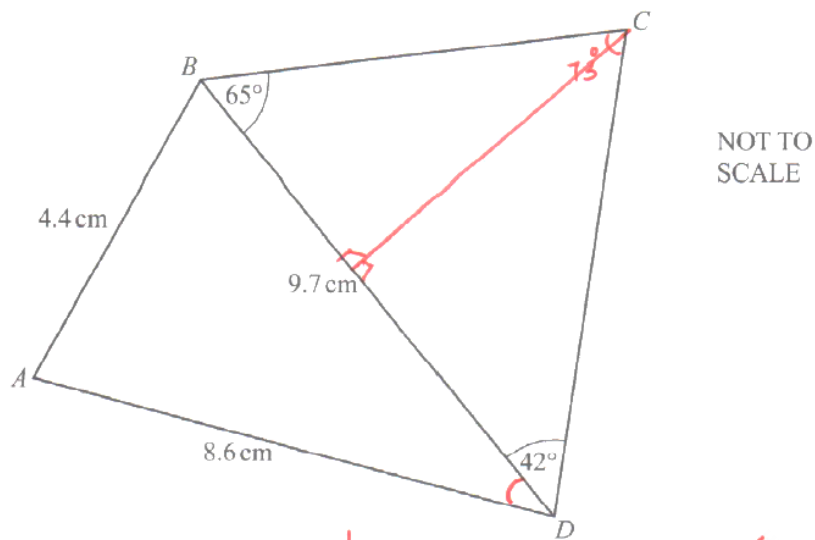
Calculate the value of x .

$$\begin{aligned}
 BD^2 &= AD^2 - AB^2 \\
 BD^2 &= 13.6^2 - 7.4^2 \\
 BD^2 &= 184.96 - 54.76 \\
 BD^2 &= 130.2 \\
 BD &= \sqrt{130.2} \\
 &= 11.41 \\
 &\approx \underline{\underline{11.4 \text{ cm}}}
 \end{aligned}$$

$$\begin{aligned}
 x \sin 41^\circ &= \frac{11.4}{x} \times x \\
 x \sin 41^\circ &= 11.4 \\
 x &= \frac{11.4}{\sin 41^\circ} \\
 x &= 17.376 \\
 &\approx \underline{\underline{17.38 \text{ cm}}}
 \end{aligned}$$

$$x = \underline{\underline{17.38 \text{ cm}}} \dots \dots \dots [5]$$

PapaCambridge



(a) Calculate angle ADB .

$$\cos D = \frac{b^2 + a^2 - d^2}{2 \times a \times b}$$

$$= \frac{8.6^2 + 9.7^2 - 4.4^2}{2 \times 9.7 \times 8.6}$$

$$\frac{73.96 + 94.09 - 19.36}{166.84}$$

$$= \frac{148.69}{166.84}$$

$$= 0.891213138$$

$$\cos^{-1} = 26.97^\circ$$

$$\approx 27^\circ$$

Angle $ADB = \dots 27^\circ \dots [3]$

(b) Calculate DC .

$$\angle BCD = 180^\circ - (65^\circ + 42^\circ)$$

$$= 73^\circ$$

$$\frac{9.7}{\sin 73^\circ} = \frac{DC}{\sin 65^\circ}$$

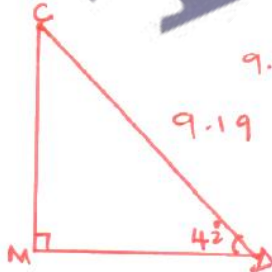
$$DC = \frac{9.7 \sin 65^\circ}{\sin 73^\circ}$$

$$9.1928$$

$$\approx 9.19$$

$DC = \dots 9.19 \dots \text{cm} [4]$

(c) Calculate the shortest distance from C to BD .



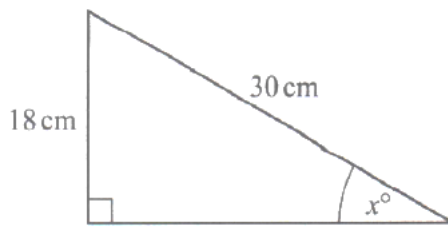
$$9.19 \times \sin 42^\circ = \frac{CM}{9.19} \times 9.19$$

$$CM = 9.19 \sin 42^\circ$$

$$= 6.1493$$

$$\approx 6.15$$

$\dots 6.15 \dots \text{cm} [3]$

NOT TO
SCALE

The diagram shows a right-angled triangle.

Show that the value of x is 36.9, correct to 1 decimal place.

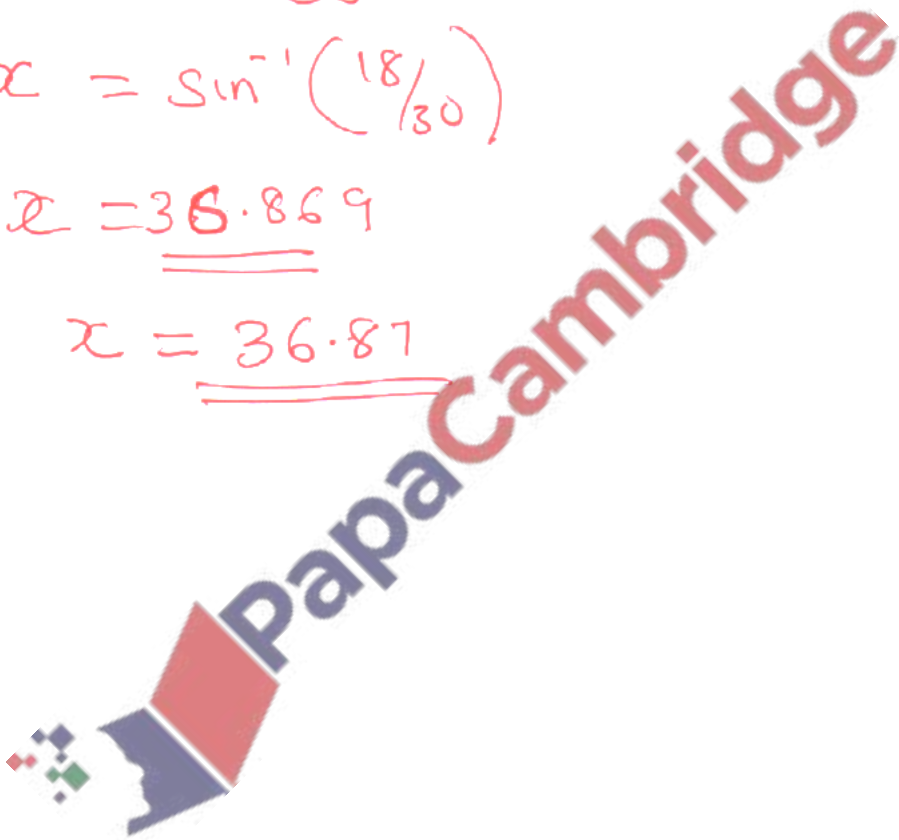
$$\sin x = \frac{18}{30}$$

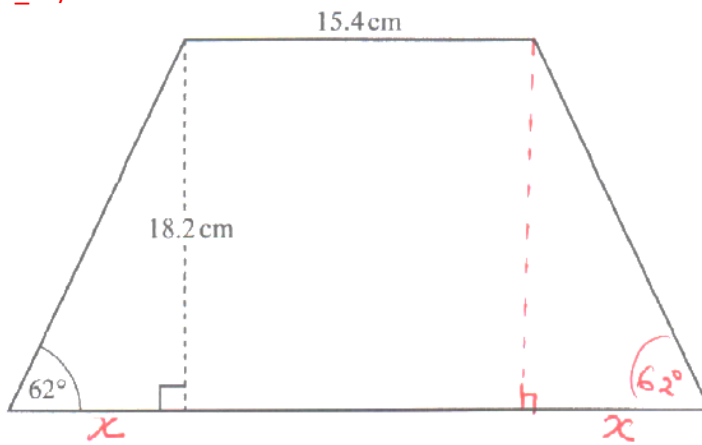
$$x = \sin^{-1}\left(\frac{18}{30}\right)$$

$$x = \underline{\underline{36.869}}$$

$$x = \underline{\underline{36.87}}$$

[2]



NOT TO
SCALE

The diagram shows a trapezium.
The trapezium has one line of symmetry.

Work out the area of the trapezium.

$$\tan 62 = \frac{18.2}{x}$$

$$x \tan 62 = 18.2$$

$$x = \frac{18.2}{\tan 62} = 9.68$$

$$2x = \underline{\underline{19.35}}$$

$$= 19.35 + 15.4 + 15.4$$

$$34.75 + 15.4 = 50.15$$

$$\frac{1}{2} (50.15) \times 18.2 = 456.365$$

$$= \underline{\underline{456.4}}$$

456.4

..... cm² [4]

16. June/2021/Paper_22/No.23

Find all the solutions of $4 \sin x = 3$ for $0^\circ \leq x \leq 360^\circ$.

$$\sin x = \frac{3}{4}$$

$$x = \sin^{-1}\left(\frac{3}{4}\right)$$



$$x = \underline{\underline{48.6^\circ}}$$

$$180 - 48.6^\circ = \underline{\underline{131.4^\circ}} \quad \underline{\underline{48.6^\circ, 131.4^\circ}} \dots [2]$$

17. June/2021/Paper_23/No.23

solveupapers.co.uk

11

A triangle has sides of length 11 cm, 10 cm and 9 cm.



Calculate the largest angle in the triangle.

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab}$$

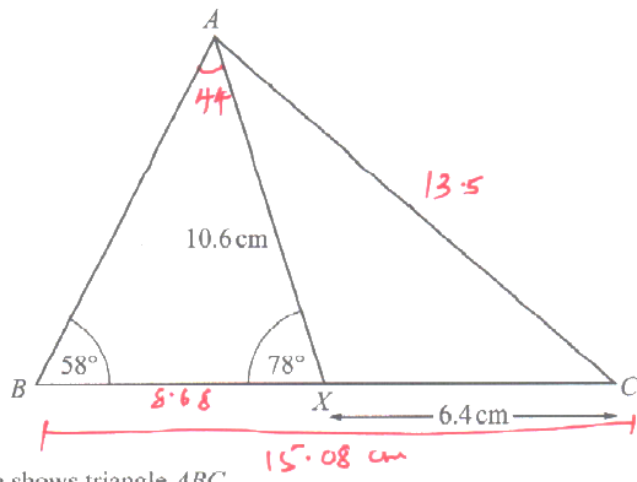
$$\cos c = \frac{10^2 + 9^2 - 11^2}{2 \times 10 \times 9}$$

$$\cos c = \frac{100 + 81 - 121}{180}$$

$$\cos c = \frac{60}{180}$$

$$\cos^{-1} c = \cos^{-1} 0.333 = \underline{\underline{70.528^\circ}}$$

$$\underline{\underline{70.528^\circ}} \dots [4]$$



NOT TO SCALE

The diagram shows triangle ABC .
 X is a point on BC .
 $AX = 10.6$ cm, $XC = 6.4$ cm, angle $ABC = 58^\circ$ and angle $AXB = 78^\circ$.

Angles in straight line add up to 180°
 $\angle AXC = 180 - 78 = 102^\circ$

(a) Calculate AC .

Use cosine rule $AC^2 = AX^2 + XC^2 - 2(AX)(XC) \cos 102^\circ$
 $AC^2 = 10.6^2 + 6.4^2 - 2 \times 10.6 \times 6.4 \cos 102^\circ$
 $AC^2 = 112.36 + 40.96 - 135.68 \cos 102^\circ$
 $AC^2 = 153.32 - (-28.2094)$
 $AC^2 = \sqrt{181.529}$
 $AC = 13.473$
 $AC = 13.5$ cm

$AC = 13.7$ cm [4]

(b) Calculate BX .

$180 - (56 + 78) = 44^\circ$
 $\frac{BX}{\sin 44} = \frac{10.6}{\sin 58}$
 $BX = \frac{10.6 \sin 44}{\sin 58}$
 $BX = 8.682$
 $BX = 8.68$

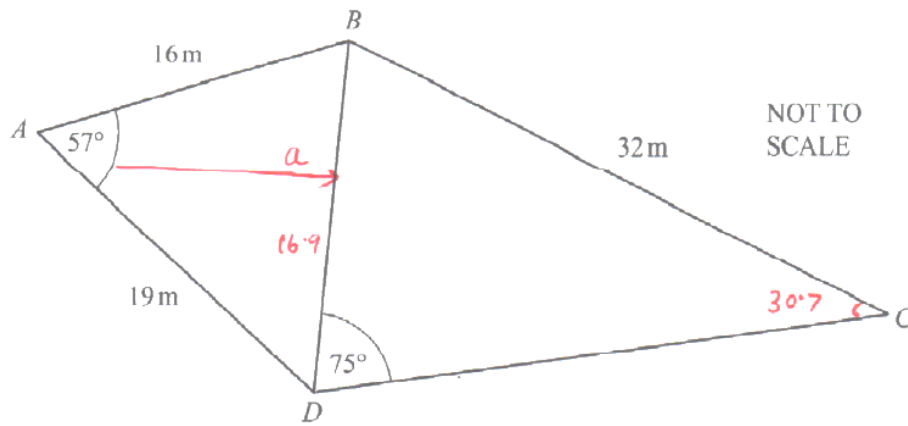
$BX = 8.68$ cm [4]

(c) Calculate the area of triangle ABC .

Area of Triangle = $\frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 13.5 \times 15.08 \sin 50.2^\circ$
 $= 6.75 \times 15.08 \sin 50.2^\circ$
 $= 78.20$ cm²

$\angle ACB = \frac{10.6}{13.5} = \frac{\sin 102}{\sin 102}$
 $= 10.6 \sin 102$
 $= 50.2^\circ$

78.20 cm² [3]



The diagram shows a quadrilateral $ABCD$ made from two triangles, ABD and BCD .

(a) Show that $BD = 16.9$ m, correct to 1 decimal place.

cosine rule.

$$\begin{aligned}
 a^2 &= b^2 + d^2 - 2bd \cos A \\
 a^2 &= 19^2 + 16^2 - 2 \times 19 \times 16 \cos 57^\circ \\
 a^2 &= 361 + 256 - 608 \cos 57^\circ \\
 a^2 &= 617 - 608 \cos 57^\circ \\
 a^2 &= 617 - 331.14053 \\
 a^2 &= \sqrt{285.859} = 16.907 \\
 &= 16.9
 \end{aligned}$$

[3]

(b) Calculate angle CBD .

sine rule.

$$\begin{aligned}
 \frac{\sin 75^\circ}{32} &= \frac{\sin c}{16.9} \\
 \sin c &= \frac{16.9 \sin 75^\circ}{32} \\
 \sin^{-1} c &= 30.67 \\
 &= 30.7
 \end{aligned}$$

Since angles in a triangle add up to 180° .

$$\begin{aligned}
 \angle CBD &= 180^\circ - (75^\circ + 30.7^\circ) \\
 &= 74.3^\circ
 \end{aligned}$$

Angle $CBD = 74.3^\circ$ [4]

(c) Find the area of the quadrilateral $ABCD$.

$$\begin{aligned}
 \text{Area of triangle } ABD &= \frac{1}{2} \times ab \sin C \\
 &= \frac{1}{2} \times 16 \times 19 \sin 57^\circ \\
 &= 127.478 \text{ cm}^2
 \end{aligned}$$

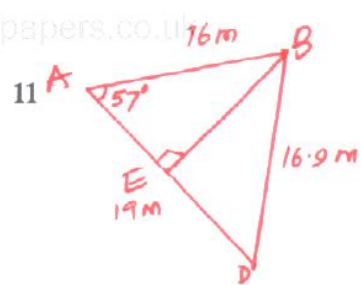
$$\begin{aligned}
 \text{Area of triangle } BCD &= \frac{1}{2} \times 16.9 \times 32 \sin 74.3^\circ \\
 &= 260.31 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Total area of } ABCD & \\
 &= 127.478 + 260.31 \\
 &= 387.788 \\
 &= 388 \text{ m}^2 \\
 &= 388 \text{ m}^2 \text{ [3]}
 \end{aligned}$$

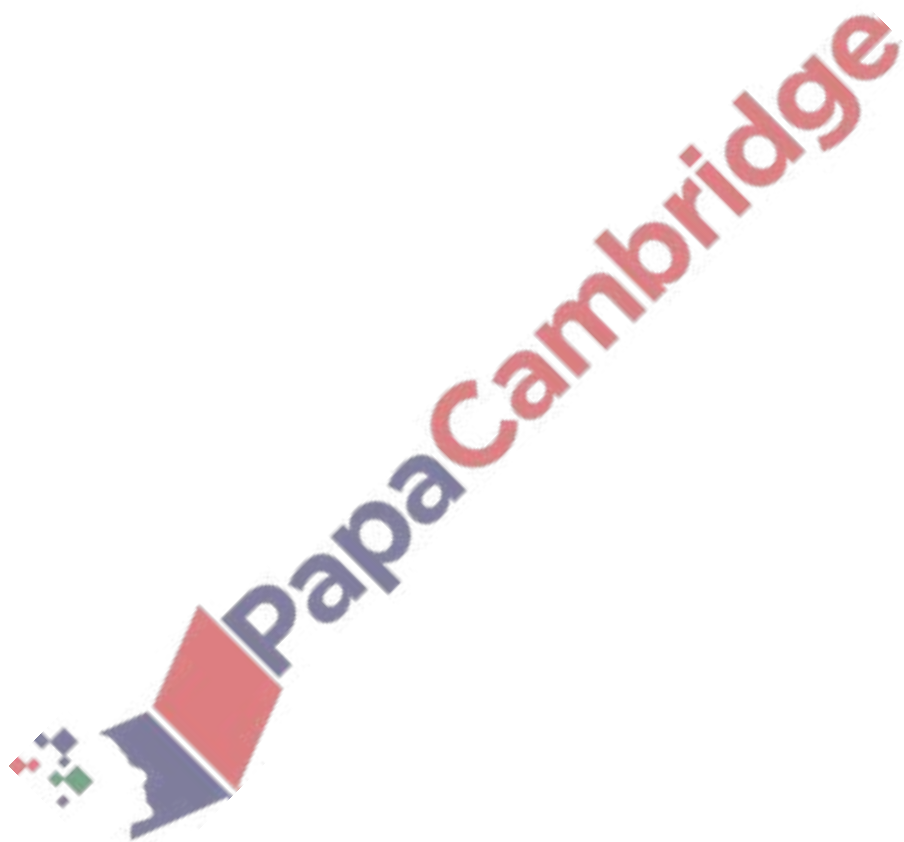
(d) Find the shortest distance from B to AD .

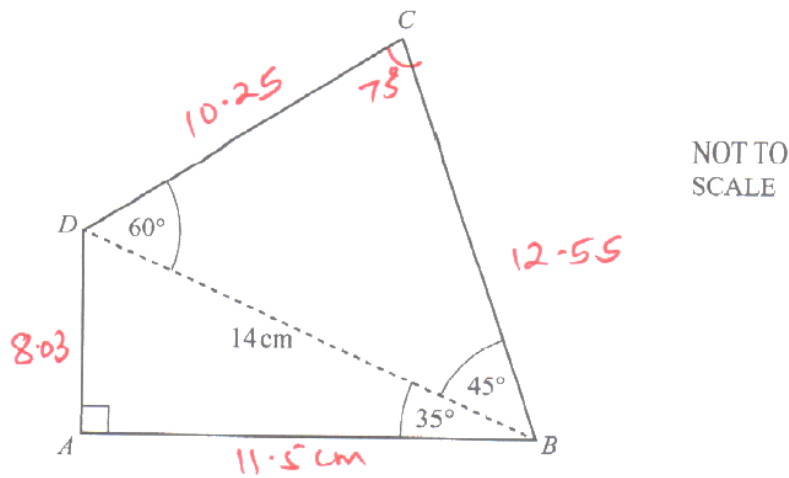
$$16 \times \frac{BE}{16} = \sin 57^\circ \times 16$$

$$BE = 16 \sin 57^\circ \\ = \underline{13.42}$$



$$\underline{13.42} \text{ m [3]}$$





Calculate the perimeter of the quadrilateral $ABCD$.

Length of $AD =$
Using trigonometric ratios $AD =$

$$14 \times \sin 35^\circ = \frac{AD \times 14}{14 \text{ cm}}$$

$$AD = 14 \sin 35$$

$$AD = \underline{8.03}$$

$$AB = 14 \times \cos 35^\circ = \frac{AB \times 14}{14 \text{ cm}}$$

$$AB = 11.468$$

$$= \underline{11.5 \text{ cm}}$$

Since angles in a triangle
sum up to 180° .

$$\begin{aligned} \angle DCB &= 180^\circ - (60 + 45) \\ &= 180^\circ - 105 \\ &= \underline{75^\circ} \end{aligned}$$

Using sine rule length
of $DC =$

$$\frac{14 \text{ cm}}{\sin 75^\circ} = \frac{CD}{\sin 45^\circ} \quad (\text{Cross multiply})$$

$$\frac{14 \sin 45}{\sin 75} = \frac{CD \sin 75}{\sin 75}$$

$$CD = \underline{10.25 \text{ cm}}$$

$$\frac{\sin 60^\circ}{BC} = \frac{\sin 45^\circ}{10.25}$$

$$BC = \frac{10.25 \sin 60}{\sin 45}$$

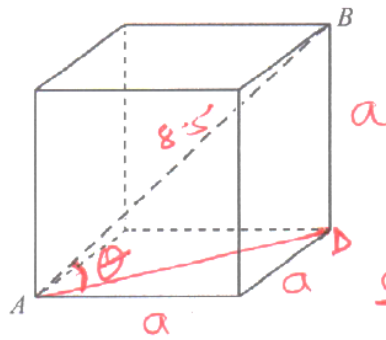
$$BC = \underline{12.55 \text{ cm}}$$

$$42.33$$

..... cm [7]

$$\begin{aligned} \text{Perimeter} &= 8.03 + 11.5 + 10.25 \\ &\quad + 12.55 \\ &= \underline{42.33 \text{ cm}} \end{aligned}$$

(b)



NOT TO SCALE

Since it is a cube all sides are equal.

The diagram shows a cube.
The length of the diagonal AB is 8.5 cm.

(i) Calculate the length of an edge of the cube.

$$A^2 = a^2 + a^2$$

$$= \underline{\underline{2a^2}}$$

$$AD^2 + BD^2 = AB^2$$

$$2a^2 + a^2 = 8.5^2$$

$$3a^2 = 8.5^2$$

$$\frac{3a^2}{3} = \frac{72.25}{3}$$

$$a^2 = \sqrt{24.083}$$

$$a = 4.9074$$

$$= \underline{\underline{4.91 \text{ cm}}}$$

4.91

..... cm [3]

(ii) Calculate the angle between AB and the base of the cube.



$$\sin \theta = \frac{4.91}{8.5}$$

$$\sin^{-1} \theta = 0.5776$$

$$\approx 35.28$$

$$= \underline{\underline{35.3}}$$

35.3°

..... [3]