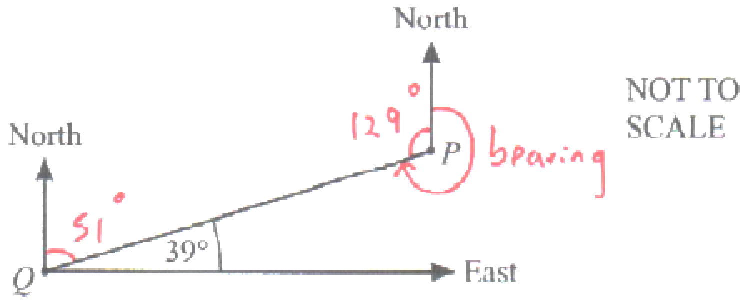
Trigonometry – 2023 Nov IGCSE 0580 Math

1. Nov/2023/Paper_0580/12,22/No.13, 7



Find the bearing of Q from P.

$\angle P = 90° - 39° = 51°$

Co-interior angles add up to $180°$

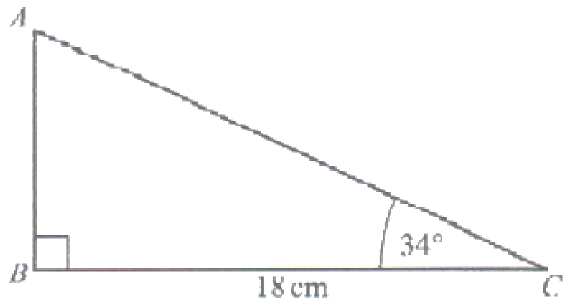
$\angle P = 180° - 51° = 129°$

Bearing $= 360° - 129° = 231°$

$231°$ .... [2]

2. Nov/2023/Paper_0580/12/No.21

ABC is a right-angled triangle.



Calculate AC.

Using trigonometric ratios (SOHCAHTOA)

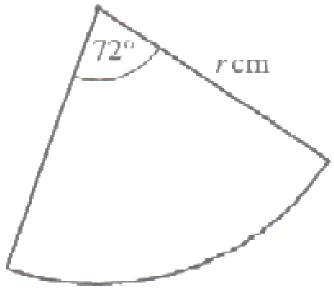
$\cos 34° = \frac{18}{AC}$

$\Rightarrow AC = \frac{18}{\cos 34°}$

$= 21.7\text{cm}$ (3sf)

$AC =$ ....21.7........ cm [3]

3. Nov/2023/Paper_0580/13/No.20



NOT TO SCALE

The diagram shows a sector of a circle with radius r cm and sector angle 72°.
The arc length is 9.35 cm.

Calculate the value of r.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

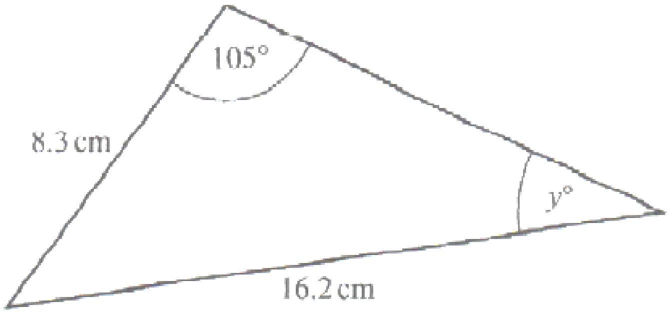
$$\Rightarrow \frac{72}{360} \times 2 \times \pi \times r = 9.35$$

$$\frac{0.4\cancel{\pi} r}{0.4\cancel{\pi}} = \frac{9.35}{0.4\pi}$$

$\therefore r = 7.44$ cm (3 s.f.)

$r =$ 7.44 .... [2]

4. Nov/2023/Paper_0580/21/No.18



NOT TO SCALE

Calculate the value of y.

Using Sine rule

$\frac{8.3}{\sin y°} = \frac{16.2}{\sin 105°}$

$\sin y° = \frac{8.3 \times \sin 105°}{16.2}$

$\sin y° = 0.494888$

$y° = \sin^{-1}(0.494888)$

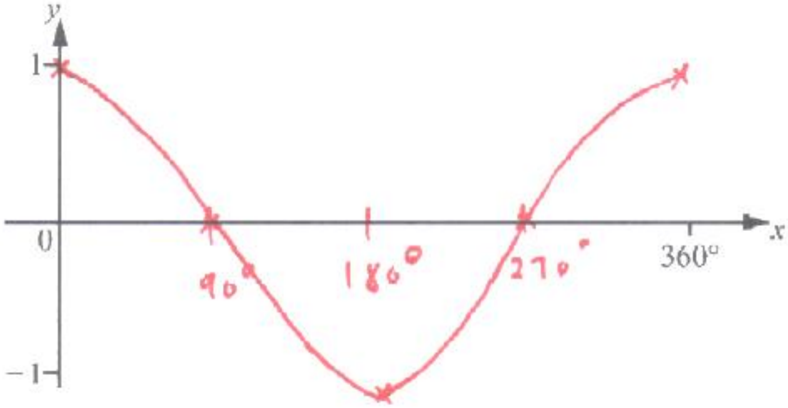
$= 29.66°$

$\approx 29.7°$ (1dp)

$y =$ 29.7 [3]

5. Nov/2023/Paper_0580/21/No.19

(a)



Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$. [2]

Table of values

x	0	90°	180°	270°	360°
y	1	0	−1	0	1

(b) When $\cos x = 0.21$, find the **reflex** angle x.

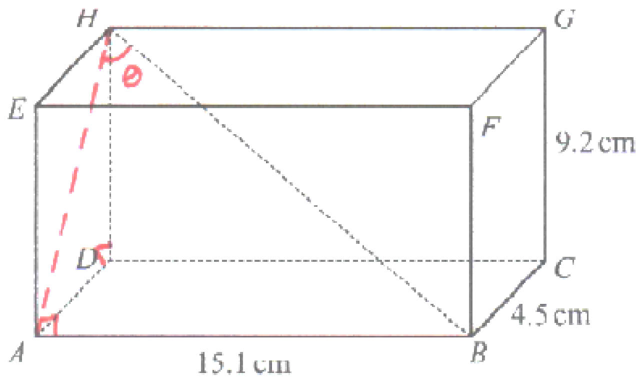
$x = \cos^{-1}(0.21)$

$x = 77.9°, \; (360° - 77.9°)$

$\therefore x = 282.1°$

282.1° .... [2]

6. Nov/2023/Paper_0580/21/No.21



NOT TO SCALE

The diagram shows a cuboid $ABCDEFGH$.
$AB = 15.1$ cm, $BC = 4.5$ cm and $CG = 9.2$ cm.

Calculate the angle that the diagonal BH makes with the face $ADHE$.

Let θ be the angle BH makes with the face ADHE

Using Pythagora's theorem

$AH = \sqrt{9.2^2 + 4.5^2} = 10.24$ cm

Using trigonometric ratios (SOHCAHTOA)

$\tan\theta = \frac{15.1}{10.24}$

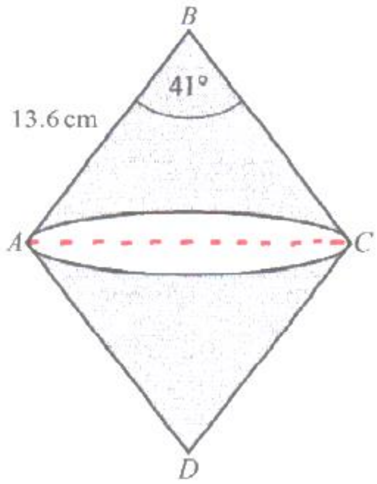
$\tan\theta = 1.47438$

$\theta = \tan^{-1}(1.47438)$

$\theta = 55.9°$ (1dp)

.... $55.9°$ [4]

7. Nov/2023/Paper_0580/21/No.22



NOT TO SCALE

$ABCD$ is a rhombus with side length 13.6 cm.
Angle $ABC = 41°$.
BAC is a sector of a circle with centre B.
DAC is a sector of a circle with centre D.

Calculate the shaded area.

Shaded area = Area of the rhombus − Area of the 2 segments.

Area of the rhombus $= 2\left(\frac{1}{2} \times 13.6 \times 13.6 \times \sin 41°\right)$

$= 121.34 \text{ cm}^2$

Area of one segment = Area of the sector − Area of the triangle

$= \left(\frac{41}{360} \times \pi \times 13.6^2\right) - \left(\frac{1}{2} \times 13.6 \times 13.6 \times \sin 41°\right)$

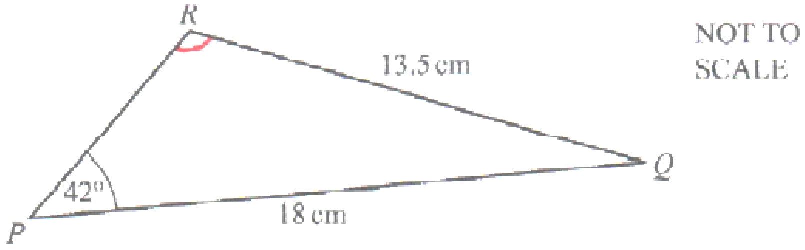
$= 5.505 \text{ cm}^2$

Shaded area $= 121.34 - 2(5.505)$ .... 110 cm^2 [4]

$= 121.34 - 11.01$

$= 110.33 \text{ cm}^2 \approx 110 \text{ cm}^2$ (3sf)

8. Nov/2023/Paper_0580/23/No.19



Calculate the obtuse angle *PRQ*.

Using Sine rule :

$$\frac{QR}{\sin \angle QPR} = \frac{PQ}{\sin \angle PRQ}$$

$$\frac{13.5}{\sin 42^\circ} = \frac{18}{\sin \angle PRQ}$$

$$\therefore \sin \angle PRQ = \frac{18 \times \sin 42^\circ}{13.5}$$

$$\sin \angle PRQ = 0.89217$$

$$\angle PRQ = \sin^{-1}(0.89217)$$

$$= 63.1^\circ$$

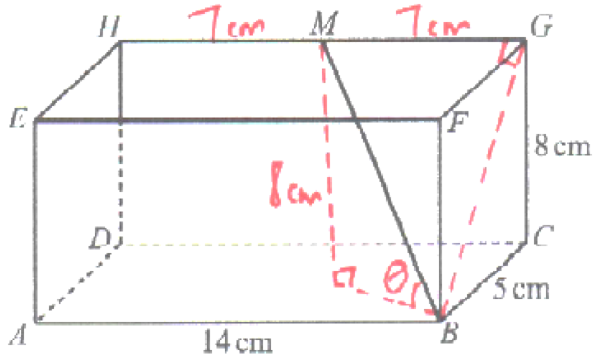
But $\angle PRQ$ is obtuse

$$\therefore \angle PRQ = 180^\circ - 63.1^\circ$$

$$= 116.9^\circ$$

Angle *PRQ* = 116.9° [4]

9. Nov/2023/Paper_0580/23/No.21



NOT TO SCALE

The diagram shows a cuboid $ABCDEFGH$.
$AB = 14$ cm, $BC = 5$ cm and $CG = 8$ cm.
M is the midpoint of HG.

(a) Calculate BM.

Using Pythagora's theorem:

$BG = \sqrt{5^2 + 8^2} = \sqrt{89}$

$HM = MG = \frac{1}{2} HG = \frac{1}{2}(14) = 7$cm

$BM = \sqrt{(\sqrt{89})^2 + 7^2} = \sqrt{138}$

$= 11.7$cm

........11.7........ cm [3]

(b) Calculate the angle that BM makes with the base $ABCD$.

Let θ be the angle that BM makes with the base ABCD.

Using trigonometric ratios (SOHCAHTOA)

$\sin\theta = \frac{8}{11.7}$

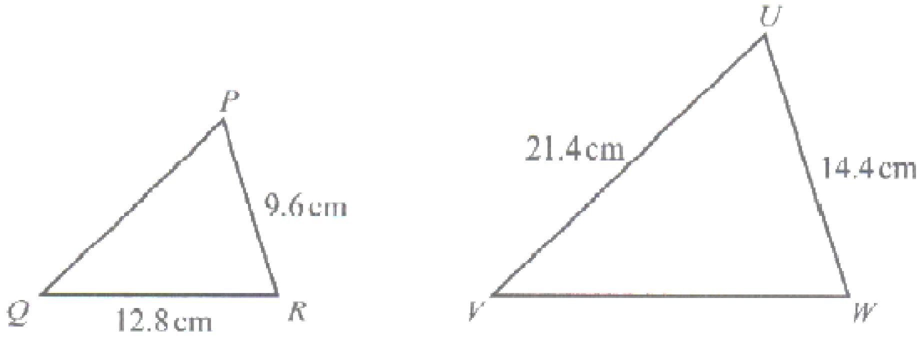
$\theta = \sin^{-1}\left(\frac{8}{11.7}\right)$

$= 42.9°$ (1dp)

........42.9°........ [3]

10. Nov/2023/Paper_0580/31/No.10

(a)



NOT TO SCALE

Triangle *PQR* is mathematically similar to triangle *UVW*.

Calculate *VW*.

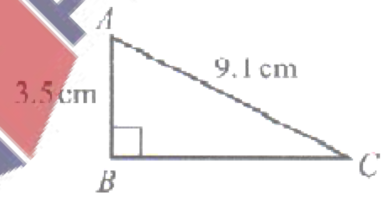
$$\frac{14.4}{9.6} = \frac{VW}{12.8}$$

$$\frac{9.6\,VW}{9.6} = \frac{14.4 \times 12.8}{9.6}$$

$$VW = \frac{14.4 \times 12.8}{9.6} = 19.2\text{ cm}$$

VW = ...19.2........ cm [2]

(b) *ABC* is a right-angled triangle.



NOT TO SCALE

Calculate *BC*.

Using Pythagoras's theorem

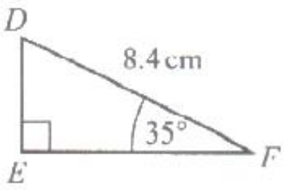
$$BC = \sqrt{9.1^2 - 3.5^2}$$

$$= \sqrt{70.56}$$

$$= 8.4\text{ cm}$$

BC = ...8.4........ cm [3]

(c) *DEF* is a right-angled triangle.



NOT TO SCALE

Calculate *EF*.

Using trigonometric ratios (SOHCAHTOA)

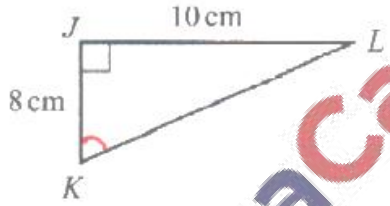
$\cos 35° = \frac{EF}{8.4}$

$EF = 8.4 \times \cos 35°$

$= 6.88$ cm (3sf)

EF = 6.88 .... cm [2]

(d) *JKL* is a right-angled triangle.



NOT TO SCALE

Calculate angle *JKL*.

$\tan \angle JKL = \frac{10}{8}$

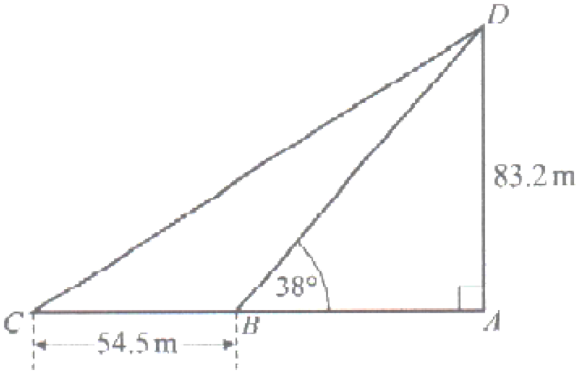
$\angle JKL = \tan^{-1}\left(\frac{10}{8}\right)$

$= 51.3°$ (1dp)

Angle *JKL* = 51.3° .... [2]

11. Nov/2023/Paper_0580/41/No.5

(a)



NOT TO SCALE

ACD is a right-angled triangle.
B is on AC and $BC = 54.5$ m.
$AD = 83.2$ m and angle $ABD = 38°$.

Calculate angle ACD.

Using trigonometric ratios (SOHCAHTOA)

$\tan 38° = \frac{83.2}{AB} \Rightarrow AB = \frac{83.2}{\tan 38°}$

$AB = 106.49$ m

$\tan \angle ACD = \frac{83.2}{54.5 + 106.49}$

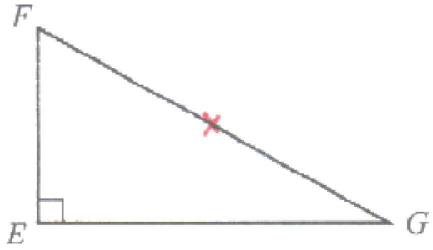
$\tan \angle ACD = 0.5168$

$\angle ACD = \tan^{-1}(0.5168)$

$= 27.3°$

Angle ACD = 27.3° [5]

(b)

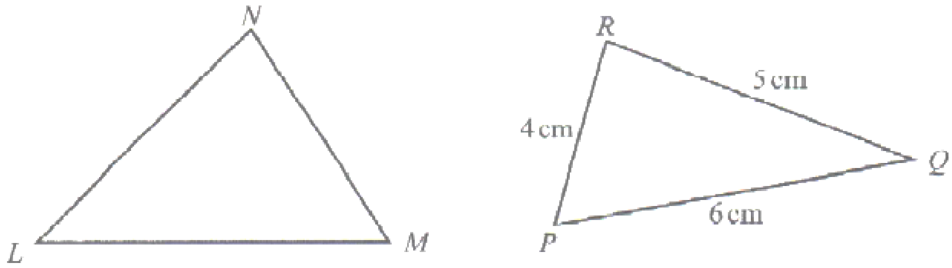


EFG is a right-angled triangle.
A circle can be drawn that passes through the three vertices of the triangle.

On the diagram, mark the position of the centre of the circle with a cross.
Explain how you decide.

Angle in a Semi-Circle is 90°, So the centre of the circle is the midpoint of FG.
[2]

(c)



In triangle LMN, the ratio angle L : angle M : angle $N = 4 : 5 : 6$.
In triangle PQR, $PQ = 6$ cm, $PR = 4$ cm and $QR = 5$ cm.

Calculate the difference between the largest angle in triangle PQR and the largest angle in triangle LMN.

For triangle LMN the largest angle is represented by the largest ratio.

Total ratio $= 4 + 5 + 6 = 15$

Angles in a triangle add up to $180°$.

$\angle N = \frac{6}{15} \times 180° = 72°$

For triangle PQR :

Using cosine rule : $PQ^2 = RP^2 + RQ^2 - 2(RP)(RQ)\cos \angle R$

$\cos \angle R = \frac{RP^2 + RQ^2 - PQ^2}{2 \times RP \times RQ}$

$= \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}$

.... $10.8°$ [7]

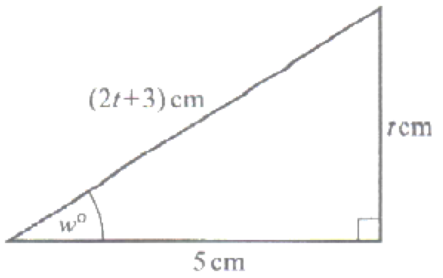
$\cos \angle R = 0.125$

$\angle R = \cos^{-1}(0.125)$

$= 82.8°$

Difference $= 82.8° - 72° = 10.8°$

12. Nov/2023/Paper_0580/42/No.6



NOT TO SCALE

The diagram shows a right-angled triangle.

Find the value of w.

Using Pythagora's theorem

$5^2 + t^2 = (2t+3)^2$

$25 + t^2 = 2t(2t+3) + 3(2t+3)$

$25 + t^2 = 4t^2 + 6t + 6t + 9$

$25 + t^2 = 4t^2 + 12t + 9$

$\Rightarrow 4t^2 + 12t + 9 - 25 - t^2 = 0$

$3t^2 + 12t - 16 = 0$

Solving for t using the quadratic formula

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-12 \pm \sqrt{12^2 - 4(3)(-16)}}{2 \times 3}$

$= \dfrac{-12 \pm \sqrt{336}}{6}$

But $t > 0$, $t = \dfrac{-12 + \sqrt{336}}{6} = 1.055$

Using trigonometric ratios

$\tan w° = \dfrac{1.055}{5}$

$\tan w° = 0.211$

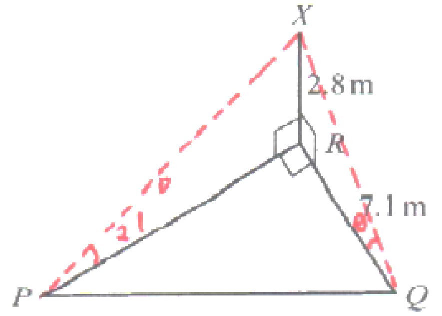
$w° = \tan^{-1}(0.211)$

$w° = 11.9°$ (1dp)

$w =$ 11.9 [7]

13. Nov/2023/Paper_0580/42/No.7

(a)



NOT TO SCALE

The diagram shows a right-angled triangle PQR on horizontal ground.
X is vertically above R and the angle of elevation of X from P is 21°.
$XR = 2.8$ m and $RQ = 7.1$ m.

(i) Calculate the angle of elevation of X from Q.

$\tan\theta = \dfrac{2.8}{7.1}$

$\theta = \tan^{-1}\left(\dfrac{2.8}{7.1}\right)$

$\theta = 21.5°$

21.5° [2]

(ii) Calculate PQ.

Using trigonometric ratios

$\tan 21° = \dfrac{2.8}{PR}$

$PR = \dfrac{2.8}{\tan 21°} = 7.29$ m

Using Pythagora's theorem:

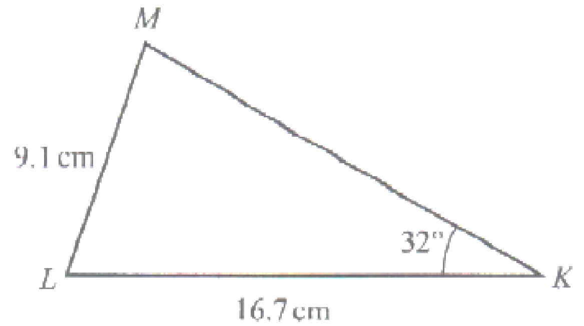
$PQ = \sqrt{PR^2 + QR^2}$

$= \sqrt{7.29^2 + 7.1^2}$

$= 10.2$ m (3sf)

10.2 m [3]

(b)



Calculate the acute angle *KML*.

Using Sine rule :

$$\frac{9.1}{\sin 32°} = \frac{16.7}{\sin \angle KML}$$

$$\sin \angle KML = \frac{16.7 \times \sin 32°}{9.1}$$

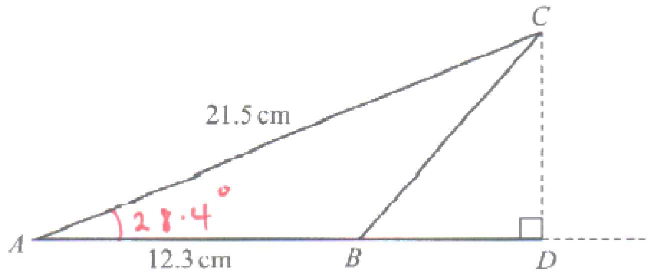
$\sin \angle KML = 0.972489$

$\angle KML = \sin^{-1}(0.972489)$

$= 76.5°$ (1dp)

Angle *KML* = 76.5° [3]

(c)



NOT TO SCALE

The area of triangle ABC is 62.89 cm^2.

(i) Show that angle $BAC = 28.4°$, correct to 1 decimal place.

$\text{Area} = \frac{1}{2} \times AB \times AC \times \sin \angle BAC$

$\Rightarrow \frac{1}{2} \times 12.3 \times 21.5 \times \sin \angle BAC = 62.89$

$\frac{132.25 \sin \angle BAC}{132.25} = \frac{62.89}{132.25}$

$\sin \angle BAC = 0.4756$

$\angle BAC = \sin^{-1}(0.4756)$

$= 28.4° \text{ (1dp)}$

As required.

[2]

(ii) Calculate BC.

Using Cosine rule:

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos \angle BAC$

$BC^2 = 12.3^2 + 21.5^2 - 2(12.3)(21.5)\cos 28.4°$

$BC^2 = 148.29$

$BC = \sqrt{148.29}$

$\therefore BC = 12.2\text{ cm}$ (3sf)

.... 12.2 cm [3]

(iii) AB is extended to a point D such that angle $BDC = 90°$.

Calculate BD.

$\cos 28.4° = \frac{AD}{21.5}$

$\Rightarrow AD = 21.5 \times \cos 28.4°$

$= 18.9\text{ cm}$ (3sf)

$BD = AD - AB$

$= 18.9 - 12.3 = 6.6\text{ cm}$ .... 6.6 cm [3]