Algebra and graphs – 2022 IGCSE 0580

1. June/2022/Paper-11/No.16

$v = 3 - 5t$

(a) Work out the value of v when $t = 4$.

$V = 3 - 5t$

Substitute $t = 4$

$V = 3 - 5(4)$

$V = 3 - 20$

$V = -17$

$v =$ .... -17 .... [1]

(b) Make t the subject of the formula.

$V = 3 - 5t$

$\frac{\cancel{5}t}{\cancel{5}} = \frac{3 - V}{5}$

$t = \frac{3 - V}{5}$

$t =$ .... $\frac{3 - V}{5}$ .... [2]

2. June/2022/Paper-11/No.18

Factorise completely.

$14xy - 7y^2$

$14xy - 7y^2$

Common factor = $7y$

$7y(2x - y)$

.... $7y(2x - y)$ .... [2]

3. June/2022/Paper-12/No.12

(a) The total cost of n bags of flour is $\$d$.

Write down an expression for the cost of one bag of flour.

One bag $= \frac{d}{n}$

$ $\frac{d}{n}$ .... [1]

(b) A bag of rice costs $\$r$ and a bag of almonds costs $\$a$.
Pedro buys x bags of rice and y bags of almonds.

Write down an expression for the change that Pedro receives from a $20 note.

Rice $= r$

Almonds $= a$

Pedro $= (x \times r) + (y \times a)$

$xr + ay$

Change $= 20 - xr + ay$

$ $20 - xr + ay$ .... [2]

4. June/2022/Paper-12/No.15

The nth term of a sequence is $n^2 + 12$.

(a) Find the first three terms of this sequence.

When $n = 1$ $n^2 + 12 = 1^2 + 12 = 13$

When $n = 2$ $n^2 + 12 = 2^2 + 12 = 16$

When $n = 3$ $n^2 + 12 = 3^2 + 12 = 21$

13 ...., 16 ...., 21 .... [2]

(b) Is 5196 a term in this sequence?
Give a reason for your decision.

$n^2 + 12 = 5196$

$n^2 = 5196 - 12$

$n^2 = 5184$

$n = \sqrt{5184} = 72$

Yes .... because n is a Positive Integer. ....

.... [2]

5. June/2022/Paper-12/No.20

(a) Simplify.

$$3(2a-b)-b$$

$3(2a-b)-b$

$6a-3b-b$

$\underline{6a-4b}$

$6a-4b$ [2]

(b) Factorise.

$$x^2-8xy$$

x is common factor.

x^2-8xy

$\underline{x(x-8y)}$

$x(x-8y)$ [1]

6. June/2022/Paper-13/No.3

Simplify.

$$3x-4x+7x$$

$3x+7x-4x$

$10x-4x$

$6x$

$6x$ [1]

7. June/2022/Paper-13/No.12

Simplify.

(a) $y^3 \div y^5$

$y^3 \div y^5 = y^{3-5}$

$= \underline{y^{-2}}$

y^{-2} [1]

(b) $7x^0$

Any value raised to power of zero is always 1.

$x^0=1$ so, $7x^0 = \underline{7}$

7 [1]

8. June/2022/Paper-13/No.15

Factorise completely.

$$18px-27p$$

Common factor $= 9p$

$18px-27p$

$\underline{9p(2x-3)}$

$9p(2x-3)$ [2]

9. June/2022/Paper-13/No.16

The nth term of a sequence is $n^2 - 1$.

Find the first three terms of this sequence.

Substitute $n = 1$

$n^2 - 1 = 1^2 - 1$

$= \underline{\underline{0}}$

$n = 2 \quad n^2 - 1$

$2^2 - 1$

$4 - 1 = \underline{\underline{3}}$

$n^2 - 1$

$3^2 - 1$

$9 - 1$

$= \underline{\underline{8}}$

.......0......., ...3......., ...8....... [2]

10. June/2022/Paper-13/No.19

Find the lowest common multiple (LCM) of 32 and 40.

2	32	40
2	16	20
2	8	10
2	4	5
2	2	5
5	1	5
	1	1

$L.C.M = 2 \times 2 \times 2 \times 2 \times 2 \times 5$

$= \underline{\underline{160}}$

.......160....... [2]

11. June/2022/Paper-21/No.8(b)

(b) Rearrange the formula to find t in terms of s and a.

$2 \times S = \frac{1}{2} at^2 \times 2$

$\frac{2S}{a} = \frac{at^2}{a}$

$t^2 = \frac{2S}{a}$

$t = \underline{\underline{\sqrt{\frac{2S}{a}}}}$

$t =$ ....$\sqrt{\frac{2S}{a}}$....... [2]

12. June/2022/Paper-21/No.9

Factorise completely.

$14xy - 7y^2$

$14xy - 7y^2$

$\underline{\underline{7y(2x - y)}}$

.......$7y(2x-y)$....... [2]

13. June/2022/Paper-21/No.15

$$4^x = \frac{1}{64}$$

Find the value of x.

$4^x = \frac{1}{64}$

$4^x = 64^{-1}$

$4^x = 4^{3(-1)}$

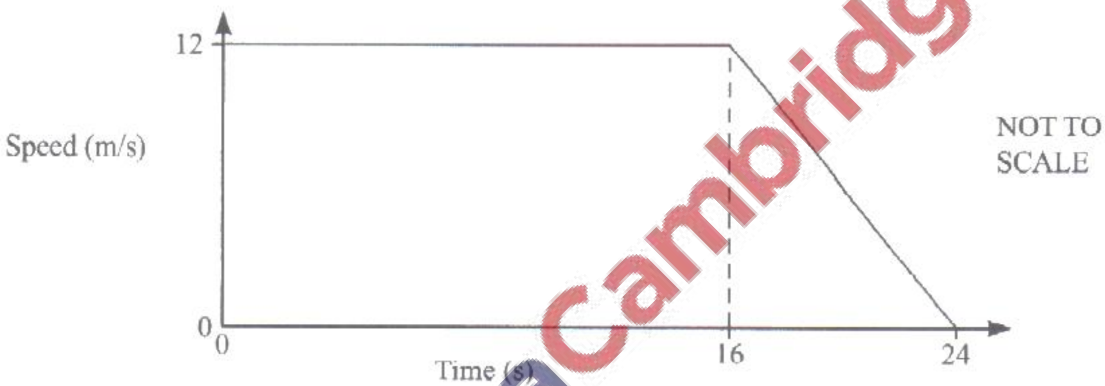
$4^x = 4^{-3}$

$x = -3$

(Make the Indices to the same base and simplify.)

$x =$ .... -3 .... [1]

14. June/2022/Paper-21/No.20



The diagram shows the speed–time graph for 24 seconds of a car journey.

Calculate

(a) the deceleration of the car in the final 8 seconds,

$a = \frac{(12-0)}{(16-24)} = \frac{12}{-8}$

$= -1.5 \text{ m/s}^2$

For deceleration, we Ignore the sign.

.... 1.5 .... m/s² [1]

(b) the total distance travelled during the 24 seconds.

Distance $= \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}(24+16) \times 12$

$= \frac{1}{2} \times 40 \times 12$

$= 240 \text{ m}$

.... 240 .... m [2]

15. June/2022/Paper-21/No.21

Factorise completely.

$$1 - q - a + aq$$

$1-q-a+aq$

$1-a-q+aq$

$1(1-a)-q(1-a)$

$(1-q)(1-a)$

$(1-q)(1-a)$ [2]

16. June/2022/Paper-21/No.22

Simplify fully $(216y^{216})^{\frac{2}{3}}$.

$(216y^{216})^{2/3}$

$(y^{216})^{2/3} = y^{216\times 2/3}$

$= y^{144}$

obtain cube root of 216 then square

$(\sqrt[3]{216})^2 = 6^2$

$= 36$

$36y^{144}$

$36y^{144}$ [2]

17. June/2022/Paper-21/No.23

$$x^2 + 8x + 10 = (x + p)^2 + q$$

(a) Find the value of p and the value of q.

$x^2+8x+10 = x^2+2px+p^2+q$

$\frac{2p}{2} = \frac{8}{2}$

$p = 4$

$p^2+q = 10$

$4^2+q=10$

$16+q=10$

$q=10-16$

$q=-6$

$p =$ 4

$q =$ −6 [2]

(b) Solve.

$$x^2 + 8x + 10 = 30$$

$x^2+8x+10-30=0$

$x^2+8x-20=0$

$P=-20$

$S=8$

$(+10,-2)$

$x^2+10x-2x-20=0$

$x(x+10)-2(x+10)=0$

$(x+10)(x-2)=0$

$x=-10$ $x=2$

$x =$ −10 or $x =$ 2 [2]

18. June/2022/Paper-21/No.27

The line $y = x + 1$ intersects the graph of $y = x^2 - 3x - 11$ at the points A and B.

Find the coordinates of A and the coordinates of B.
You must show all your working.

Since $y = x+1$ and $y = x^2-3x-11$

Equate both functions

$x+1 = x^2 - 3x - 11$

$x^2 - 3x - x - 11 - 11 = 0$

$x^2 - 4x - 12 = 0$

Product $= -12$ $(-6, 2)$

Sum $= -4$

$x^2 - 6x + 2x - 12 = 0$

$x(\cancel{x-6}) + 2(\cancel{x-6}) = 0$

$x - 6 = 0$ | $x + 2 = 0$

$x = 6$ | $x = -2$

Substitute $x =$ when $x = 6$, | when $x = -2$

$y = x + 1$ | $y = x + 1$

$y = 6 + 1$ | $y = -2 + 1$

$y = 7$ | $y = -1$

A (-2 , -1)

B (6 , 7) [4]

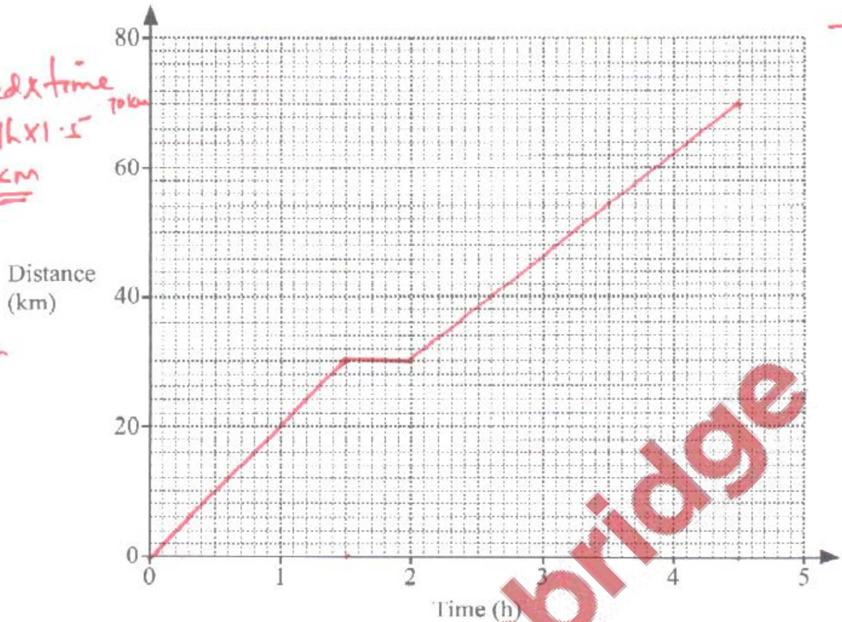
19. June/2022/Paper-22/No.12

Annette cycles a distance of 70 km from Midville to Newtown.
Leaving Midville, she cycles for 1 hour 30 minutes at a constant speed of 20 km/h and then stops for 30 minutes.
She then continues the journey to Newtown at a constant speed of 16 km/h.

Distance = speed × time
(d_1) = 20 km/h × 1.5
= 30 km

Remaining distance
= (70 − 30) km
= 40 km

Time d_2 = D/S
= 40 km / 16 km/hr
= 2 hr 30 min



(a) On the grid, draw the distance–time graph for the journey. [3]

(b) Calculate the average speed for the whole journey.

Total distance covered = 70 km
Total time taken = (1 hr 30 mins + 2 hr. 30 mins + 30 mins)
= 4 hrs 30 mins (4.5 hrs)

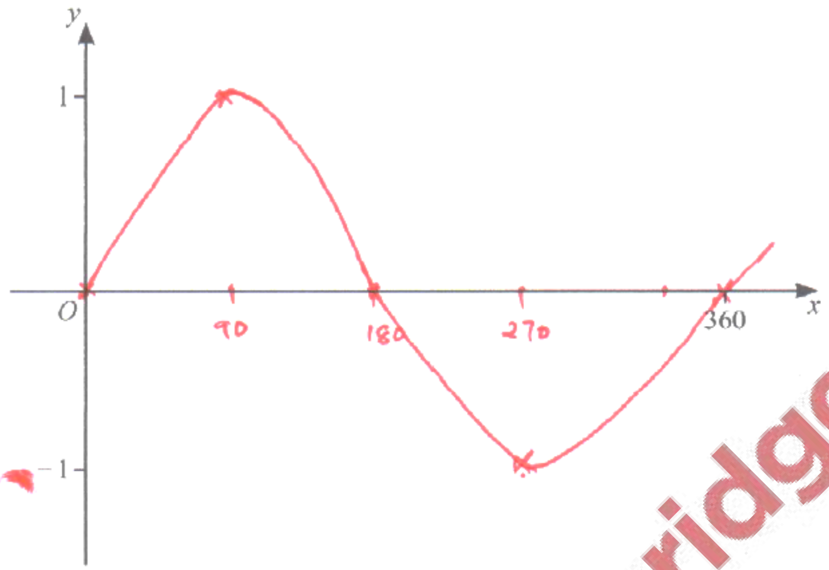
Average speed = Total distance / Total time taken
= 70 km / 4.5 hrs
= 15.55 ≈ 15.6 km/hr

........15.6........ km/h [3]

20. June/2022/Paper-22/No.17

x	0	90	180	270	360
$\sin x$	0	1	0	-1	0

(a) Sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.



[2]

(b) Solve the equation $3\sin x + 1 = 0$ for $0° \leqslant x \leqslant 360°$.

$3\sin x + 1 = 0$

$\frac{\cancel{3}\sin x}{\cancel{3}} = \frac{-1}{3}$

$\sin x = -\frac{1}{3}$

$x = \sin^{-1}\left(-\frac{1}{3}\right) = -19.47$

$= -19.5°$

$180° + 19.5° = 199.5°$

$360° - 19.5° = 340.5°$

$x =$ 199.5° or $x =$ 340.5° [3]

21. June/2022/Paper-22/No.20

Factorise completely.

(a) $2m + 3p - 8km - 12kp$

$2m - 8km + 3p - 12kp$

$2m(1 - 4k) + 3p(1 - 4k)$

$(2m + 3p)(1 - 4k)$

$(2m+3p)(1-4k)$ .... [2]

(b) $5x^2 - 20y^2$

$5[x^2 - 4y^2]$

$x^2 - 4y^2 \rightarrow$ difference of two squares

$5[(x-2y)(x+2y)]$

$= 5[(x-2y)(x+2y)]$

$5[(x-2y)(x+2y)]$ .... [3]

22. June/2022/Paper-23/No.13

Factorise completely.

(a) $18px - 27p$

Common factor; $9p$

$18px - 27p$

$9p(2x - 3)$

$9p(2x-3)$ .... [2]

(b) $mt - n - m + nt$

$mt - m - n + nt$

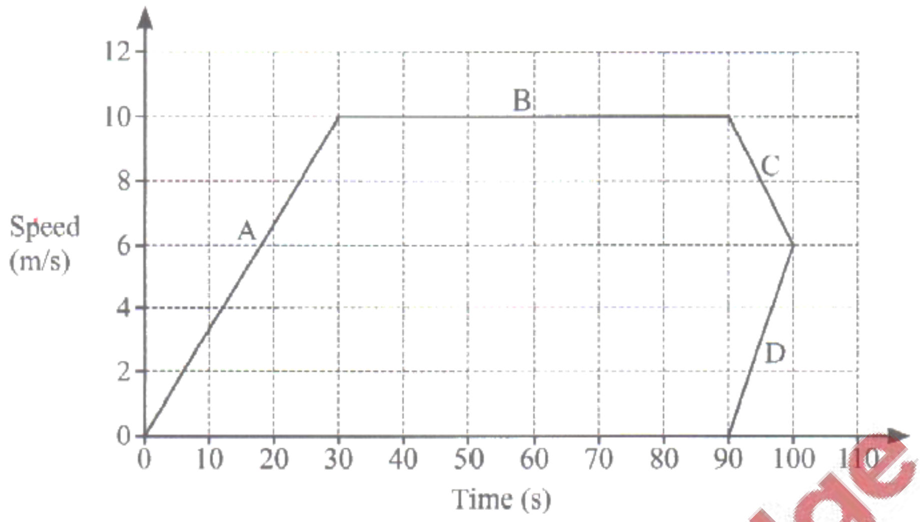
$m(t-1) + n(t-1)$

$(m+n)(t-1)$

$(t-1)(m+n)$ .... [2]

23. June/2022/Paper-23/No.16

Abdul draws this speed–time graph for a journey.
The graph has four sections A, B, C and D.



Complete these statements about the speed–time graph.

Section ....D....... cannot be correct.

Section ....B....... shows constant speed.

Section ....C....... shows deceleration.

Section A shows acceleration of ......$0.33\dot{3}$...... m/s^2.

$v = 10$ m/s
$u = 0$ m/s

$a = \frac{v-u}{t} = \frac{10-0}{30} = \underline{0.33\dot{3} \text{ m/s}^2}$

The distance travelled in the first 30 seconds of the journey is ......150...... m. [4]

Distance travelled $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 10$

$= \underline{150\text{m}}$

24. June/2022/Paper-23/No.22

Simplify.

$$\frac{5x-x^2}{25-x^2}$$

$$\frac{5x-x^2}{25-x^2} = \frac{x(\cancel{5-x})}{(\cancel{5-x})(5+x)} = \frac{x}{5+x}$$

$25-x^2 =$ difference of two squares.

$25-x^2 = (5-x)(5+x)$

$\frac{x}{5+x}$ .... [3]

25. June/2022/Paper-23/No.25

$$m^{-\frac{1}{4}} = 27m^{-1}$$

Find the value of m.

$m^{-1/4} = 27m^{-1}$

$\frac{m^{-1/4}}{m^{-1}} = 27$

$m^{-1/4-(-1)} = 27$

$m^{-1/4+1} = 27$

$m^{3/4} = 27$

$m = (27)^{4/3}$

$m = (\sqrt[3]{27})^4$

$m = (3)^4$

$m = \underline{81}$

$m =$ ......81...................................... [3]

26. June/2022/Paper_31/No.6

(a) A football team has w wins and d draws.
The team scores 3 points for each win and 1 point for each draw.

Write an expression, in terms of w and d, for the total number of points scored by the team.

Let win $= w$

$(w \times 3) + d = 3w + d$

$3w + d$ .... [2]

(b) Athletic, Rovers and United are three football teams.

Athletic have a point score of x.
Rovers have 12 points more than Athletic's point score. $(x+12)$
United have 3 points fewer than twice Athletic's point score. $(2x-3)$

The total point score of all three teams is 121.

Use this information to write down an equation in terms of x.
Solve your equation to work out the point score for each team.

Rovers $= x+12$

United $= (2x-3)$

Athletic $= x$

$x + (x+12) + (2x-3) = 121$

$4x + 9 = 121$

$4x = 121 - 9$

$\frac{4x}{4} = \frac{112}{4}$

$x = 28$

Rovers $= 28 + 12$
$= 40$

United $= 2(28) - 3$
$= 56 - 3$
$= 53$

Athletic ....28.......... points

Rovers ....40.......... points

United ....53.......... points [5]

(c) Simplify.

(i) $4a-3b+5a+6b$

$4a+5a+6b-3b$

$9a+3b$

$9a+3b$ .... [2]

(ii) $6(2x+1)-5(x-2)$

$6(2x+1)-5(x-2)$

$12x+6-5x+10$

$12x-5x+6+10$

$7x+16$

$7x+16$ .... [2]

(d) Solve the simultaneous equations.
You must show all your working.

$$3x+5y=11$$
$$2x-3y=20$$

$3x+5y=11 \times 3$ (i)

$2x-3y=20 \times 5$ (ii)

Using elimination Multiply equation (i) by 3 and (ii) by 5

$9x+15y=33$

$10x-15y=100$

$\frac{19x}{19} = \frac{133}{19}$

$x=7$

Substitute $x=7$

$3x+5y=11$

$3(7)+5y=11$

$21+5y=11$

$5y=11-21$

$\frac{5y}{5}=\frac{-10}{5}$

$y=-2$

$x =$ ....7....

$y =$ ....-2.... [4]

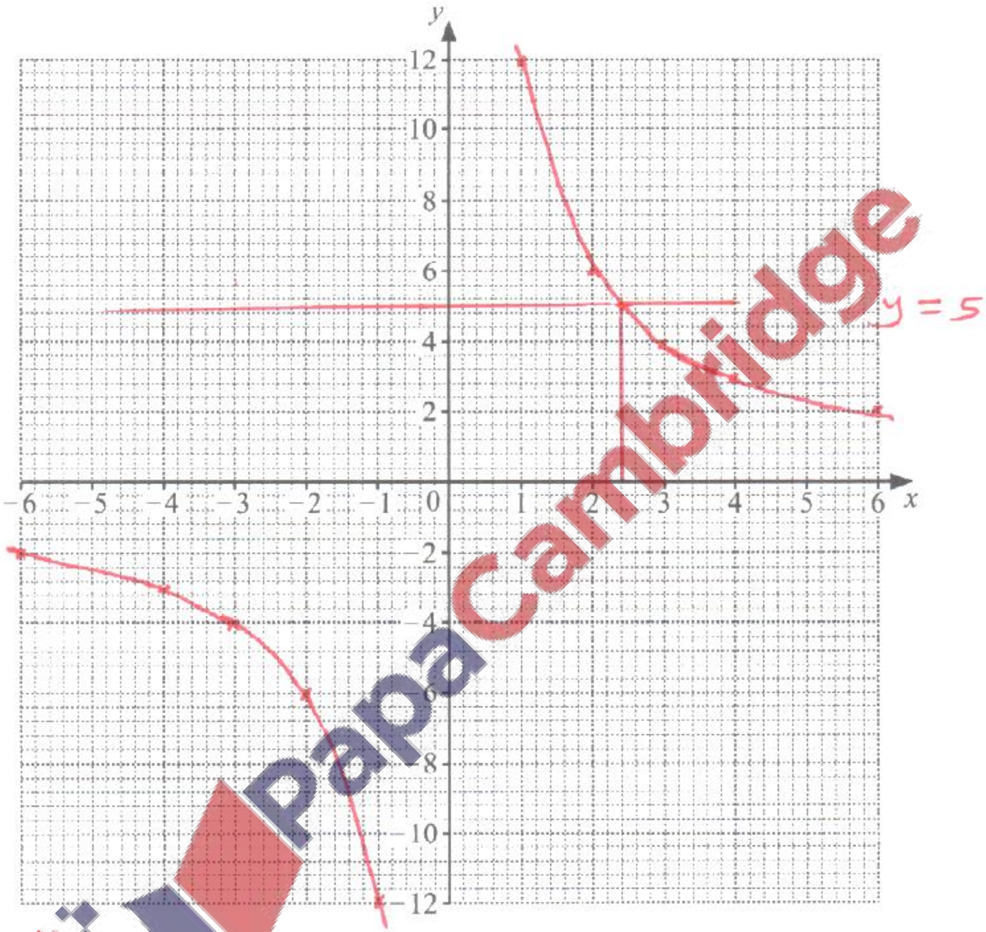
27. June/2022/Paper_31/No.9

(a) Complete the table of values for $y = \frac{12}{x}$, $x \neq 0$.

x	−6	−4	−3	−2	−1		1	2	3	4	6
y	−2	−3	−4	−6	−12		12	6	4	3	2

[3]

(b) On the grid, draw the graph of $y = \frac{12}{x}$ for $-6 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 6$.



[4]

(c) On the grid, draw the line $y = 5$. [1]

(d) Use your graph to solve the equation $\frac{12}{x} = 5$.

$y = \frac{12}{x}$

$\frac{12}{x} = 5$ $\quad x = 2.4$

$x =$ 2.4 [1]

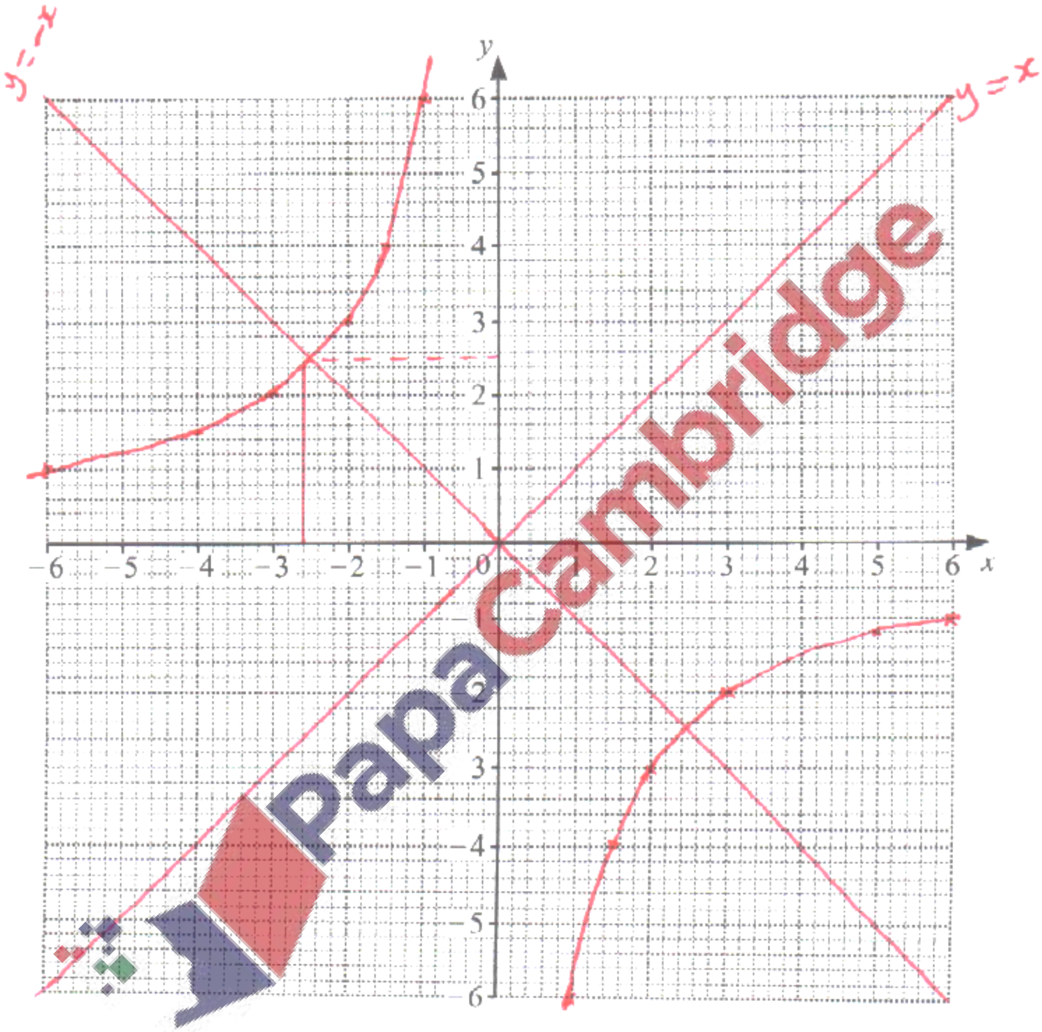
28. June/2022/Paper_32/No.2

(a) (i) Complete the table of values for $y = \frac{-6}{x}$.

x	−6	−4	−3	−2	−1.5	−1		1	1.5	2	3	5	6
y	1	1.5	2	3	4	6		−6	−4	−3	−2	−1.2	−1

[3]

(ii) On the grid, draw the graph of $y = \frac{-6}{x}$ for $-6 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 6$.



[4]

(iii) Write down the order of rotational symmetry of the graph.

.......2....... [1]

(iv) Write down the equation of each line of symmetry of the graph.

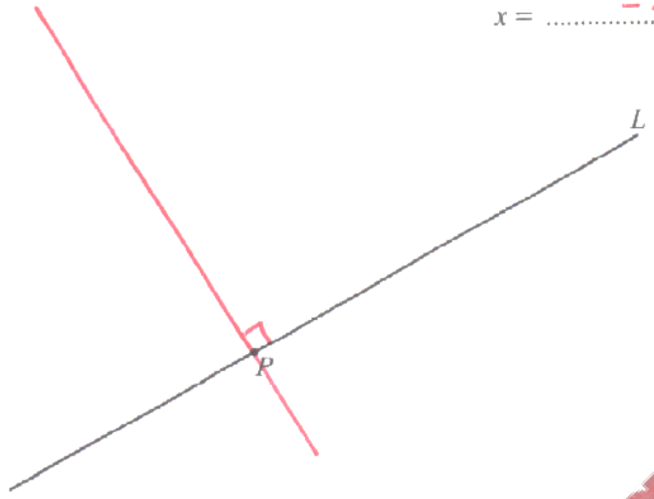
.......$y = -x$....... and ....$y = x$....... [2]

(v) On the grid, draw the line $y = 2.5$. [1]

(vi) Use your graph to solve the equation $\frac{-6}{x} = 2.5$.

$x =$ ......-2.6...... [1]

(b)



Draw a line that passes through the point P and is perpendicular to line L. [1]

(c) Find the equation of the straight line that

- is parallel to the line $y = 3x + 5$

and

- passes through the point $(1, 7)$.

Give your answer in the form $y = mx + c$.

For Parallel Lines gradients are same.

$y = 3x + 5$

gradient $= 3$

$y = mx + c$

$7 = 3(1) + c$

$7 = 3 + c$

$c = 7 - 3$

$c = \underline{4}$

$y = \underline{3x + 4}$

$y =$ ......$3x + 4$...... [2]

29. June/2022/Paper_32/No.3b(iii)

(iii) One of the interior angles of this quadrilateral is 70°.

Work out the other three interior angles.

Angles in quadrilateral sum up to 360°.

$360° - (70 + 70)$

$360 - 140$

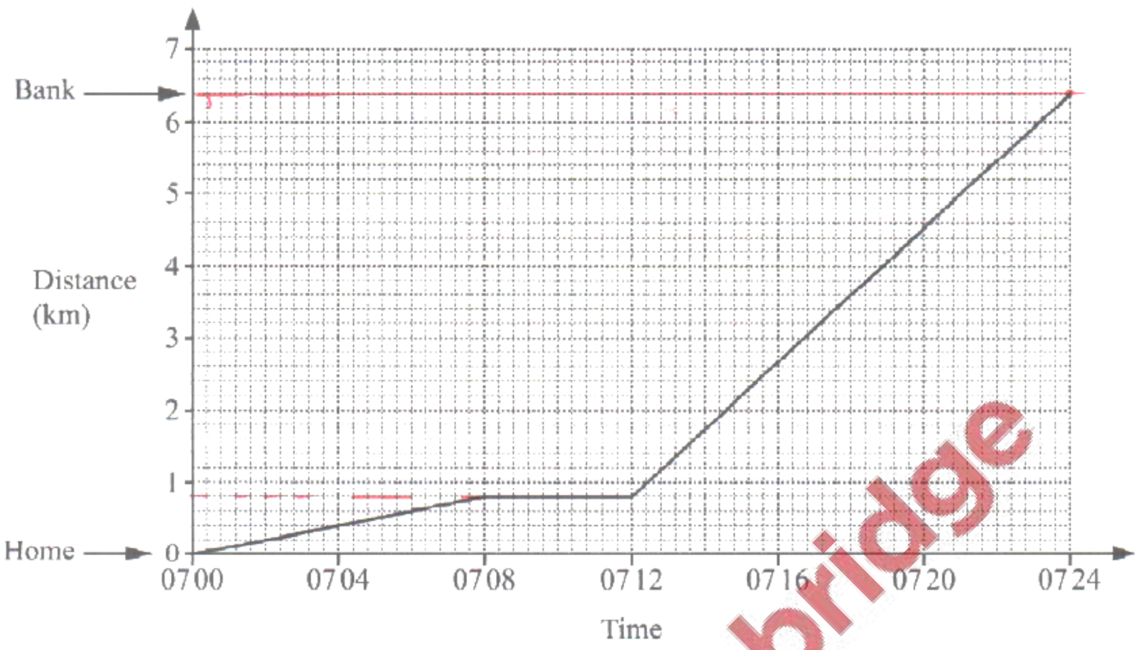
$= \frac{220°}{2} = \underline{\underline{110°}}$ (opposite angles in quadrilateral) are equal.

70° ...., 110 ...., 110 .... [2]

30. June/2022/Paper_32/No.6

Mr Vay works in a bank.

(a) The travel graph shows Mr Vay's journey from his home to the bank.



(i) Write down the distance Mr Vay travels in the first 8 minutes.

0·8 .... km [1]

(ii) Explain what is happening between 07 08 and 07 12.

He stops (He rests) .... [1]

(iii) Between which times is Mr Vay's journey the fastest?
Give a reason for your answer.

Between 07 12 .... and 07 24 ....

Reason The slope of the graph is the steepest in this time range. .... [2]

(iv) Work out Mr Vay's average speed for the whole journey.
Give your answer in kilometres per hour.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$= \frac{6.4\text{ km}}{\left(\frac{24\text{ mins}}{60}\right)}$$

$$= \frac{6.4}{0.4} = \underline{16\text{ km/hr}}$$

16 .... km/h [3]

(b) Katya takes some coins to the bank.
The table shows the number of each type of coin.

Type of coin	Number of coins
1 cent	12
5 cent	23
10 cent	17
25 cent	9
50 cent	7
1 dollar	24

$\boxed{100 \text{ cents} = 1 \text{ dollar}}$

Work out the total amount of money Katya takes to the bank.
Give your answer in dollars.

$\text{Total} = (1\times12) + (5\times23) + (10\times17) + (25\times9) + (7\times50) + (100\times24)$

$= 12 + 115 + 170 + 225 + 350 + 2400$

$= 3272$ cents

To convert to dollars $= \frac{3272}{100} = 32.72$

\$ 32.72 .... [2]

(c) Adam changes \$700 into euros at the bank.
The exchange rate is \$1 = 0.904 euros.

Work out the amount Adam receives.

$\$1 = 0.904$ $\quad 700 \times 0.904$

$\$700 = ?$ $\quad = 632.8$

632.8 .... euros [1]

(d) Clara invests \$8500 for 4 years at a rate of 1.7% per year simple interest.

Calculate the total interest earned during the 4 years.

$I = \frac{P \times R \times T}{100}$

$I = 8500 \times \frac{1.7}{100} \times 4$

$I = 578$

\$ 578 .... [2]

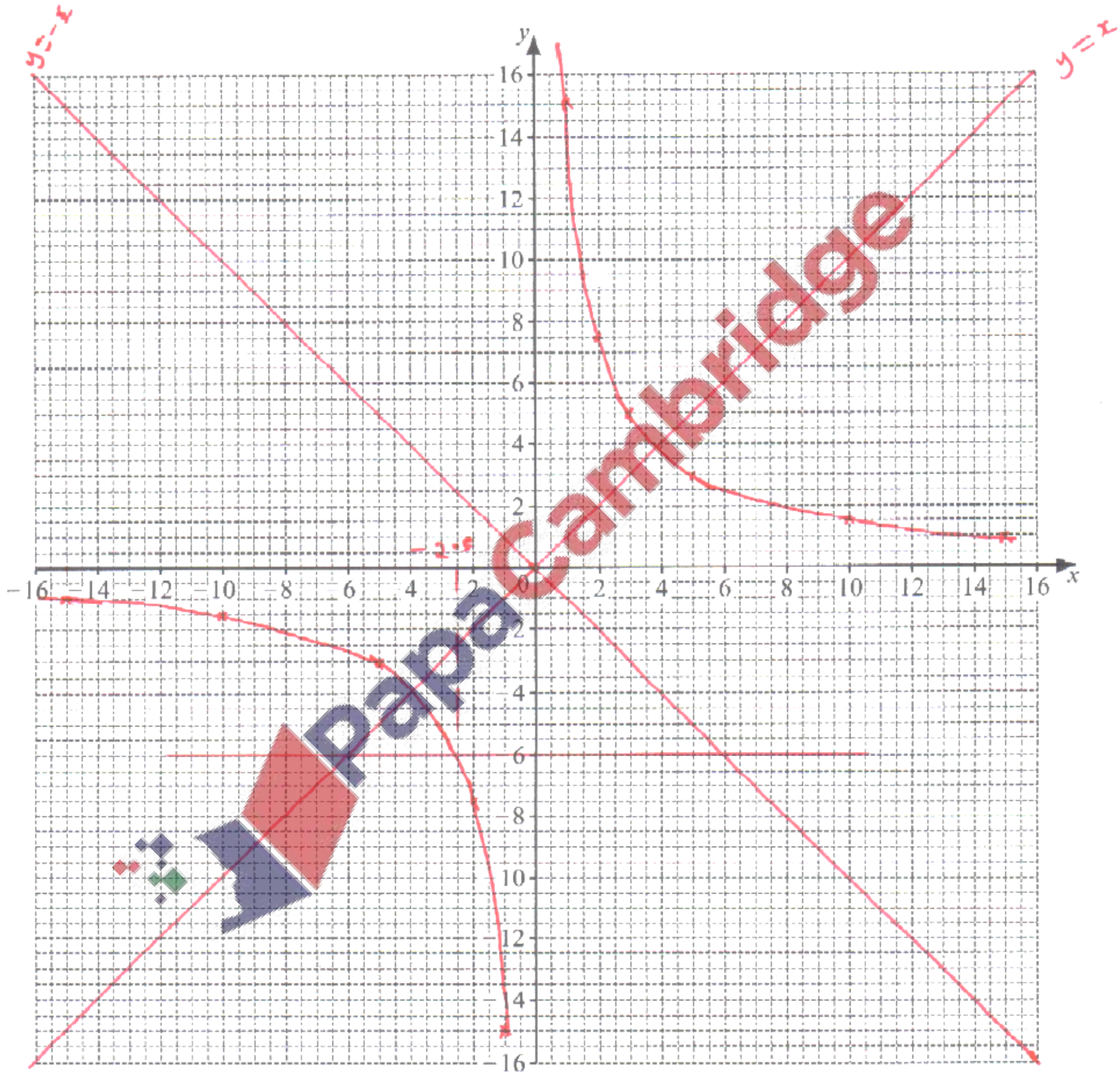
31. June/2022/Paper_33/No.7

(a) Complete the table of values for $y = \frac{15}{x}, x \neq 0$.

x	−15	−10	−5	−3	−2	−1	1	2	3	5	10	15
y	−1	−1.5	−3	−5	−7.5	−15	15	7.5	5	3	1.5	1

[3]

(b) On the grid, draw the graph of $y = \frac{15}{x}$ for $-15 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 15$.



[4]

(c) Write down the order of rotational symmetry of the graph.

2 [1]

(d) **(i)** On the grid, draw the lines of symmetry of the graph. [2]

(ii) Write down the equation of the line of symmetry that does **not** intersect the graph.

$y = -x$ [1]

(e) Use your graph to solve the equation $\frac{15}{x} = -6$.

$x =$ -2.5 [1]

32. June/2022/Paper_41/No.4

$f(x) = 2x - 1$ $\quad$ $g(x) = 3x - 2$ $\quad$ $h(x) = \frac{1}{x}, \ x \neq 0$ $\quad$ $j(x) = 5^x$

(a) Find

(i) $f(2)$,

Substitute $x = 2$

$f(2) = 2(2) - 1$
$= 4 - 1$
$= 3$

.... 3 .... [1]

(ii) $gf(2)$.

$f(2) = 3$
$g(3) = 3(3) - 2$
$= 9 - 2$
$= 7$

.... 7 .... [1]

(b) Find $g^{-1}(x)$.

$g(x) = 3x - 2$
$y = 3x - 2$
$x = 3y - 2$
$\frac{x + 2}{3} = \frac{3y}{3}$ $\quad$ $y = \frac{x+2}{3}$

$g^{-1}(x) = \frac{x+2}{3}$

$g^{-1}(x) =$ .... $\frac{x+2}{3}$ .... [2]

(c) Find x when $h(x) = j(-2)$.

$h(x) = j(-2)$
$\frac{1}{x} = 5^{-2}$
$\frac{1}{x} = \frac{1}{5^2}$

$\frac{1}{x} = \frac{1}{25}$ (cross multiply)

$x = 25$

$x =$ .... 25 .... [2]

(d) Write $f(x) - h(x)$ as a single fraction.

$f(x) = 2x - 1$
$h(x) = \frac{1}{x}$

$2x - 1 - \frac{1}{x}$

$\frac{x(2x-1) - 1}{x}$

$= \frac{2x^2 - x - 1}{x}$

.... $\frac{2x^2 - x - 1}{x}$ .... [2]

(e) Find the value of $jj(2)$.

$j(x) = 5^x$

$j(2) = 5^2$

$= 25$

$j(25) = 5^{25}$

$\approx 2.98 \times 10^{17}$

2.98×10^{17} .... [1]

(f) Find x when $j^{-1}(x) = 4$.

$j(x) = 5^x$

$x = 5^y$

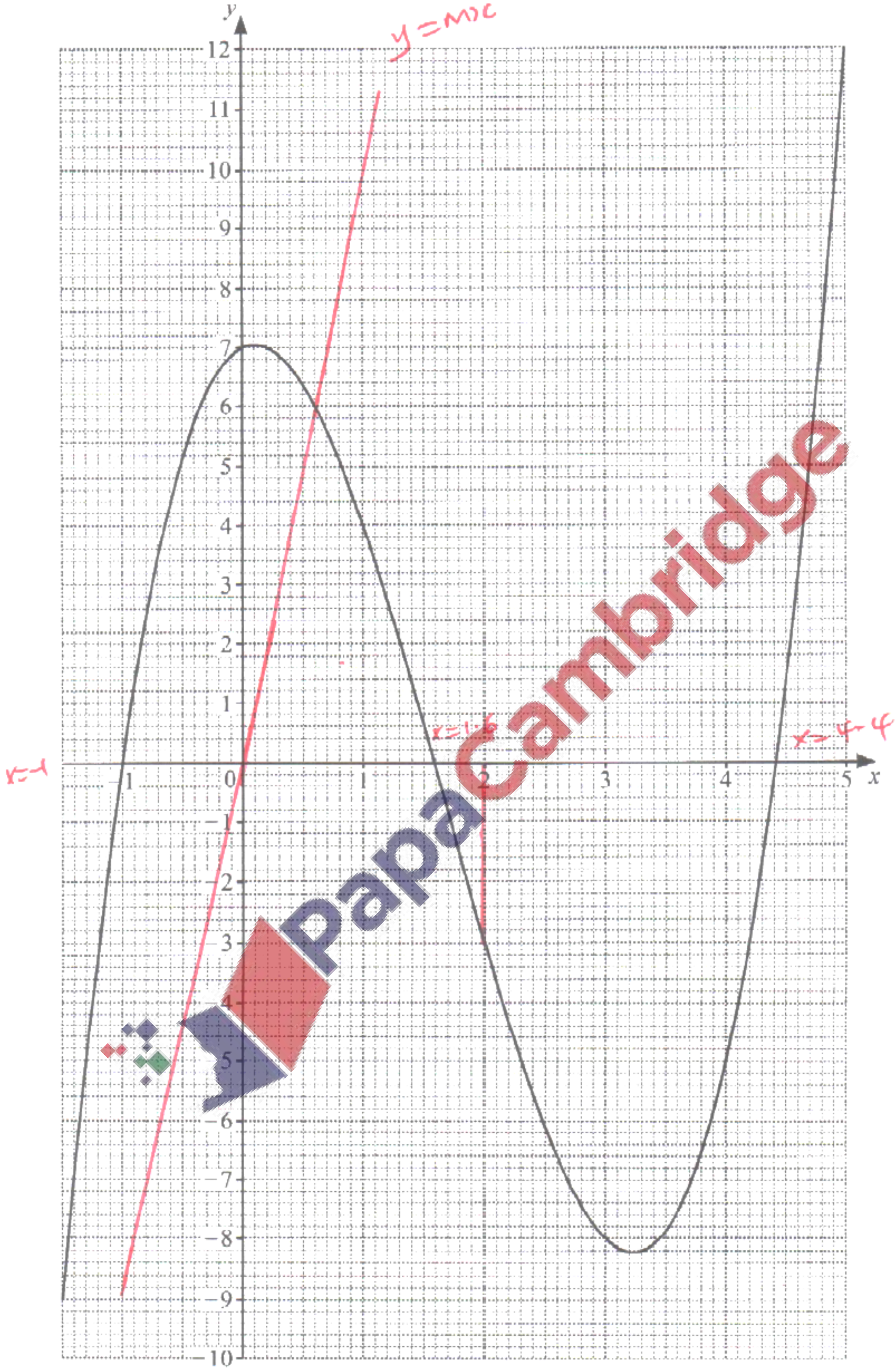
$x = 5^4$

$x = 625$

$x =$ .... 625 .... [2]

33. June/2022/Paper_41/No.6

(a)



The diagram shows the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 5$.

(i) Find f(2).

.... -3 .... [1]

(ii) Solve the equation $f(x) = 0$ for $-1.5 \leqslant x \leqslant 5$.

$x =$ ..... -1 ..... or $x =$ ..... 1.6 ..... or $x =$ ..... 4.4 ..... [3]

(iii) $f(x) = k$ has three solutions for $-1.5 \leqslant x \leqslant 5$ where k is an integer.

Find the smallest possible value of k.

$k =$ ..... -8 ..... [1]

(iv) On the grid, draw a line $y = mx$ so that $f(x) = mx$ has exactly one solution for $-1.5 \leqslant x \leqslant 5$. [2]

(b) $y = 3x^2 - 12x + 7$

(i) Find the value of $\frac{dy}{dx}$ when $x = 5$.

$\frac{dy}{dx} = 6x - 12$

Substitute $x = 5$

$6x - 12$

$6(5) - 12$

$30 - 12 = 18$

.... 18 .... [3]

(ii) Find the coordinates of the point on the graph of $y = 3x^2 - 12x + 7$ where the gradient is 0.

$\frac{dy}{dx} = 0$ $6x - 12 = 0$

$6x = 12$

$x = 2$

Substitute $x = 2$

$y = 3(2^2) - 12(2) + 7$

$y = 12 - 24 + 7$

$y = -5$

(..... 2 ..... , ... -5 .....) [2]

(c) When $y = 2x^p + qx^2$, $\frac{dy}{dx} = 14x^6 + 6x$.

Find the value of p and the value of q.

$y = 2x^p + qx^2$

$\frac{dy}{dx} = 2px^{p-1} + 2qx$

$2px^{p-1} = 14x^6$

$p - 1 = 6$

$p = 6 + 1$

$p = 7$

$2q\cancel{x} = 6\cancel{x}$

$\frac{2q}{2} = \frac{6}{2}$

$q = 3$

$p =$ ..... 7 .....

$q =$ ..... 3 ..... [2]

34. June/2022/Paper_41/No.9

(a)

The area of the rectangle is 29 cm^2 greater than the area of the square.
The difference between the perimeters of the two shapes is k cm.

Find the value of k.
You must show all your working.

Area of rectangle $= (2x+1)(x-1)$
$2x(x-1) + 1(x-1)$
$= 2x^2 - 2x + x - 1$
$= 2x^2 - x - 1$

Area of square $= x \times x$
$= x^2$

$2x^2 - x - 1 - x^2 = 29$
$x^2 - x = 30$
$x^2 - x - 30 = 0$
$P = -30$ $(-6, 5)$
$S = -1$

$x^2 - 6x + 5x - 30 = 0$
$x(x-6) + 5(x-6) = 0$
$(x-6)(x+5) = 0$
$x - 6 = 0$ | $x + 5 = 0$
$x = 6$ | $x = -5$

Ignore negative value; so $x = 6$

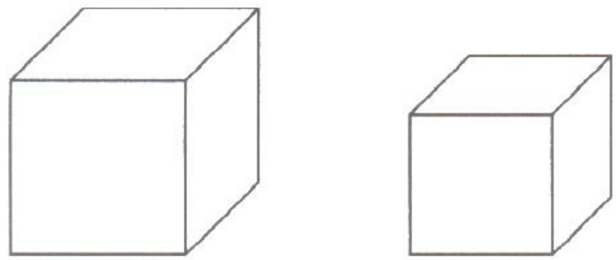
Perimeter of rectangle
$= (2x+1) + (2x+1) + (x-1) + (x-1)$
$= 6x$

Perimeter square
$= x + x + x + x$
$= 4x$
$K = 6x - 4x$
$K = 2x$

$K = 2x$
$K = 2 \times 6$
$K = 12$

$k =$ ......12...... [6]

(b)



NOT TO SCALE

The volume of the larger cube is 5 cm^3 greater than the volume of the smaller cube.

(i) Show that $3y^2 + 3y - 4 = 0$.

Volume of cube $= l^3$

Larger cube $= (y+1)^3$

$(y+1)(y+1)$

$y(y+1) + 1(y+1)$

$y^2 + y + y + 1$

$y^2 + 2y + 1$

$(y+1)(y^2 + 2y + 1)$

$y(y^2 + 2y + 1) + 1(y^2 + 2y + 1)$

$y^3 + 2y^2 + y + y^2 + 2y + 1$

$y^3 + 3y^2 + 3y + 1$

small cube.

$y \times y \times y = y^3 + 5$

$\cancel{y^3} + 3y^2 + 3y + 1 - \cancel{y^3} - 5 = 0$

$3y^2 + 3y - 4 = 0$

[4]

(ii) Find the volume of the smaller cube.
Show all your working and give your answer correct to 2 decimal places.

$3y^2 + 3y - 4 = 0$

$a = 3,\ b = 3,\ c = -4$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-3 \pm \sqrt{3^2 - 4(3)(-4)}}{6}$

$\dfrac{-3 \pm \sqrt{57}}{6}$

$y_1 = \dfrac{-3 + \sqrt{57}}{6}$

$y_1 = 0.7583$

$y_2 = \dfrac{-3 - \sqrt{57}}{6}$

$y_2 = -1.7583$

(Ignore negative value)

Volume of Large cube $=$
$(1.7583)^3 = 5.436$ cm^3

volume of small cube $=$
$(0.7583)^3 = 0.436$ cm^3

..........0.436.......... cm^3 [4]

35. June/2022/Paper_42/No.8

(a) Solve.

$$10 - 3p = 3 + 11p$$

$10 - 3 = 11p + 3p$

$\frac{7}{14} = \frac{14p}{\cancel{14}}$ $\quad p = \frac{1}{2}$ or 0.5

$p =$ ....0.5........ [2]

(b) Make m the subject of the formula.

$$mc^2 - 2k = mg$$

$mc^2 - 2k = mg$

$mc^2 - mg = 2k$

$\frac{m(\cancel{c^2 - g})}{\cancel{c^2 - g}} = \frac{2k}{c^2 - g}$

$m = \frac{2k}{c^2 - g}$

$m =$ ....$\frac{2k}{c^2 - g}$........ [3]

(c) Solve.

$$\frac{1}{x-3} + \frac{4}{2x+3} = 1$$

$\frac{1}{x-3} + \frac{4}{2x+3} = 1$

obtain the LCM of $(x-3)(2x+3)$

$\frac{2x+3 + 4(x-3)}{(x-3)(2x+3)} = (2x+3)(x-3)$

$= 2x^2 + 3x - 6x - 9$

$2x + 3 + 4x - 12 = 2x^2 - 3x - 9$

$6x - 9 = 2x^2 - 3x - 9$

$2x^2 - 3x - 6x - 9 + 9 = 0$

$2x^2 - 9x = 0$

$x(2x - 9) = 0$

$x = 0 \mid 2x - 9 = 0$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{9}{2}$

$x = 4.5$

$x =$ ....0........ or $x =$ ....4.5........ [5]

(d) Solve the simultaneous equations.
You must show all your working.

$$x + 2y = 12$$
$$5x + y^2 = 39$$

Make equation (i) x the subject.

$x + 2y = 12$

$x = \underline{12 - 2y}$

$5x + y^2 = 39$

$5(12-2y) + y^2 = 39$

$60 - 10y + y^2 = 39$

$y^2 - 10y + 60 - 39 = 0$

$y^2 - 10y + 21 = 0$

Product $= 21 \quad (-7, -3)$

Sum $= -10$

$y^2 - 7y - 3y + 21 = 0$

$y(y-7) - 3(y-7) = 0$

$(y-7)(y-3) = 0$

$y - 7 = 0 \quad | \quad y - 3 = 0$

$y = \underline{7} \quad\quad y = \underline{3}$

Substitute $x = 12 - 2y$

When $y = 7$, $x = 12 - 2(7)$

$x = 12 - 14$

$x = \underline{-2}$

When $y = 3$, $x = 12 - 2(3)$

$12 - 6$

$= \underline{6}$

$x = $ ......6...... $y = $ ......3......

$x = $ ......-2...... $y = $ ......7...... [5]

(e) Expand and simplify.

$$(2x-3)(x+6)(x-4)$$

$(2x-3)(x+6)$

$2x(x+6) - 3(x+6)$

$2x^2 + 12x - 3x - 18$

$\underline{2x^2 + 9x - 18}$

$(x-4)(2x^2 + 9x - 18)$

$x(2x^2 + 9x - 18) - 4(2x^2 + 9x - 18)$

$2x^3 + 9x^2 - 18x - 8x^2 - 36x + 72$

$= \underline{2x^3 - x^2 - 54x + 72}$

......$2x^3 - x^2 - 54x + 72$...... [3]

36. June/2022/Paper_42/No.12

12 A curve has equation $y = x^3 - kx^2 + 1$.
When $x = 2$, the gradient of the curve is 6.

(a) Show that $k = 1.5$.

$y = x^3 - kx^2 + 1$

$\frac{dy}{dx} = 3x^2 - 2kx$

Gradient $= 6$; substitute x as 2

$6 = 3(2^2) - 2k(2)$

$6 = 12 - 4k$

$4k = 12 - 6$

$\frac{4k}{4} = \frac{6}{4}$ $k = \underline{1.5}$

[5]

(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

$y = x^3 - 1.5x^2 + 1$

$\frac{dy}{dx} = 0$ For stationary Points

$\frac{dy}{dx} = 3x^2 - 3x$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

$3x = 0$ | $x - 1 = 0$

$x = \underline{0}$ | $x = \underline{1}$

Substitute x as 0

$y = 0^3 - 1.5(0^2) + 1$

$y = \underline{1}$ $(0, 1)$

When $x = 1$

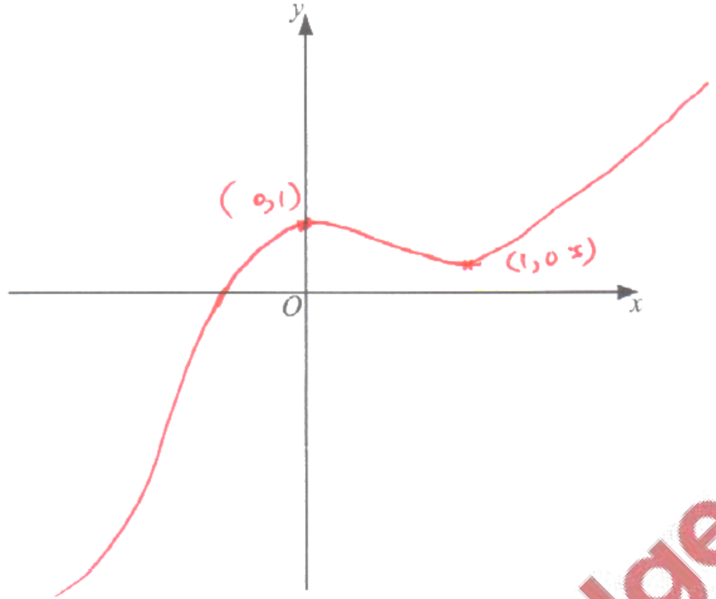
$y = 1^3 - 1.5(1^2) + 1$

$y = 1 - 1.5 + 1$

$y = \underline{0.5}$ $\underline{(1, 0.5)}$

(0 , 1) and (1 , 0.5) [4]

(c) Sketch the curve $y = x^3 - 1.5x^2 + 1$.



[2]

$$\frac{dy}{dx} = 3x^2 - 3x$$

$$\frac{d^2y}{dx^2} = 6x - 3$$

For $(0, 1)$ substitute $x = 0$

$6(0) - 3$

$0 - 3$

$= -3$ (Maximum Point)

For $(1, 0.5)$

$$\frac{d^2y}{dx^2} = 6x - 3$$

$= 6(1) - 3$

$= 6 - 3$

$= 3$ (Minimum point)

37. June/2022/Paper_43/No.6

(a) Simplify.

$$a-2b-3a+7b$$

$a-3a+7b-2b$

$-2a+5b$

$5b-2a$

$5b-2a$ [2]

(b) Expand and simplify.

$$4(x-5)-(3-2x)$$

$4(x-5)-(3-2x)$

$4x-20-3+2x$

$4x+2x-20-3$

$6x-23$

$6x-23$ [2]

(c) Write as a single fraction in its simplest form.

$$\frac{3}{x-5}-\frac{7}{2x}$$

$\frac{3}{x-5} - \frac{7}{2x}$

$\frac{3(2x)-7(x-5)}{(x-5)2x}$

$\frac{6x-7x+35}{2x(x-5)}$

$\frac{-x+35}{2x(x-5)}$

$= \frac{35-x}{2x(x-5)}$

.... [3]

(d) Solve.

$$\frac{13-4x}{3}=6-x$$

$3\times\frac{13-4x}{3}=6-x\times 3$

Multiply both sides by 3

$13-4x=3(6-x)$

$13-4x=18-3x$

$-4x+3x=18-13$

$-x=5$

$x=-5$

$x =$ -5 [3]

(e) Make x the subject of the formula.

$$y = \frac{5(p-2x)}{x}$$

$$y = \frac{5(p-2x)}{x}$$

$$x \times y = \frac{5p-10x}{x} \times x$$

Multiply both sides by x

$$yx = 5p - 10x$$

$$yx + 10x = 5p$$

$$\frac{x(\cancel{y+10})}{\cancel{y+10}} = \frac{5p}{y+10}$$

$$x = \frac{5p}{y+10}$$

$x =$ $\frac{5p}{y+10}$ .... [4]

38. June/2022/Paper_43/No.9

(a) Sketch the graph of $y = (x+1)(3-x)(3+x)$, indicating the coordinates of the points where the graph crosses the x-axis and the y-axis.

when $y = 0$

$x+1=0$

$x = -1$

$3-x=0$

$x = 3$

$3+x=0$

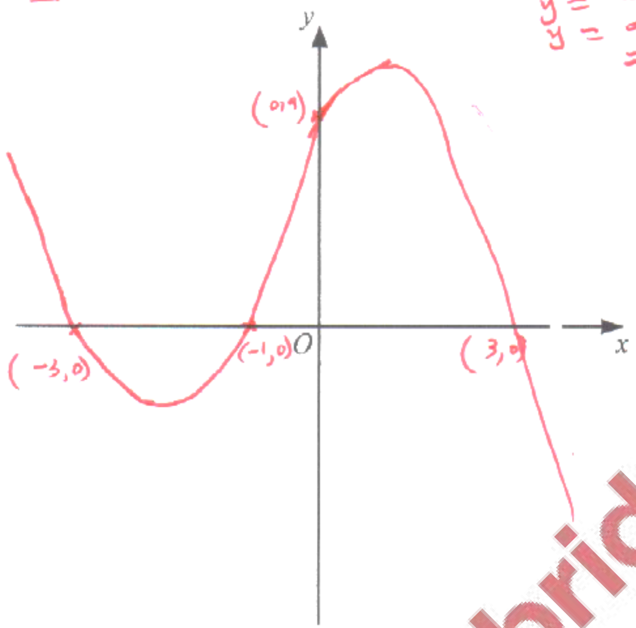
$x=-3$

When $x=0$

$y=(0+1)(3-0)(3+0)$

$y = (1)(3)(3)$

$y = 9$



[4]

(b) (i) Show that $y = (x+1)(3-x)(3+x)$ can be written as $y = 9 + 9x - x^2 - x^3$.

$(x+1)(3+x)$

$x(3+x)+1(3+x)$

$3x + x^2 + 3 + x$

$x^2 + 3x + x + 3$

$x^2 + 4x + 3$

$(3-x)(x^2+4x+3)$

$3(x^2+4x+3) - x(x^2+4x+3)$

$3x^2 + 12x + 9 - x^3 - 4x^2 - 3x$

$9 + 12x - 3x + 3x^2 - 4x^2 - x^3$

$9 + 9x - x^2 - x^3$

[2]

(ii) Calculate the x–values of the turning points of $y = 9 + 9x - x^2 - x^3$.
Show all your working and give your answers correct to 2 decimal places.

At turning point $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = 9 - 2x - 3x^2$

$9 - 2x - 3x^2 = 0$

using quadratic equation formula;

$a = -3,\ b = -2,\ c = 9$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times (-3)(9)}}{-6}$

$\frac{2 \pm \sqrt{112}}{-6}$

$x = \frac{2 \pm 10.583}{-6}$

$x_1 = \underline{\underline{-2.10}}$ or $x_2 = \underline{\underline{1.43}}$

$x =$ ......-2.10......, $x =$ ......1.43...... [7]

(iii) The equation $9 + 9x - x^2 - x^3 = k$ has one solution only when $k < a$ and when $k > b$, where a and b are integers.

Find the maximum value of a and the minimum value of b.

When $x = -2.10$

$y = 9 + 9(-2.10) - (2.10)^2 - (-2.10)^3$

$y = \underline{\underline{-5.049}}$

When $x = 1.43$

$y = 9 + 9(1.43) - (1.43^2) - (1.43^3)$

$y = 16.90$

The values of $a = -6$ and $b = 17$

$a =$ ......-6......

$b =$ ......17...... [3]