Algebra and graphs – 2023 IGCSE 0580

1.

Factorise completely.

$$8g - 2g^2$$

.... [2]

2.
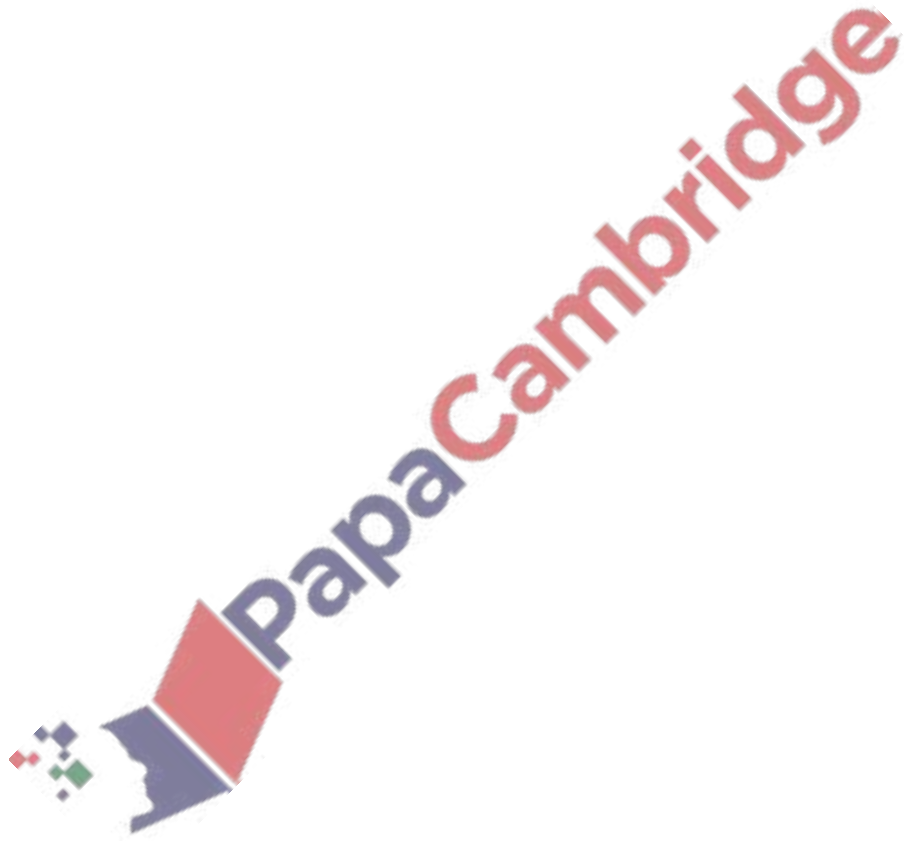
Solve.
$$\frac{25-2u}{3} = 2$$

$u =$ .... [2]

Solve the simultaneous equations.
You must show all your working.

$$3x - 2y = 19$$
$$x + y = 3$$

$x =$ ....

$y =$ .... [3]

4.
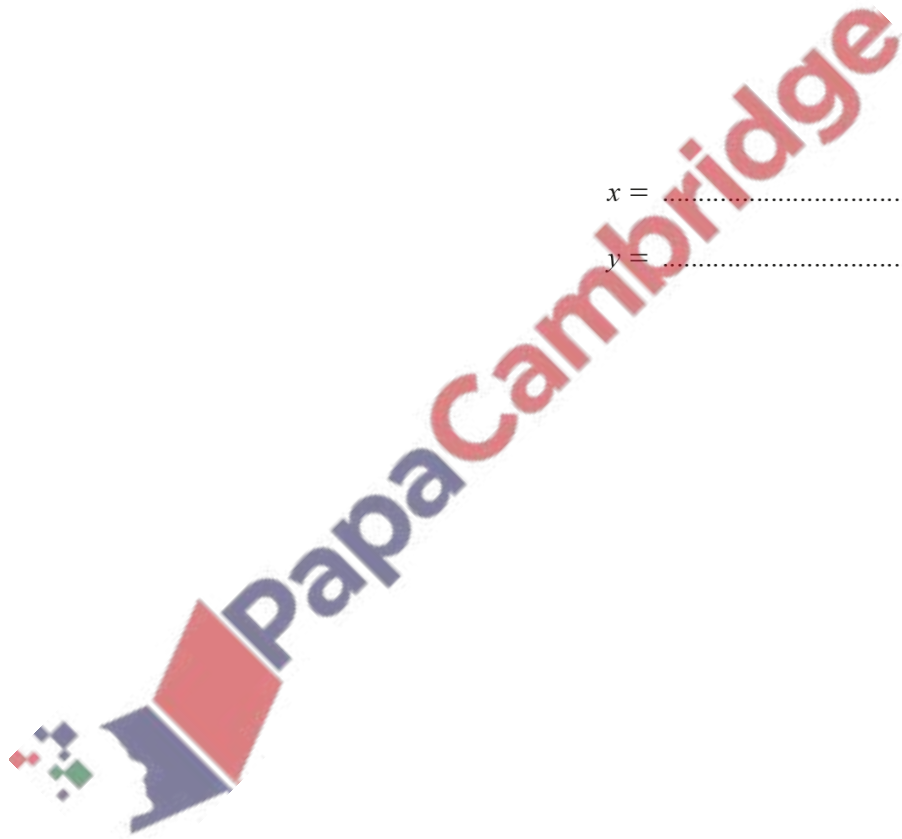
Solve.

(a) $15t + 8 = 4 - t$

$t =$ .... [2]

(b) $\dfrac{25 - 2u}{3} = 2$

$u =$ .... [2]
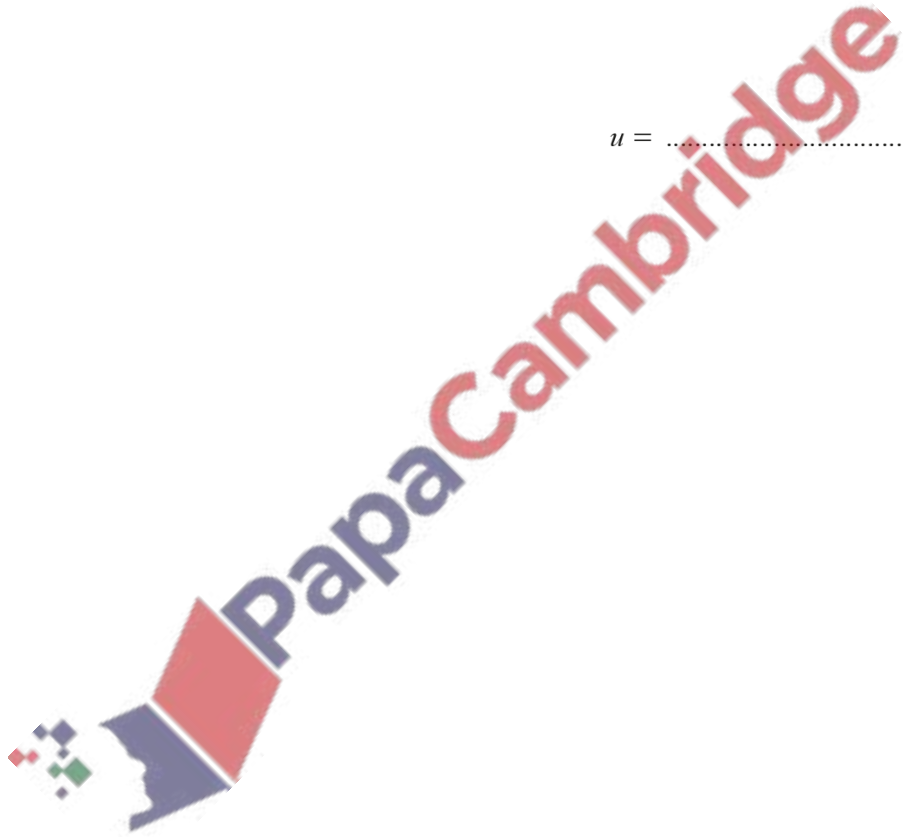
5.

Solve the simultaneous equations.
You must show all your working.

$$3x - 2y = 19$$
$$x + y = 3$$

$x =$ ....

$y =$ .... [3]

6.
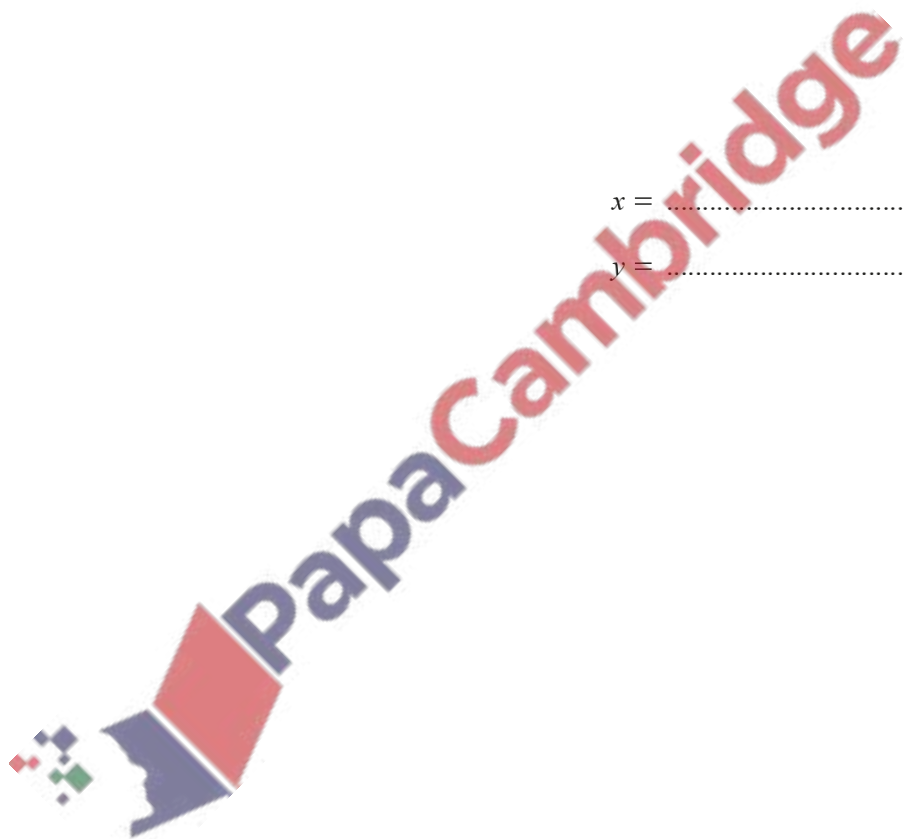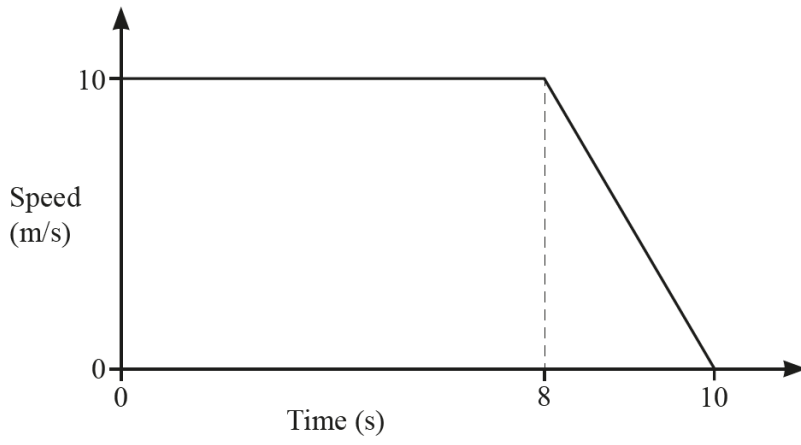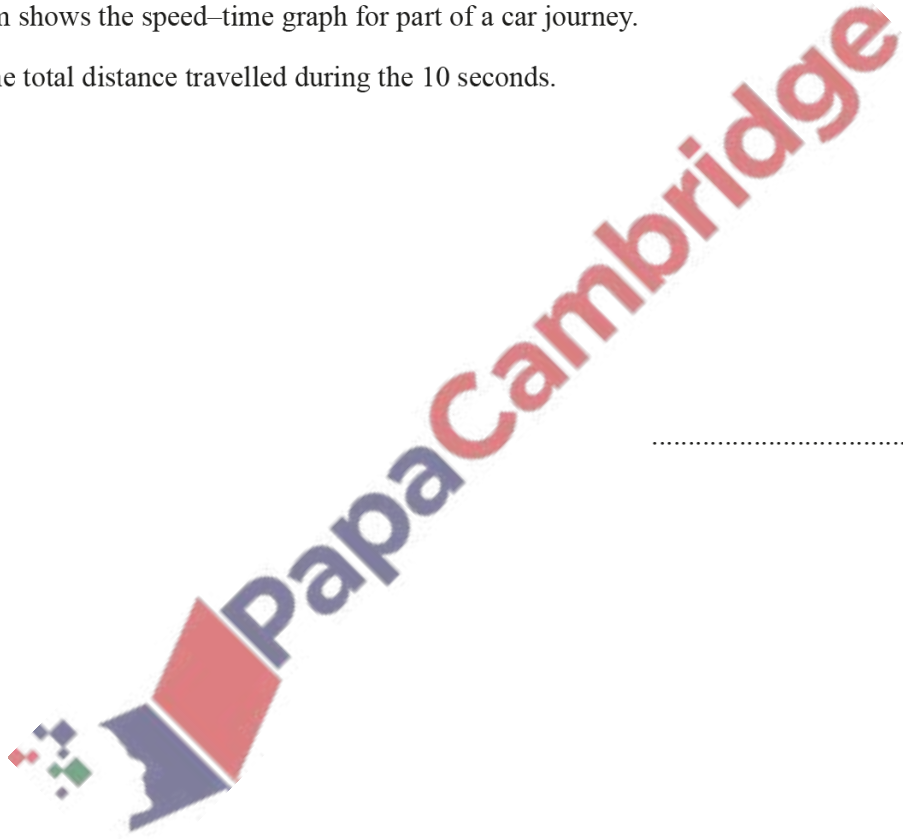

The diagram shows the speed–time graph for part of a car journey.

Calculate the total distance travelled during the 10 seconds.

.... m [2]

7.

Simplify $\left(3125w^{3125}\right)^{\frac{1}{5}}$.

.... [2]

8.
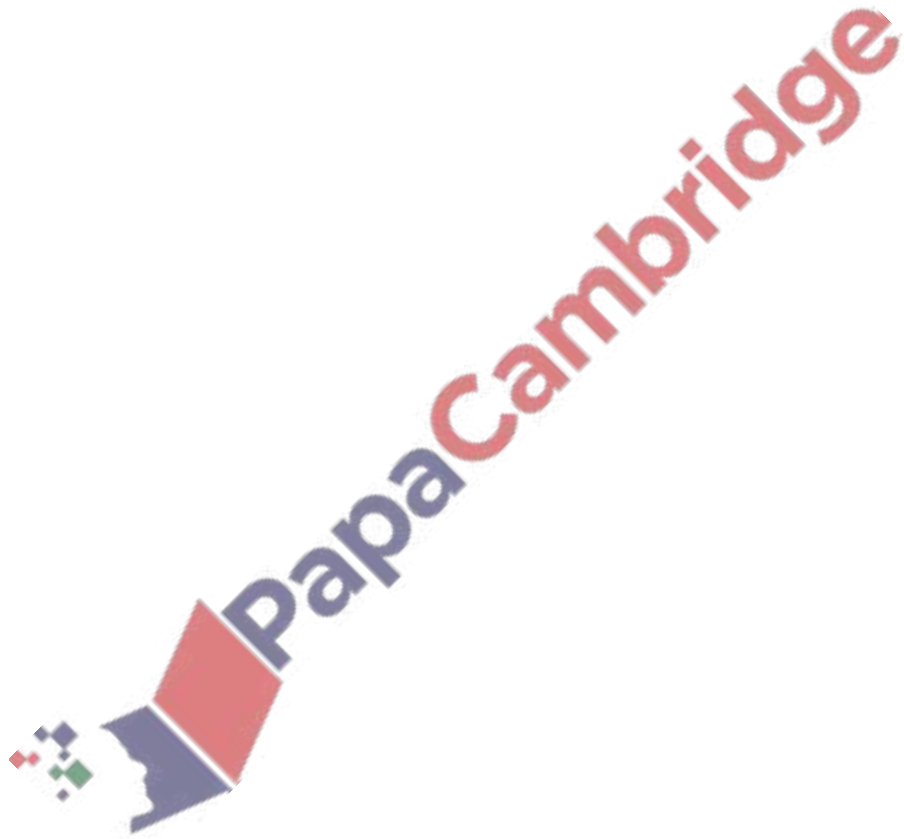
Find the values of x when $6x + y = 10$ and $y = x^2 - 3x + 10$.

$x =$ .... or $x =$ .... [3]

Find the values of x when $6x + y = 10$ and $x^2 - 3x + 10$.
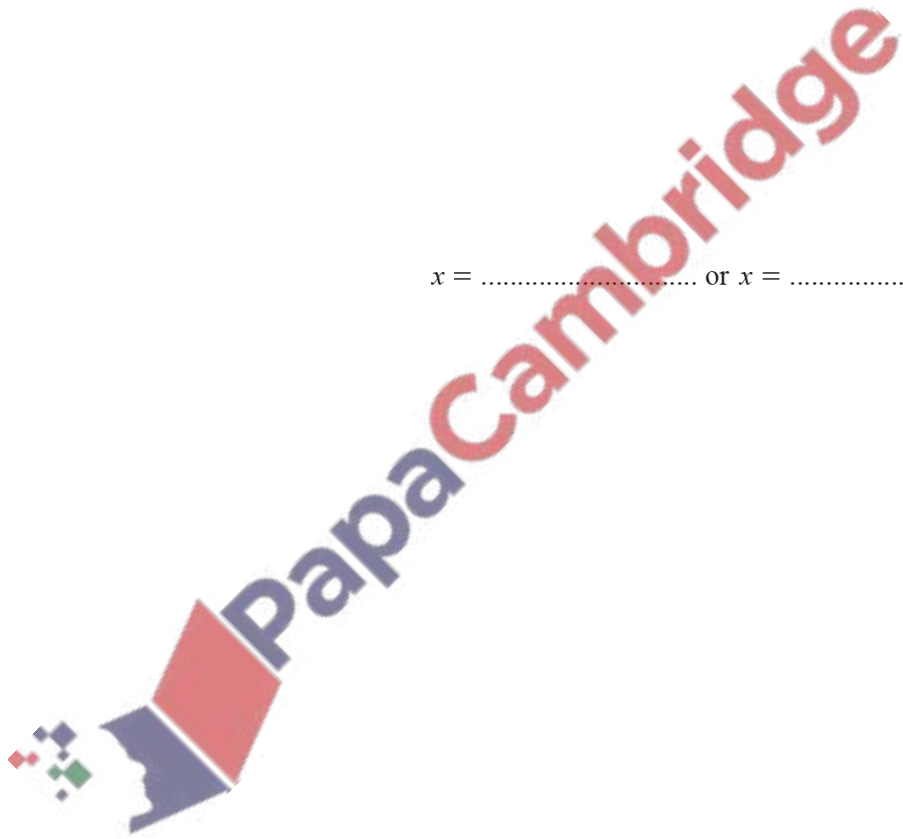
Simplify.

$$\frac{5x^2 - 19x + 12}{x^2 - 9}$$

.... [4]
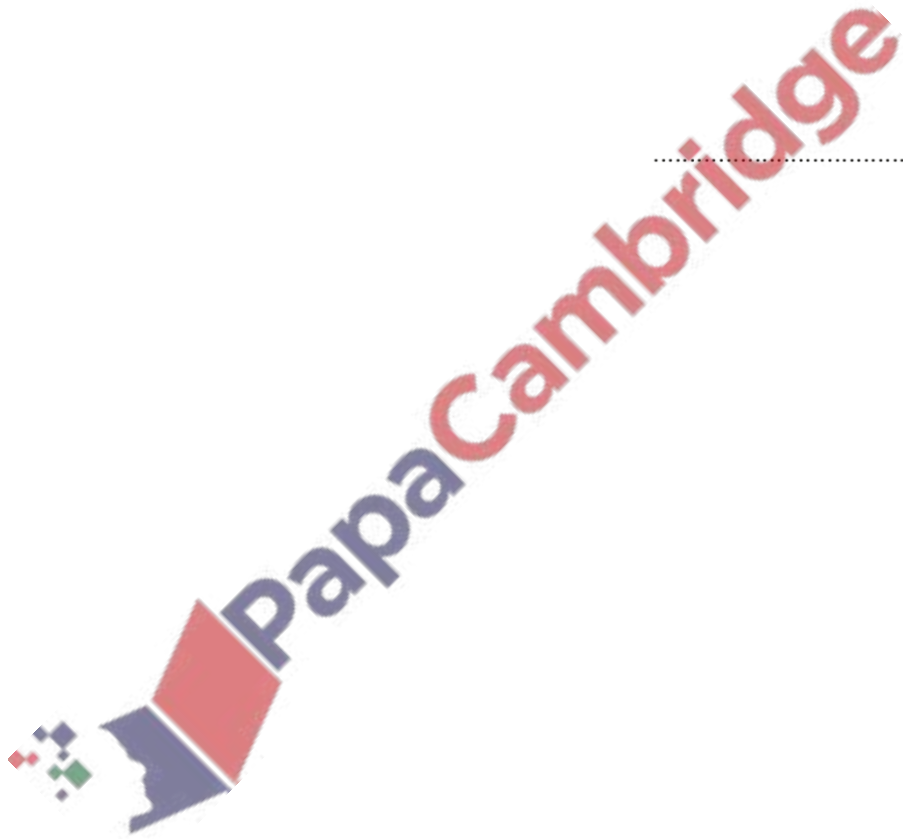
$$f(x) = x^3 + 1$$

Find $f^{-1}(x)$.

$$f^{-1}(x) = \text{...} \quad [2]$$
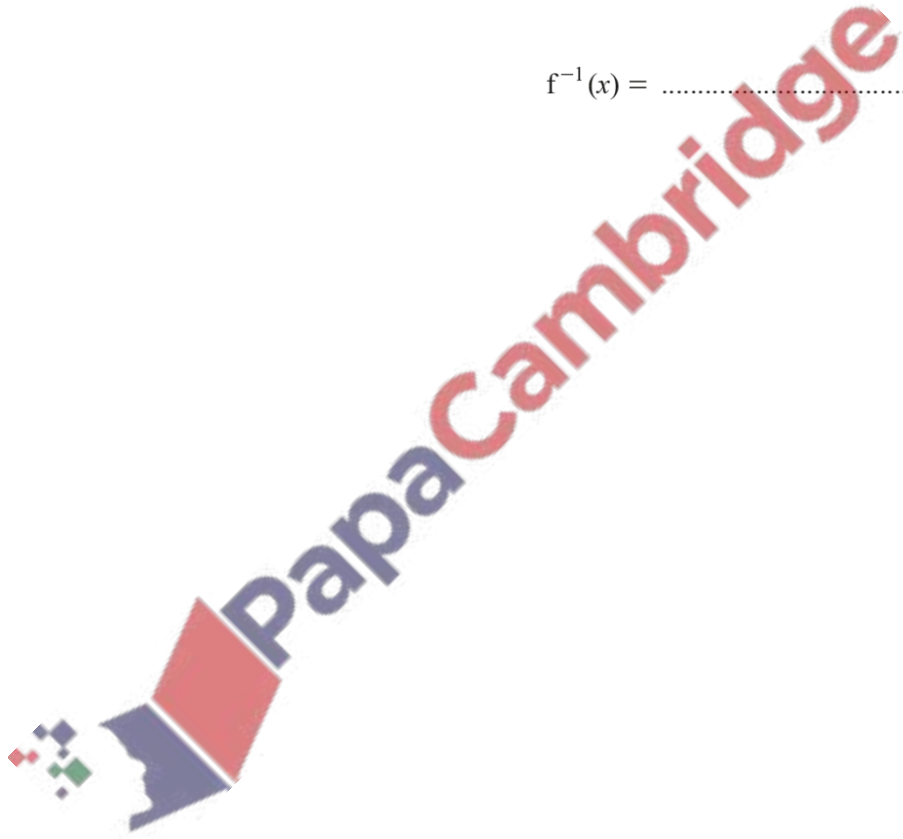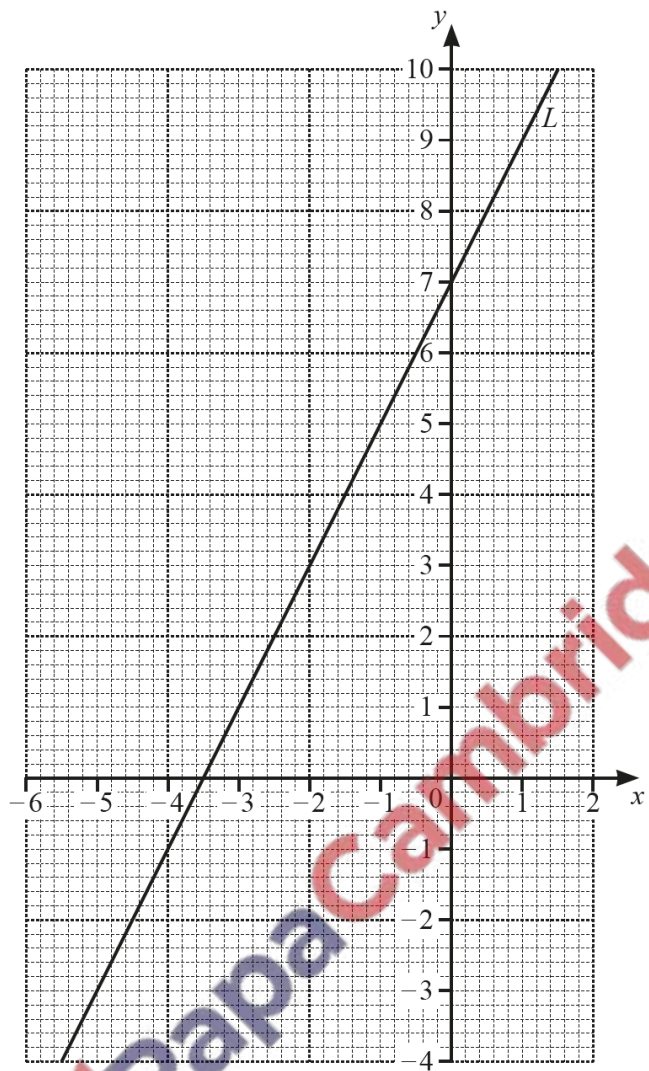
(a) Find the equation of line L in the form $y = mx + c$.

$y =$ .... [2]

(b) Write down the coordinates of the point where line L crosses the x-axis.

(..................... , ...) [1]

(c) (i) Complete the table of values for $y = x^2 + 5x + 3$.

x	-6	-5	-4	-3	-2	-1	0	1
y	9		-1			-1		

[3]

(ii) On the grid, draw the graph of $y = x^2 + 5x + 3$ for $-6 \leqslant x \leqslant 1$. [4]

(d) (i) On the grid, draw the line $y = 6$. [1]

(ii) Use your graphs to solve the equation $x^2 + 5x + 3 = 6$.

$x = $ .... or $x = $ .... [2]

(a)

NOT TO SCALE
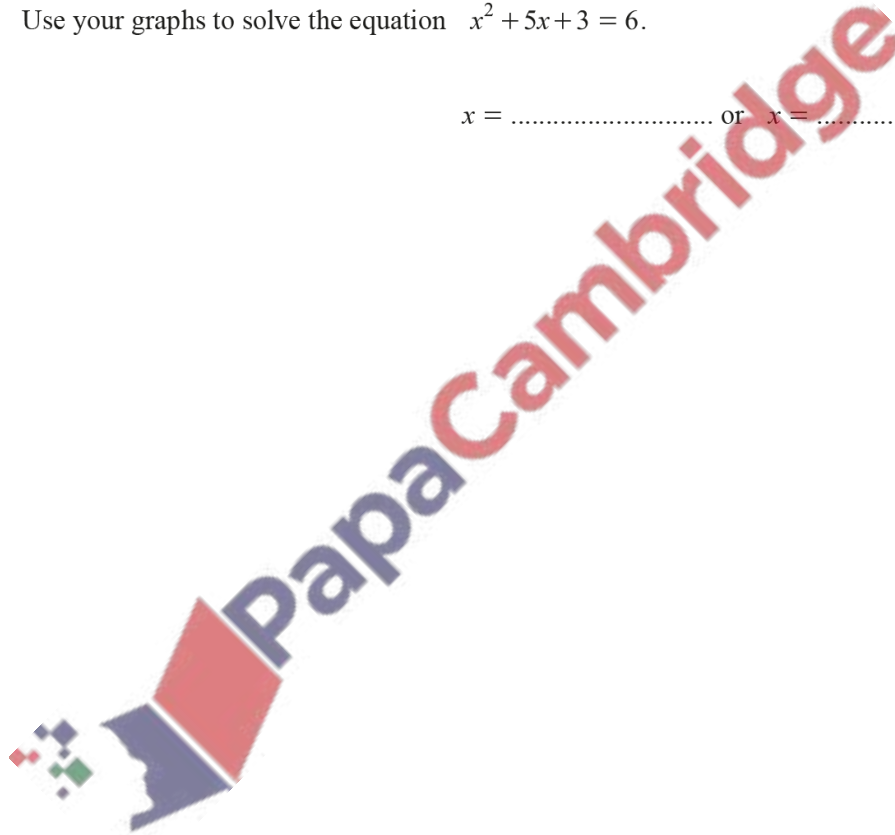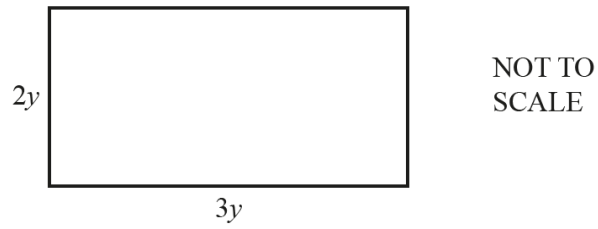
Write down an expression for the area of this rectangle.
Give your answer in its simplest form.

.... [2]

(b) In this part, all measurements are in centimetres.

NOT TO SCALE
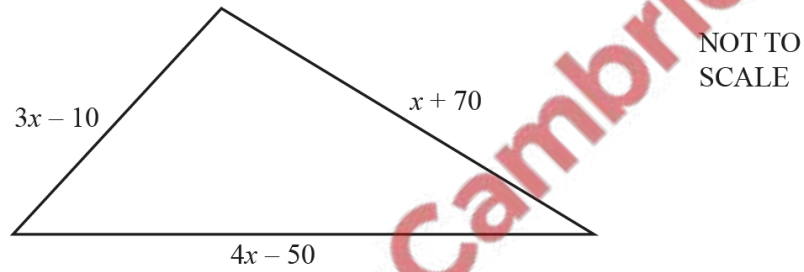
The perimeter of the triangle is 526 cm.

Find the value of x.

$x =$ .... [3]

(a) Expand and simplify.
$$(2p^2 - 3)(3p^2 - 2)$$

.... [2]

(b) $s = \frac{1}{2}(u+v)t$

(i) Find the value of s when $u = 20$, $v = 30$ and $t = 7$.

$s = $ .... [2]

(ii) Rearrange the formula to write v in terms of s, u and t.

$v = $ .... [3]

(c) Factorise completely.

(i) $2qt - 3t - 6 + 4q$

.... [2]

(ii) $x^3 - 25x$

.... [3]

$$f(x) = x^3 - 3x^2 - 4$$

(a) Find the gradient of the graph of $y = f(x)$ where $x = 1$.

.... [3]

(b) Find the coordinates of the turning points of the graph of $y = f(x)$.

(............. , ...) , (............. , ...) [4]

15
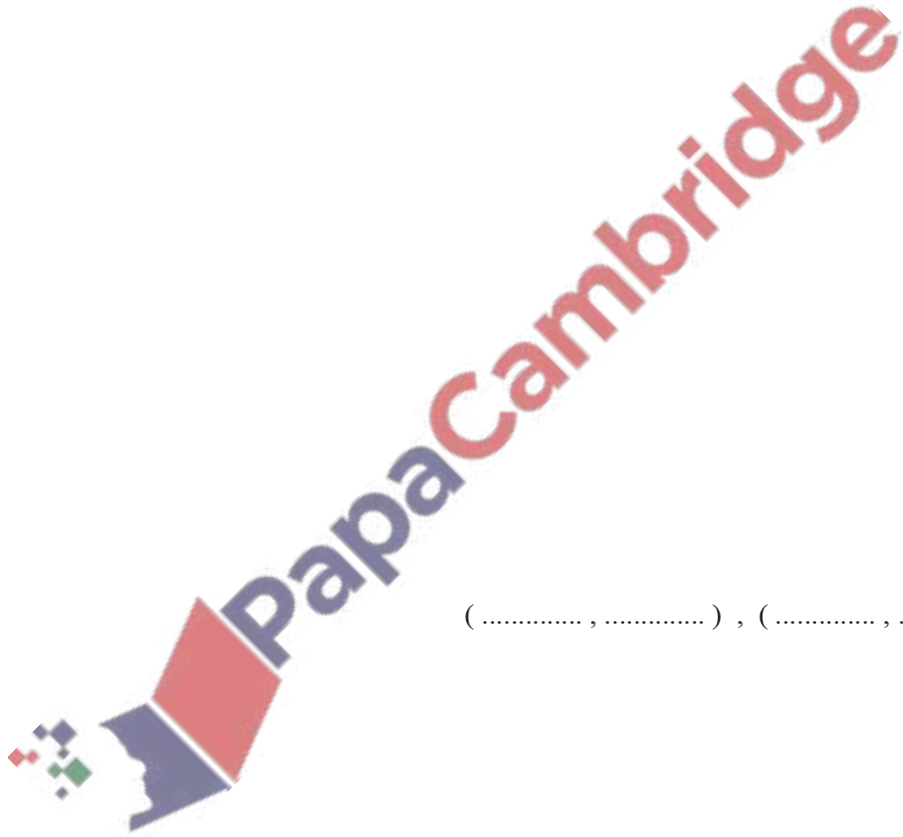
(c) Sketch the graph of $y = \text{f}(x)$.
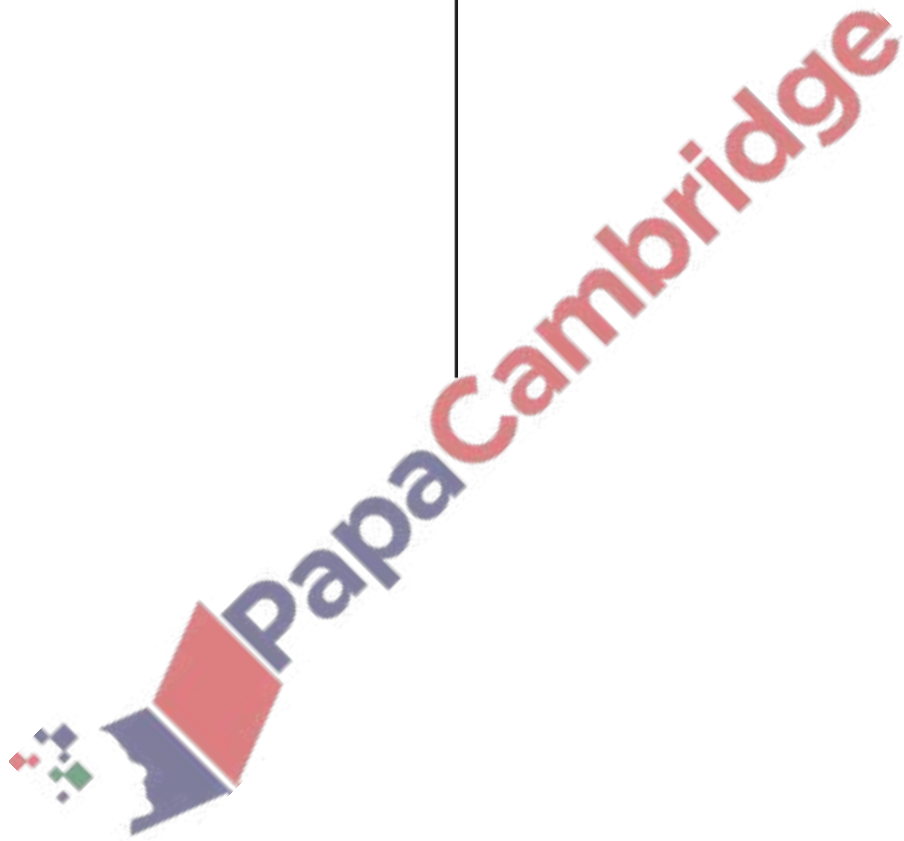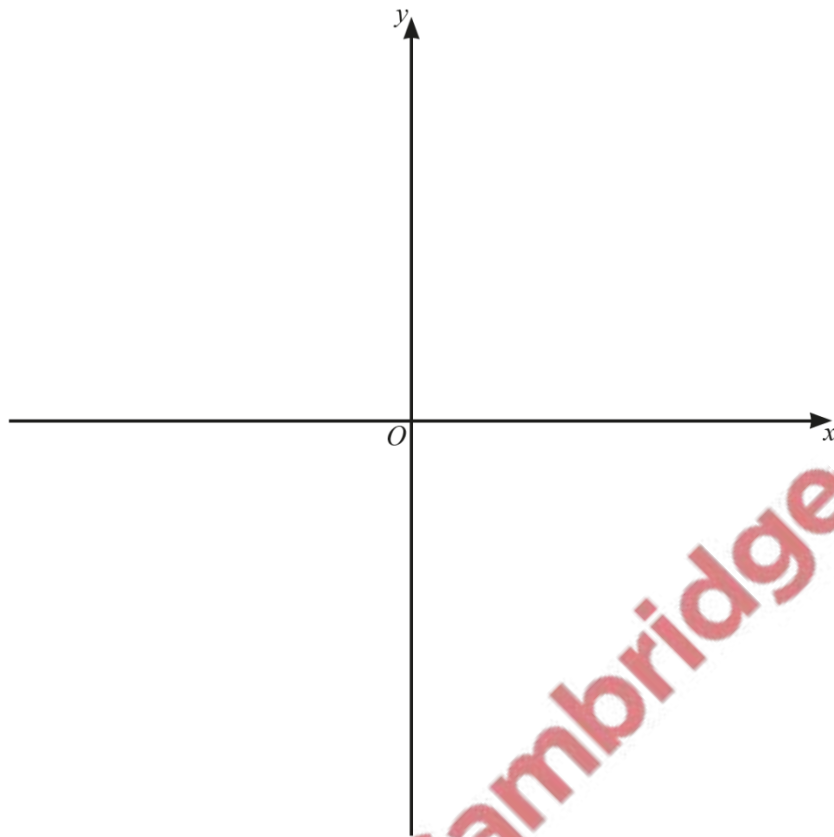
[2]

$$f(x) = 2x - 1 \qquad g(x) = 3x + 2 \qquad h(x) = \frac{1}{x}, x \neq 0 \qquad j(x) = x^2$$

(a) Find $j(-1)$.

.... [1]

(b) Find x when $f(x) + g(x) = 0$.

$x = $ .... [2]

(c) Find $gg(x)$, giving your answer in its simplest form.

.... [2]

(d) Find $hf(x) + gh(x)$, giving your answer as a single fraction in its simplest form.

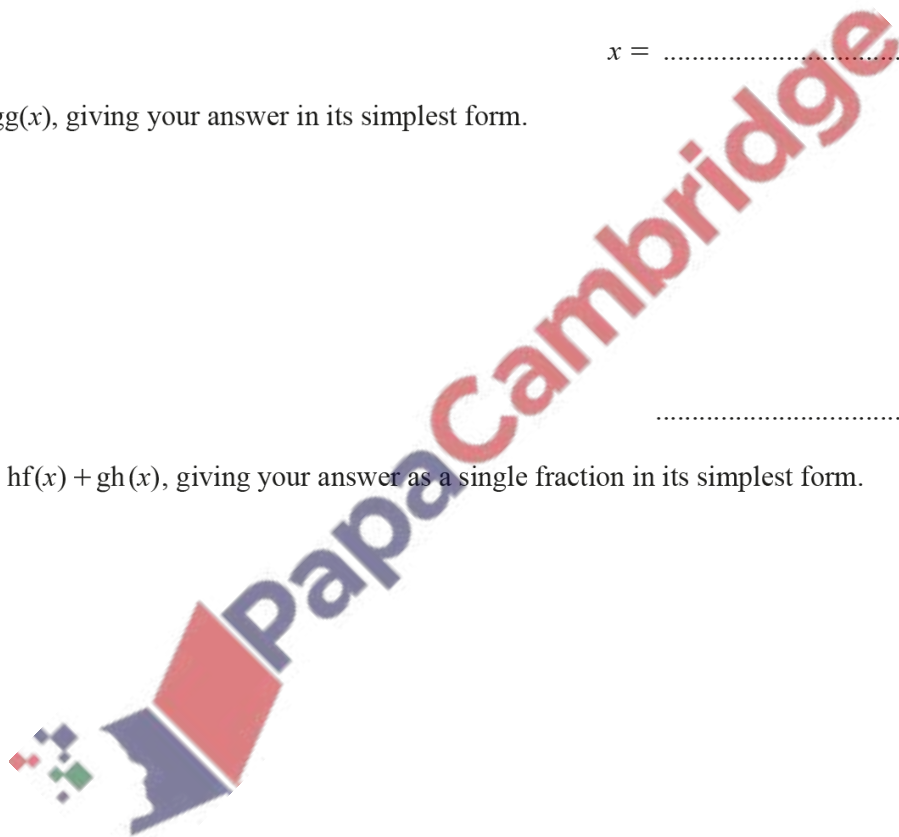... [4]

(e) When $pp(x) = x$, $p(x)$ is a function such that $p^{-1}(x) = p(x)$.

Draw a ring around the function that has this property.

$$f(x) = 2x - 1 \qquad g(x) = 3x + 2 \qquad h(x) = \frac{1}{x}, x \neq 0 \qquad j(x) = x^2$$

[1]

16.

Simplify.

$$3a - 5b - a - 6b$$

.... [2]
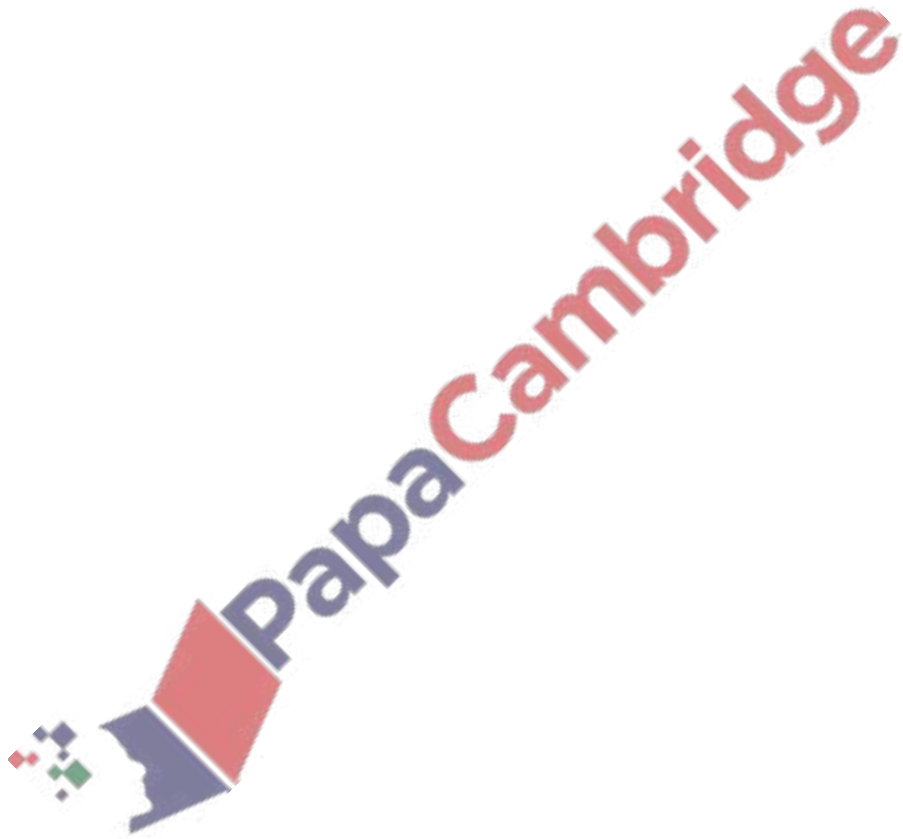
The cost of hiring a bicycle, $C, for y hours is given by the formula $C = 12 + 3.5y$.
Maria pays $36.50 to hire this bicycle.

Work out the number of hours she hires the bicycle for.

.... hours [2]

18.

Factorise completely.

$$8x^2 - 20x$$

.... [2]
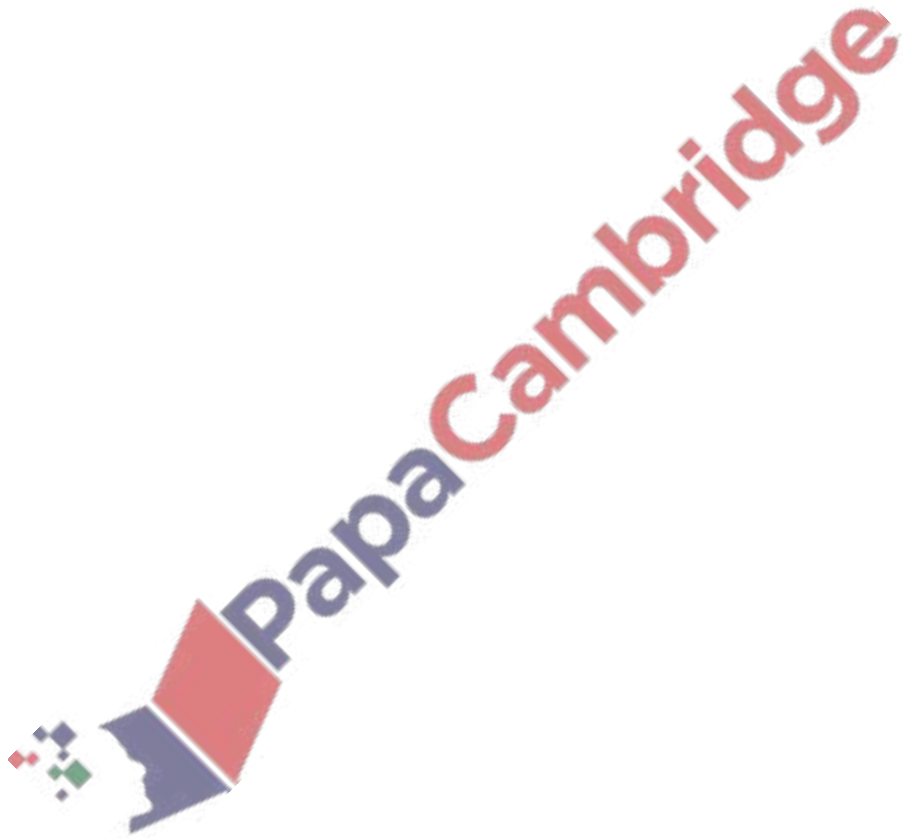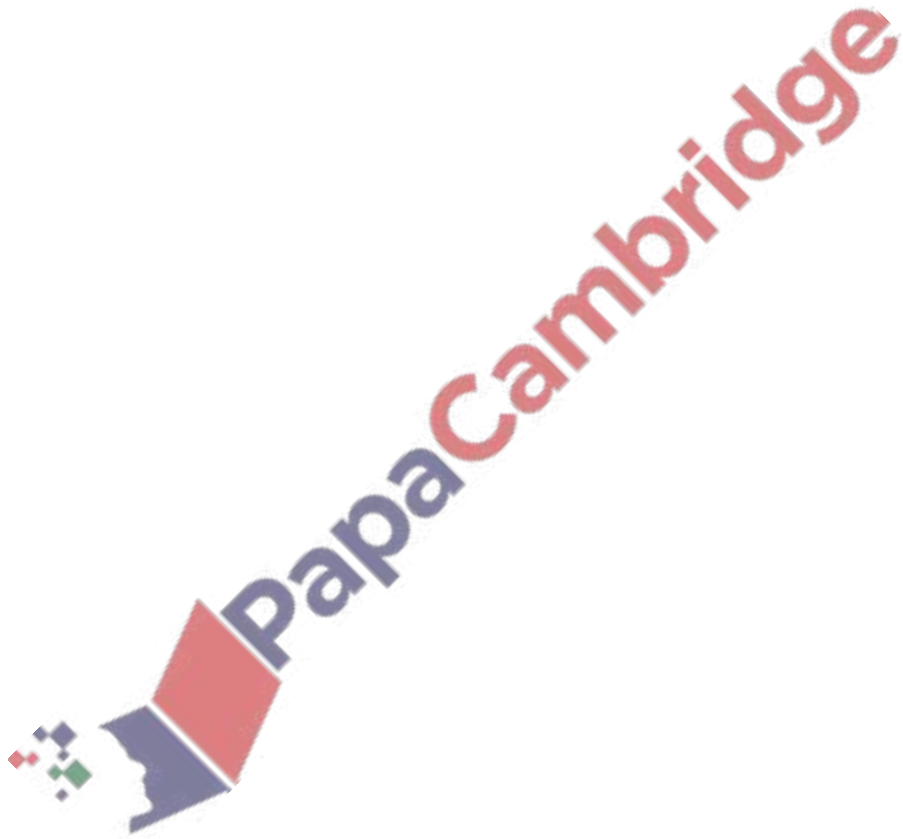
At a cinema, an adult ticket costs $a and a child ticket costs $c.

(a) Farah buys 3 adult tickets and 4 child tickets for $38.50.

Complete the equation.
$$3a + 4c = \text{.................................}$$ [1]

(b) Hana buys 6 adult tickets and 5 child tickets for $65.00.

Write down another equation in terms of a and c.

.... [1]

(c) Solve the two simultaneous equations to find the value of a and the value of c.
You must show all your working.

$a = $ ....

$c = $ .... [3]

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20.
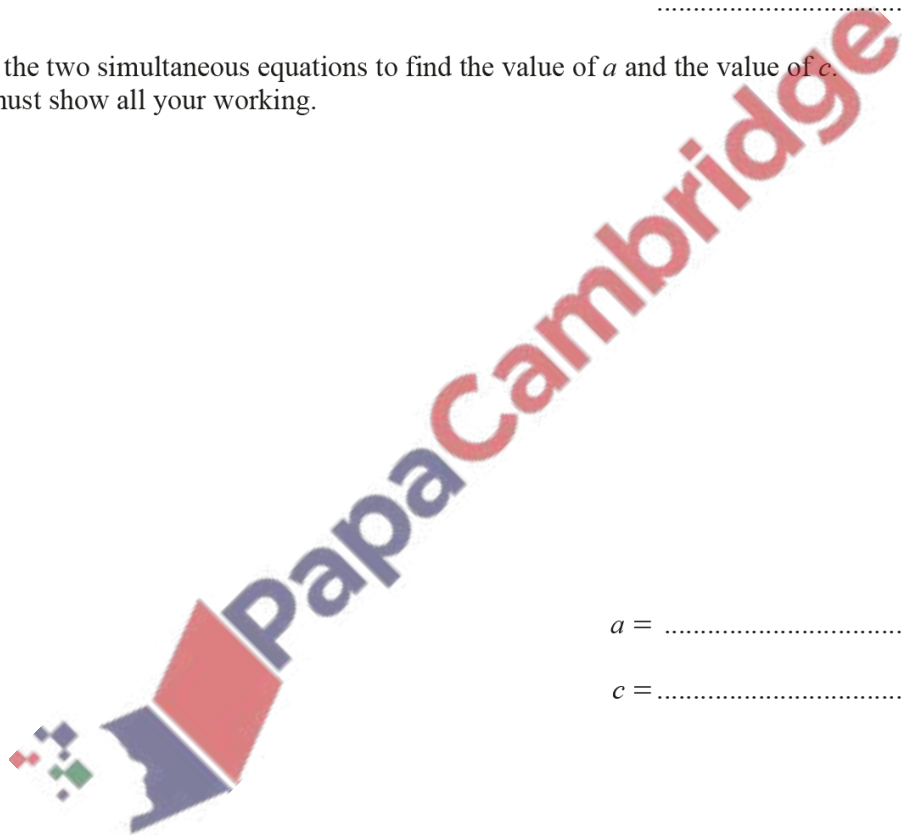
x is an integer where $x \geqslant -3$ and $x < 3$.

Write down all the possible values of x.

.... [2]
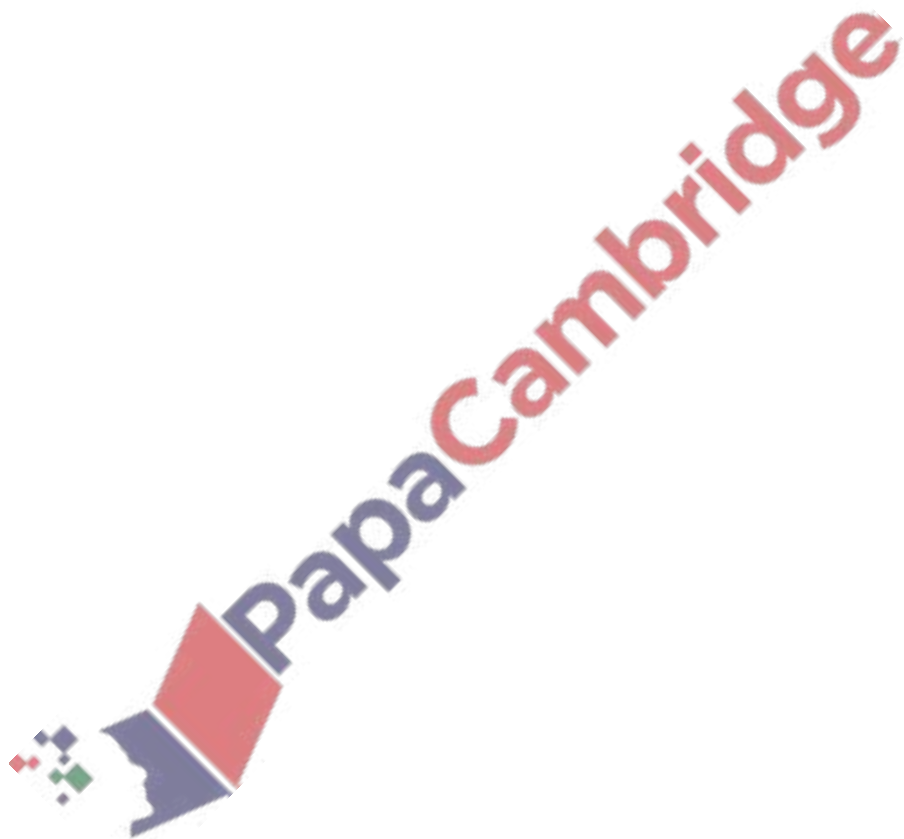
21.

Simplify.
$$18x^{12} \div 3x^3$$

.... [2]

22.

Simplify $d^8 \div d^2$.

.... [1]

23.
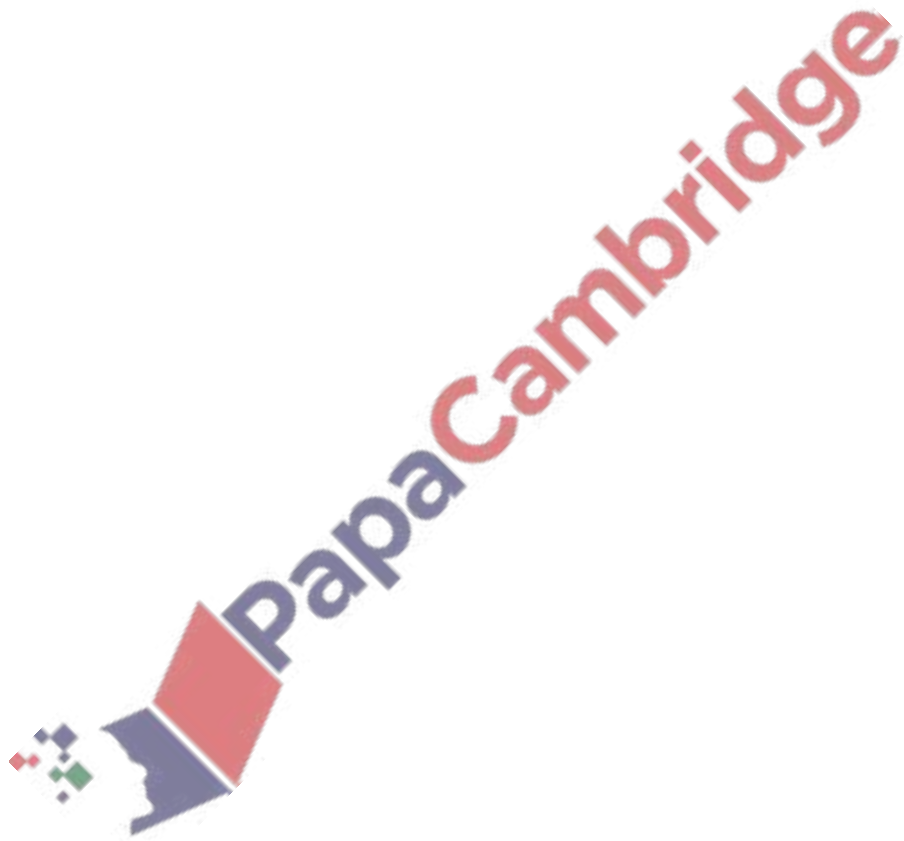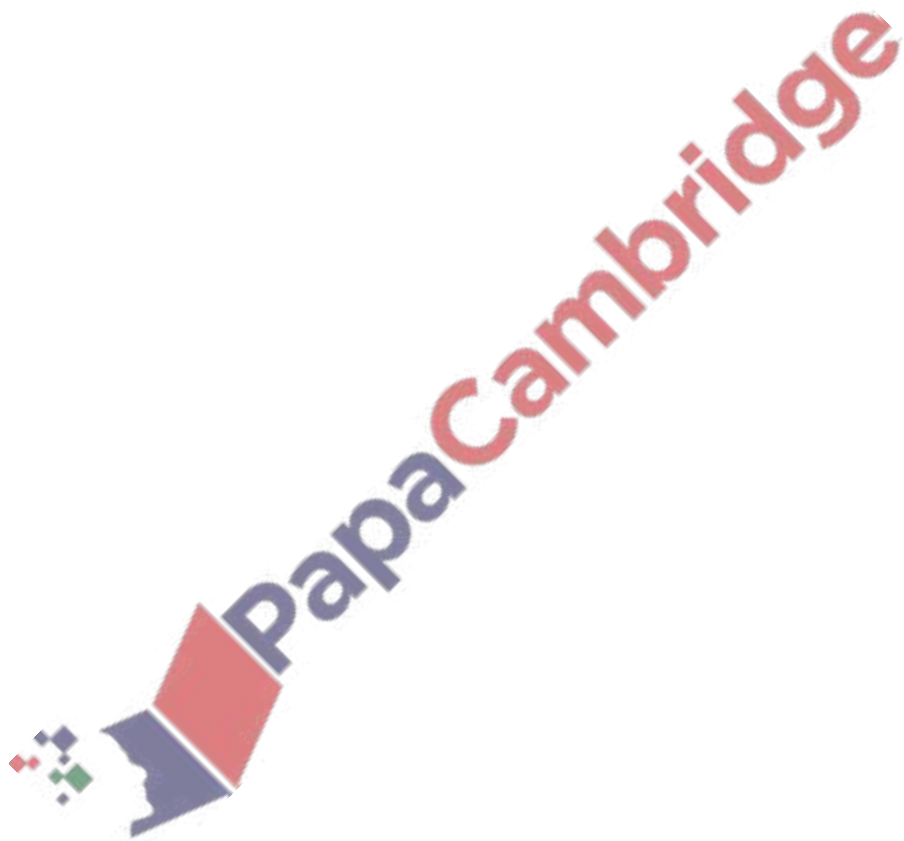
Expand and simplify.
$$(x-5)(x+8)$$

.... [2]
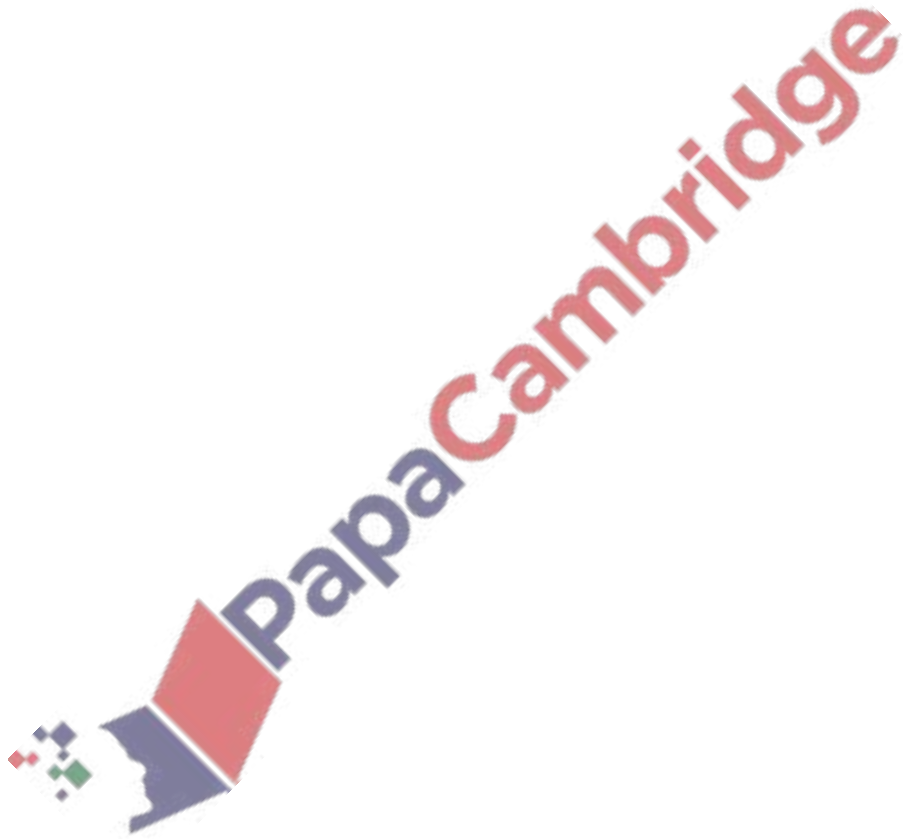
In this question, both lengths are in centimetres.

$10x - 12$

B

NOT TO
SCALE

A

C $2x + 3$

D
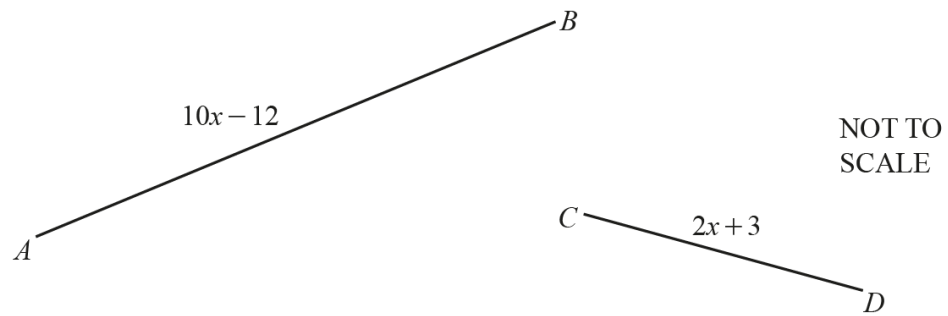
The diagram shows two lines, AB and CD.
The length of AB is $10x - 12$.
The length of CD is $2x + 3$.
Line AB is 3 times as long as line CD.

Work out the value of x.

$x =$ .... [4]

25.
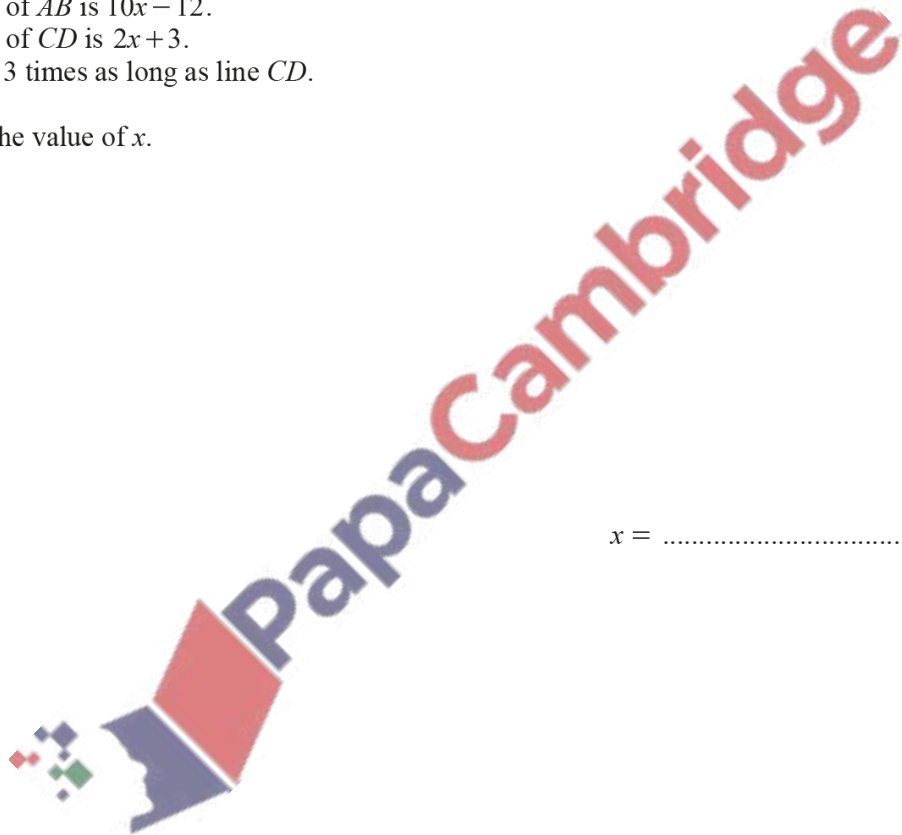
Simplify $\left(81x^{12}\right)^{\frac{3}{4}}$.

.... [2]
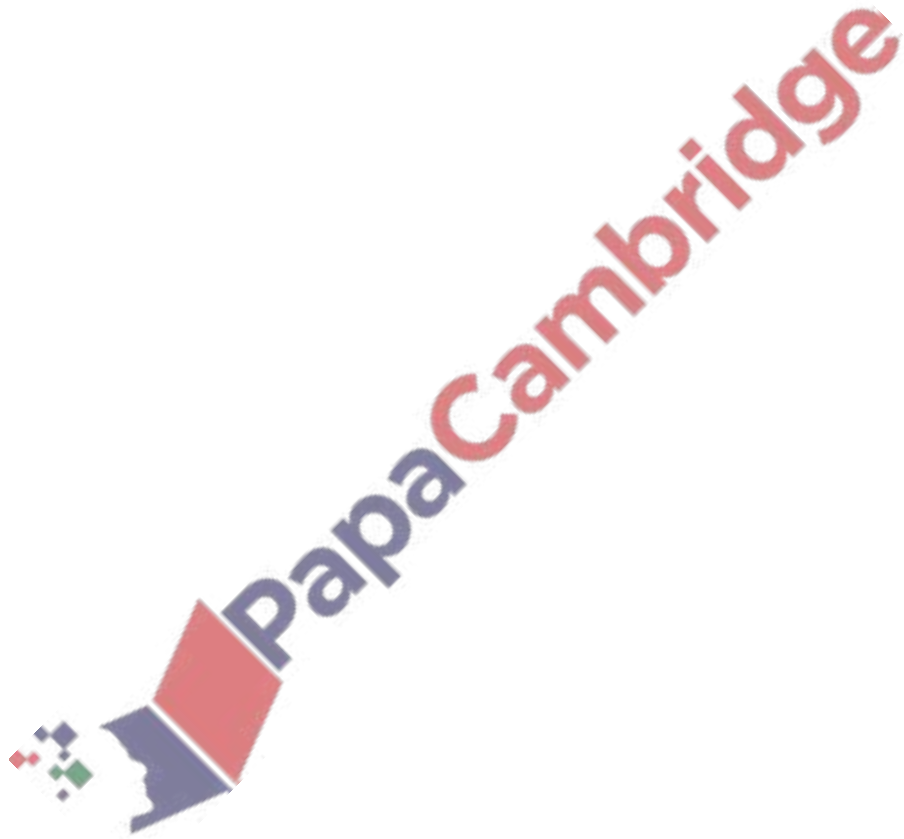
The table shows some values for $y = 3x^2 - 2x - 1$.

x	-1	-0.5	0	0.5	1	1.5
y	4		-1		0	2.75

(a) Complete the table. [1]

(b) On the grid, draw the graph of $y = 3x^2 - 2x - 1$ for $-1 \leqslant x \leqslant 1.5$.
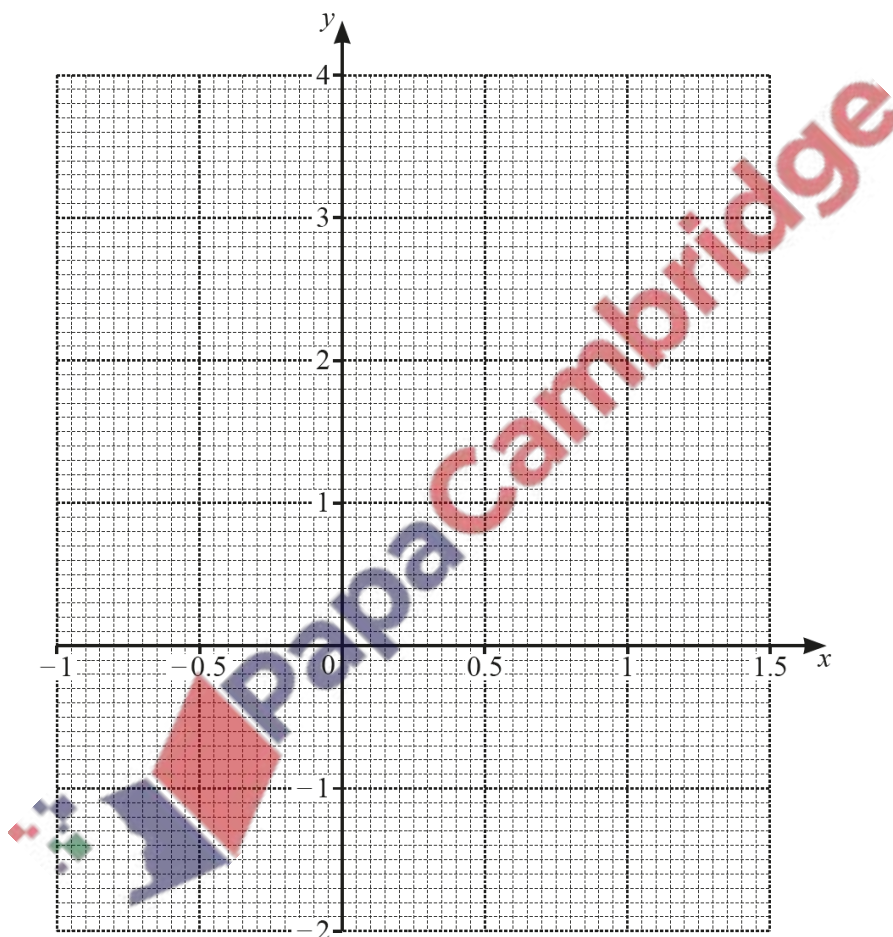
[3]

(c) By drawing a suitable straight line, solve the equation $3x^2 - 4x - 2 = 0$ for $-1 \leqslant x \leqslant 1.5$.

$x = $ .... [3]

A curve has equation $y = x^3 - 12x$.

(a) Find the gradient of the curve at the point $(1,-11)$.

.... [3]

(b) Find the coordinates of the turning points of the curve.

(................ , ...) and (................ , ...) [3]

29
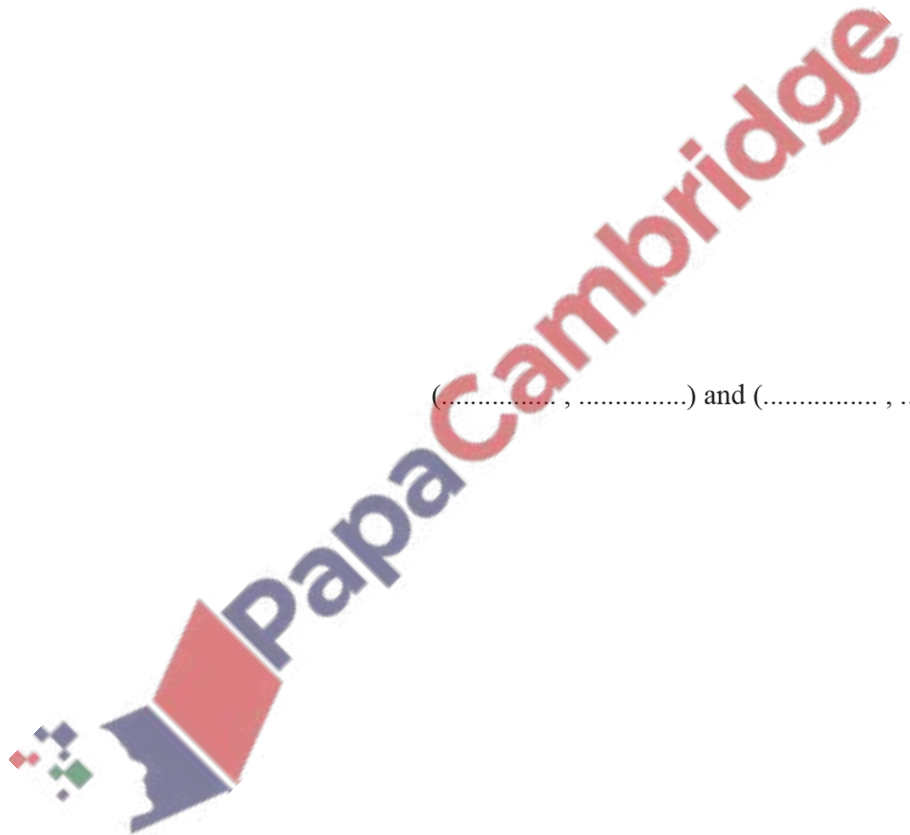
Solve.

(a) $\dfrac{30}{x} = 6$

$x =$ .... [1]

(b) $11x - 3 \geqslant 2(2x + 9)$

.... [3]
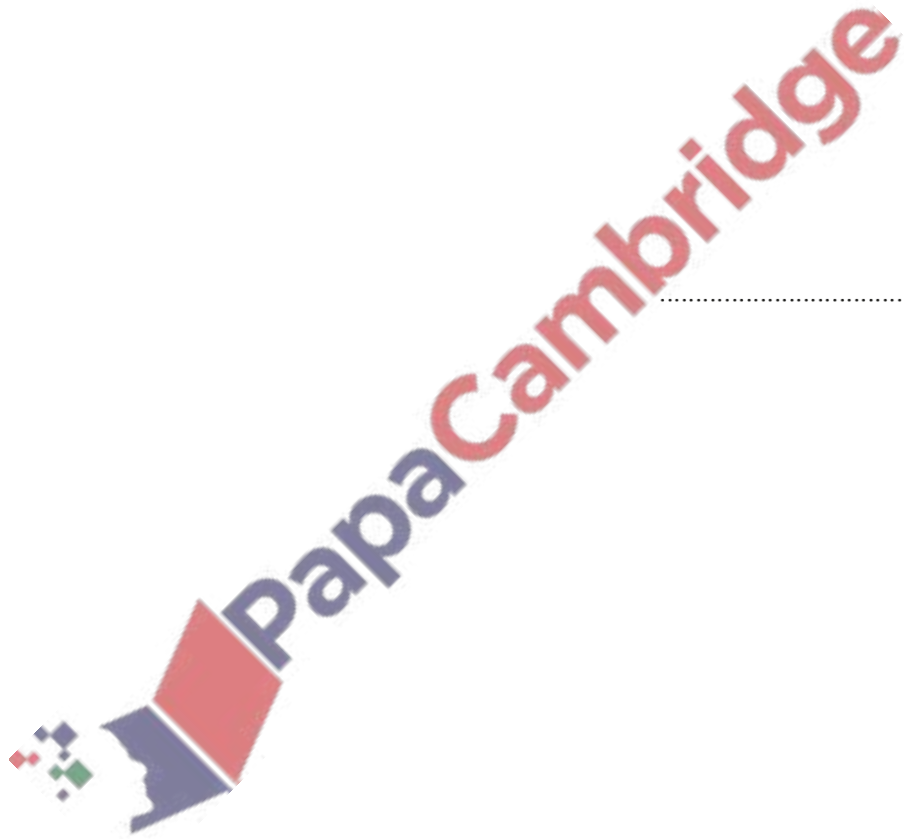
One solution of the equation $ax^2 + b = 181$ is $x = 8$.
a and b are both positive integers **greater than 1**.

(a) Find the value of b.

$b =$ .... [2]

(b) Write down the other solution of the equation $ax^2 + b = 181$.

$x =$ .... [1]
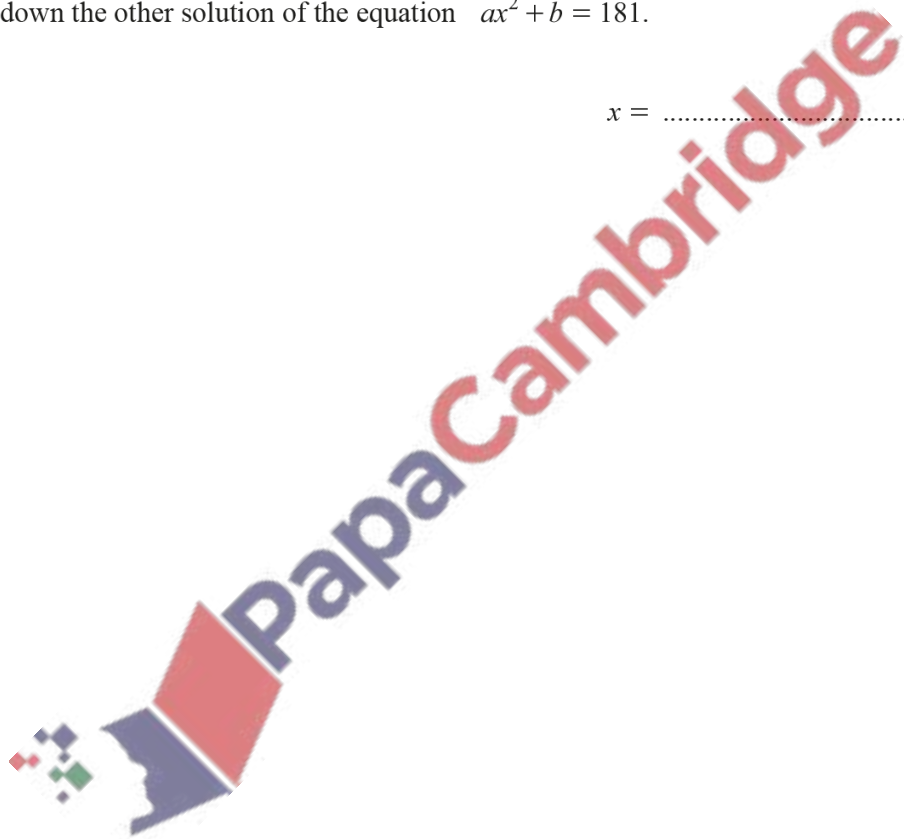
30.

$$f(x) = 5x + 2$$

Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

31.
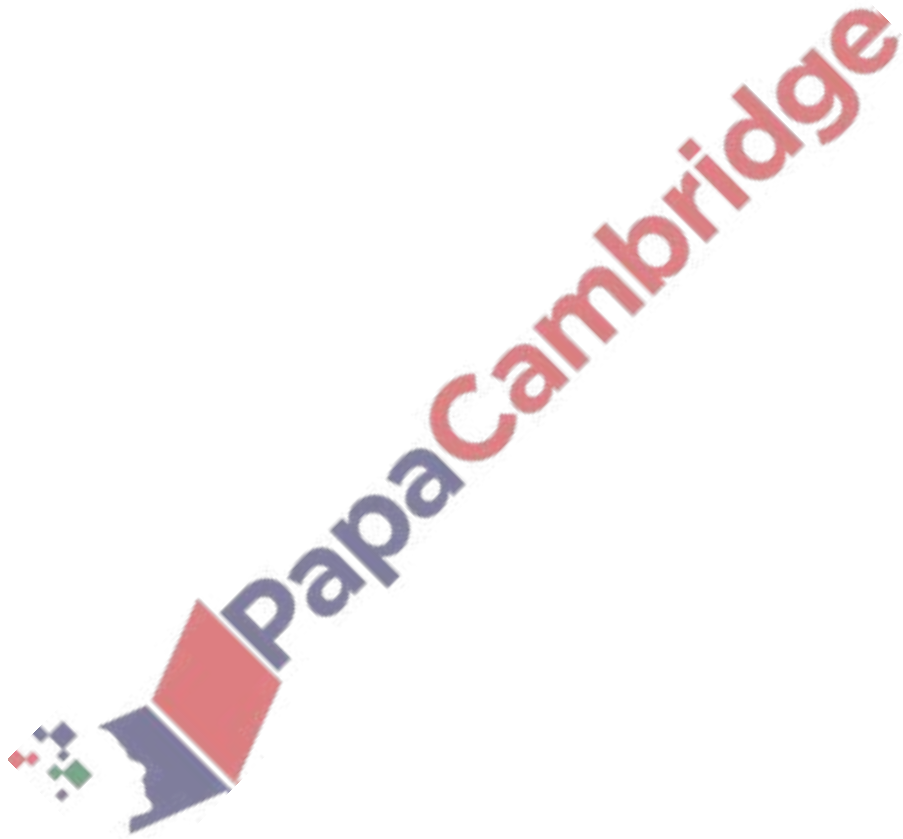
Make x the subject of the formula.

$$c = \frac{3x}{2x-5}$$

$x = $ .... [4]
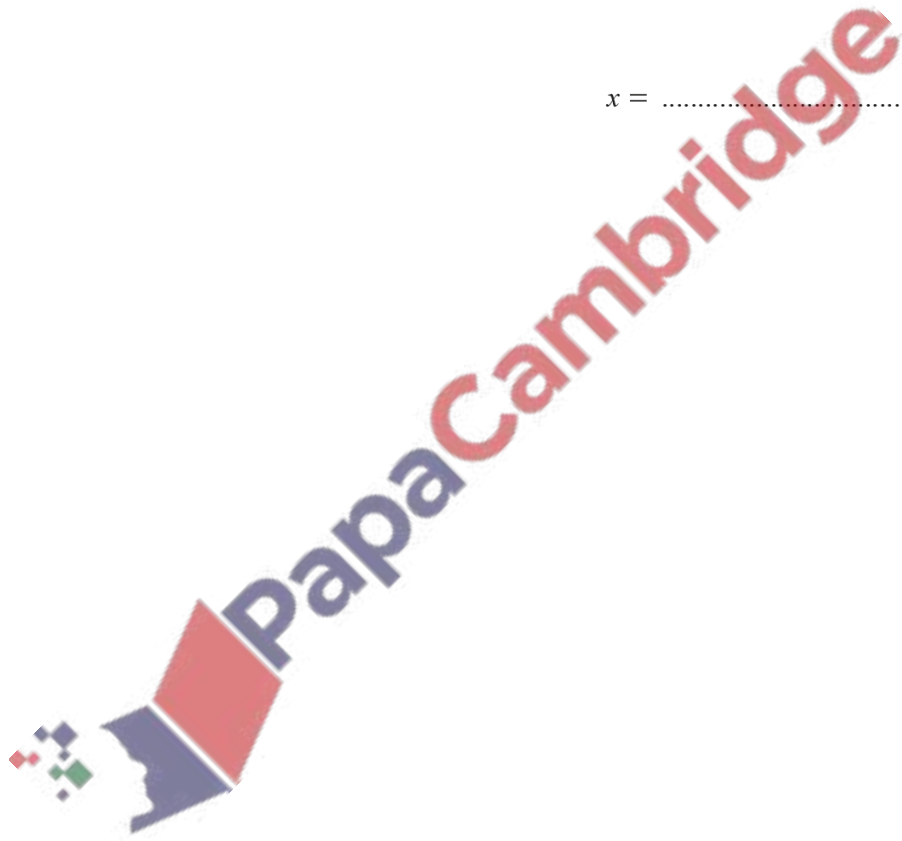
Write as a single fraction in its simplest form.

$$\frac{5}{3x+2} + \frac{4}{2x-1}$$

.... [3]
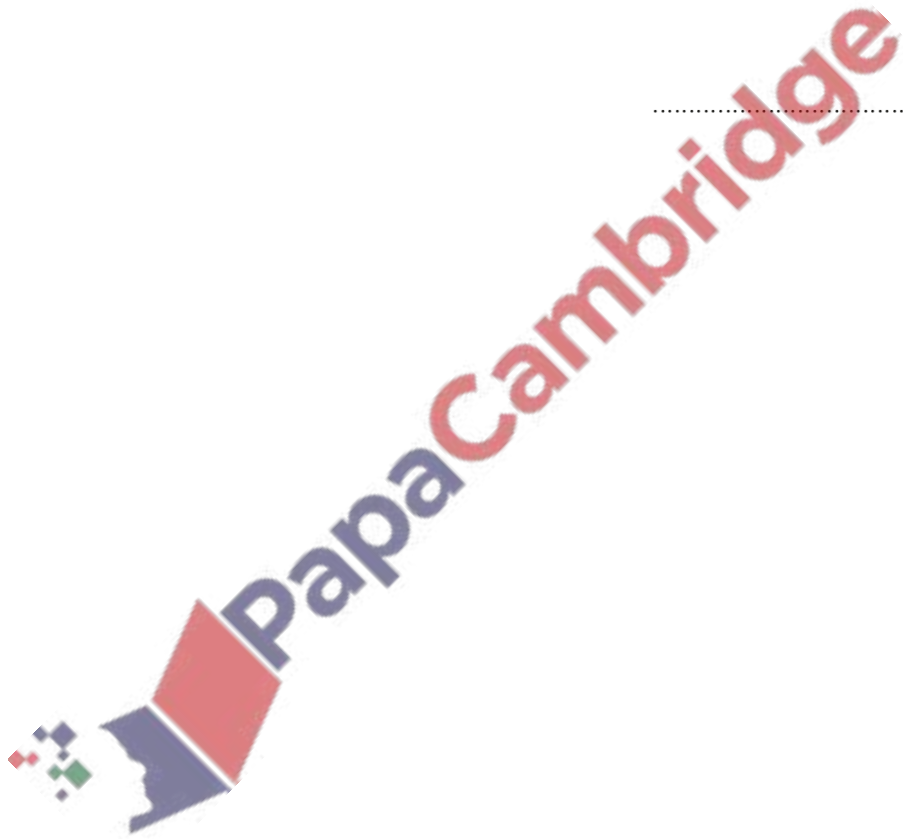
$$v = u - 9.8t$$

Find the value of v when $u = 4$ and $t = -7$.

$v =$ .... [2]
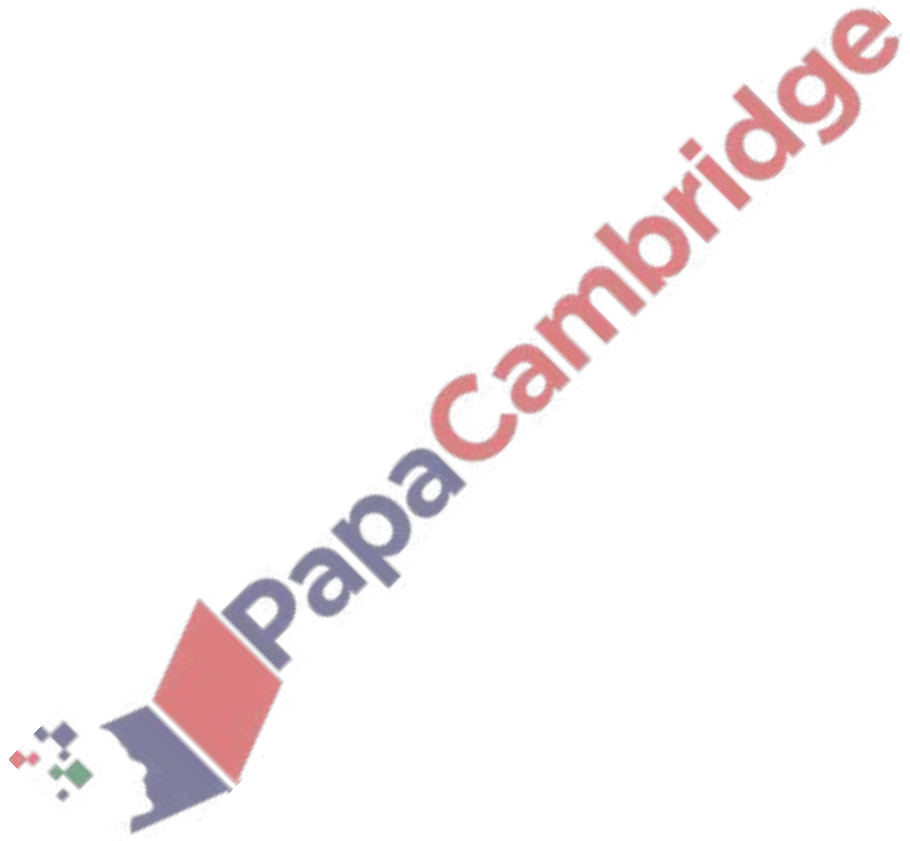
The distance–time graph shows information about Kai's journey from home to the office.
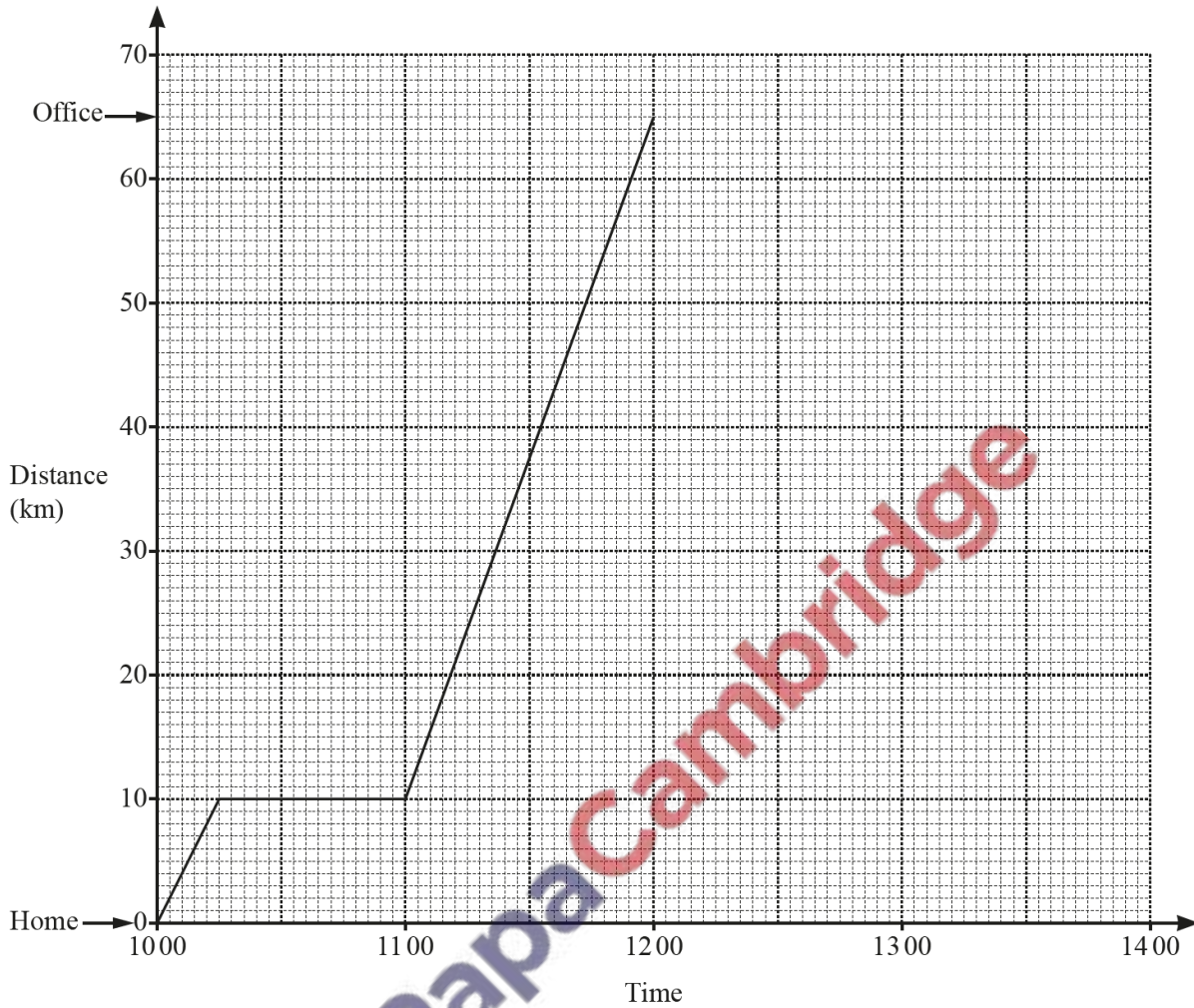
(a) Calculate the average speed, in km/h, for Kai's journey from home to the office.

.... km/h **[2]**

(b) When Kai arrives at the office, he finds his meeting is cancelled. He immediately returns home at a constant speed of 50 km/h.

Complete the distance–time graph to show his journey home. **[1]**

Rearrange the formula to make m the subject.

$$R = \frac{2(m-k)}{m}$$

$m = $ .... [4]
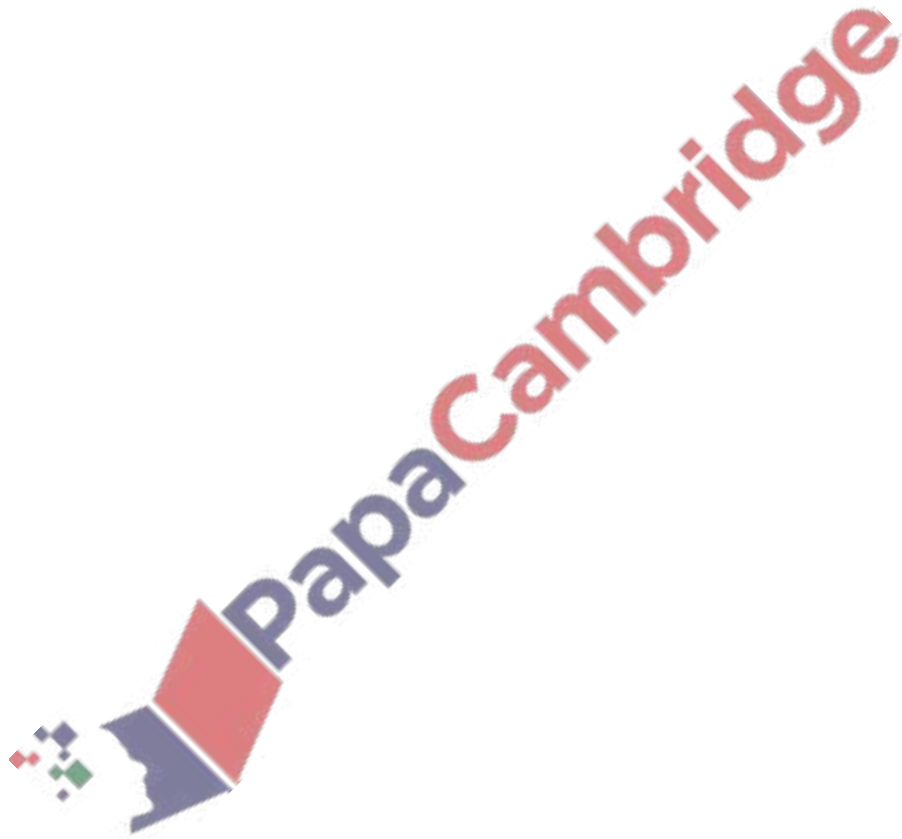
Solve the equation $x^2 + 5x - 7 = 0$.

You must show all your working and give your answers correct to 2 decimal places.

$x =$ .... or $x =$ .... [4]
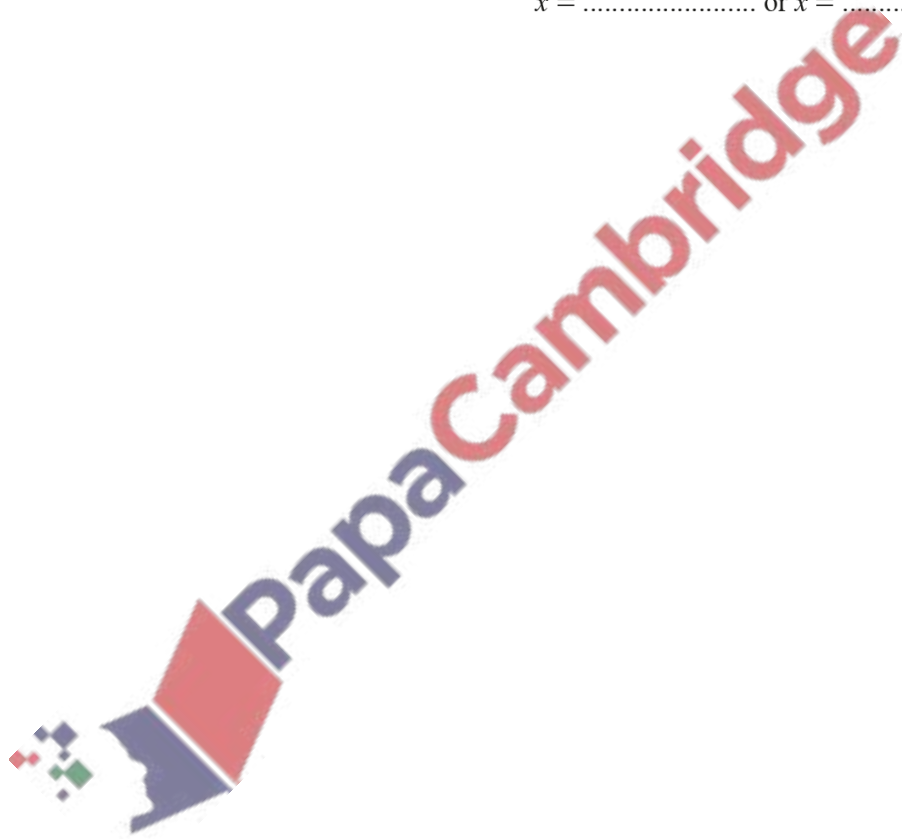
$$f(x) = 6x - 7 \qquad g(x) = x^{-3}$$

(a) Find $f(x+2)$.

Give your answer in its simplest form.

.... [2]

(b) Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

(c) Find x when $g(x) = f(22)$.

$x =$ .... [2]

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38.

Simplify.

$$\frac{2x^2 + 5x - 12}{4x^2 - 9}$$

.... [4]
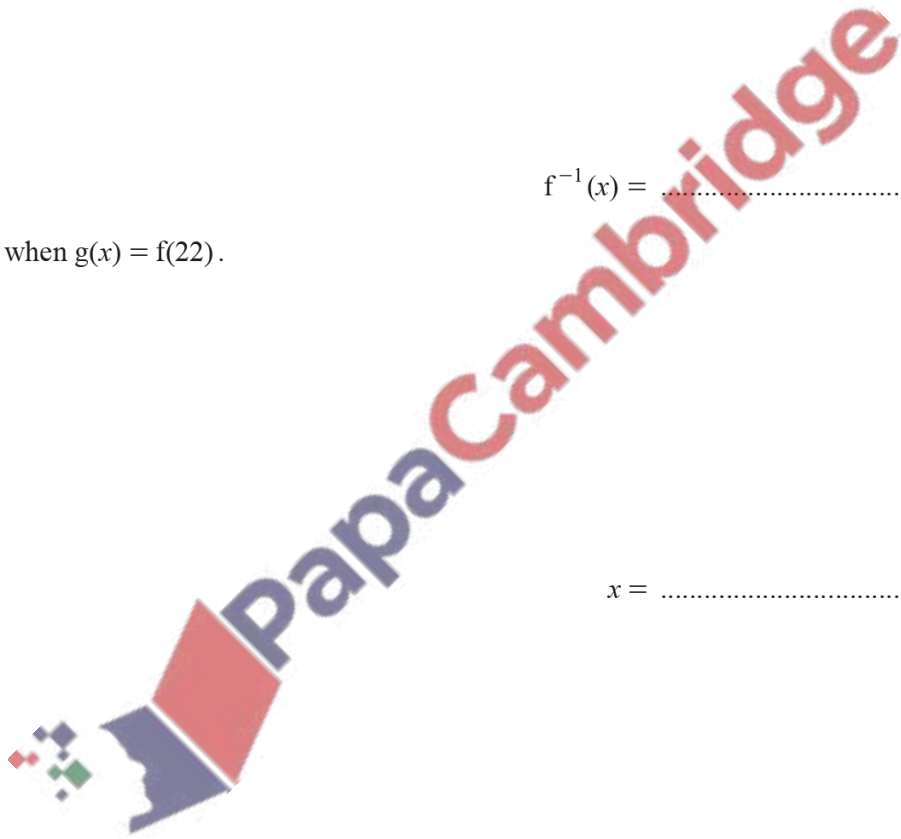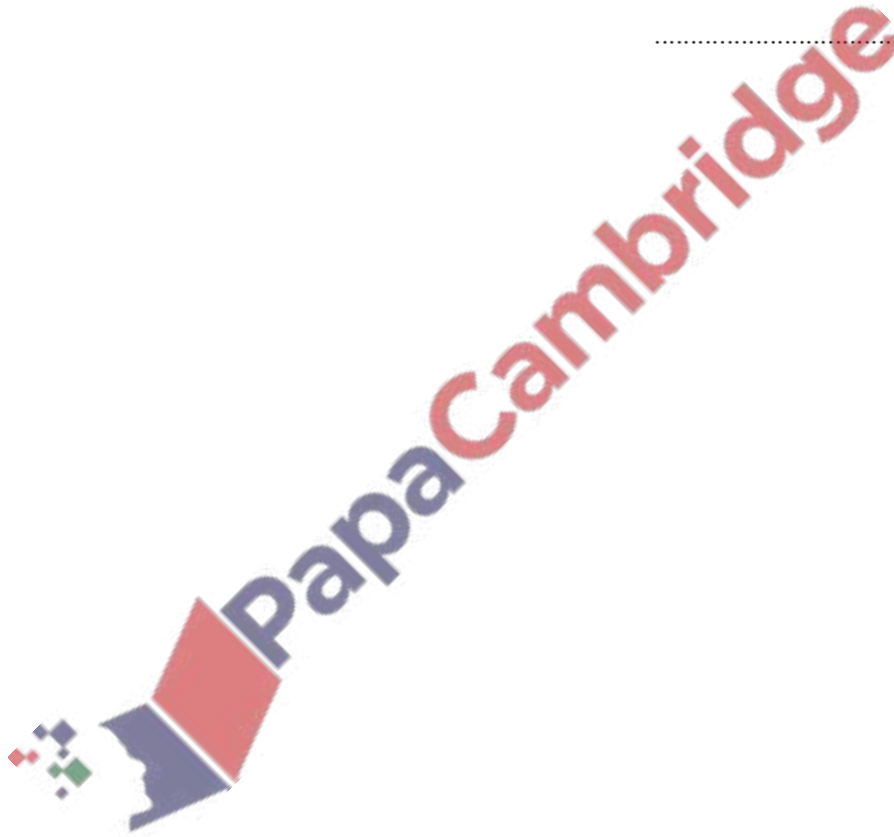
These are the first four terms of a sequence.

$$2.75 \qquad 6 \qquad 11.25 \qquad 20$$

The nth term of this sequence is $\frac{1}{4}n^3 + an^2 + bn$.

Calculate the value of a and the value of b.
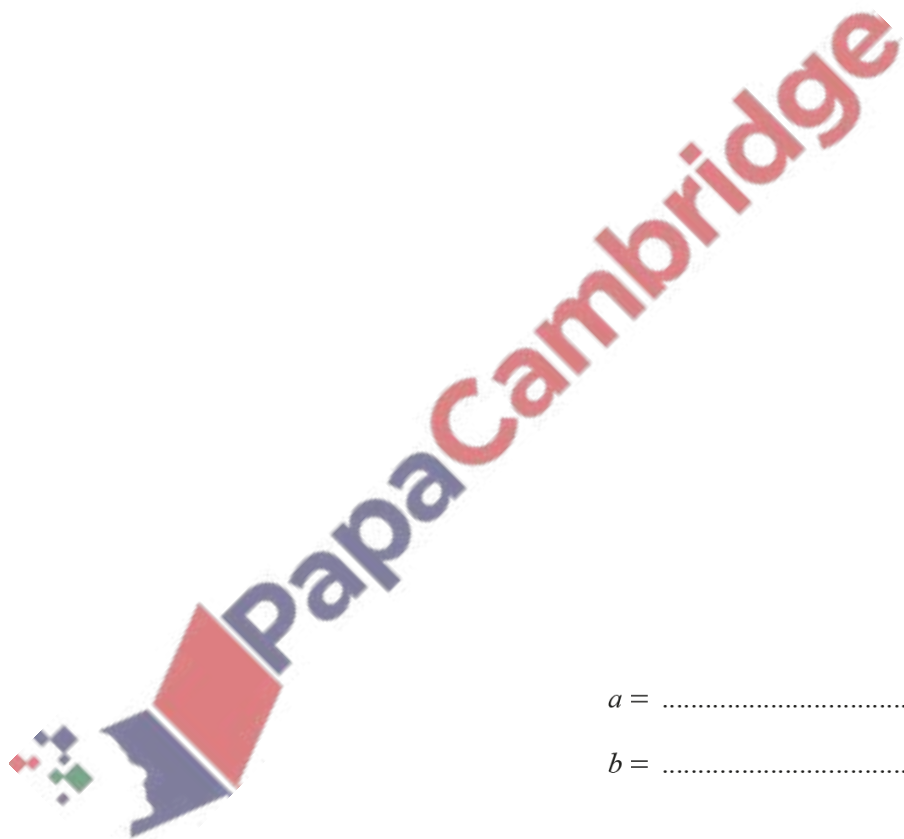
$a =$ ....

$b =$ .... [5]

(a) Complete the table of values for $y = 5 + 3x - x^2$.

x	-2	-1	0	1	2	3	4	5
y		1			7			-5

[3]

(b) On the grid, draw the graph of $y = 5 + 3x - x^2$ for $-2 \leqslant x \leqslant 5$.
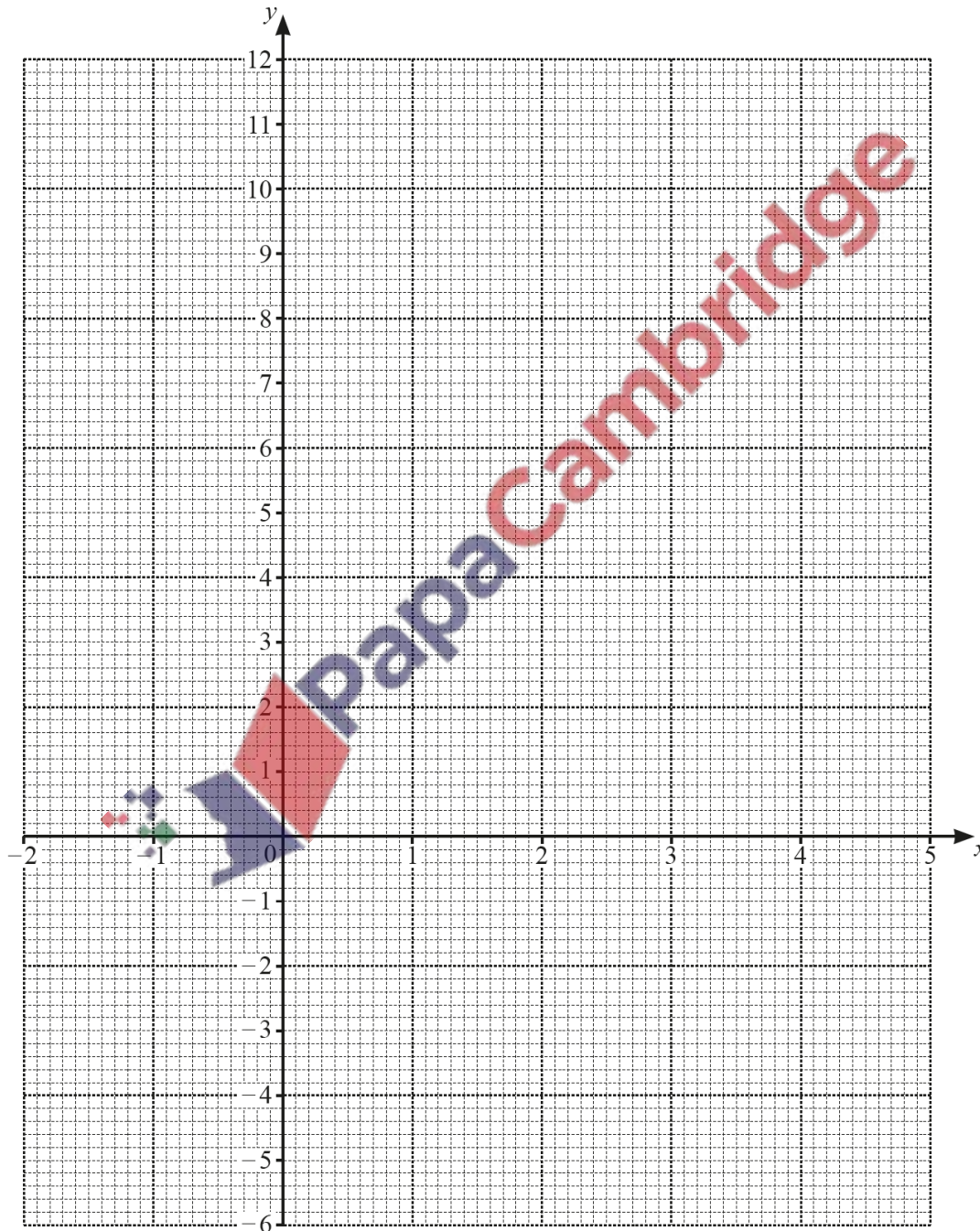
[4]

(c) Write down the equation of the line of symmetry of the graph.

.... [1]

(d) **(i)** Complete the table of values for $y = 2x + 1$.

x	-1	0	2
y			

[2]

(ii) On the grid, draw the graph of $y = 2x + 1$ for $-2 \leqslant x \leqslant 5$. [1]

(e) Write down the coordinates of the two points where the two graphs intersect.

(...................... , ...) and (...................... , ...) [3]
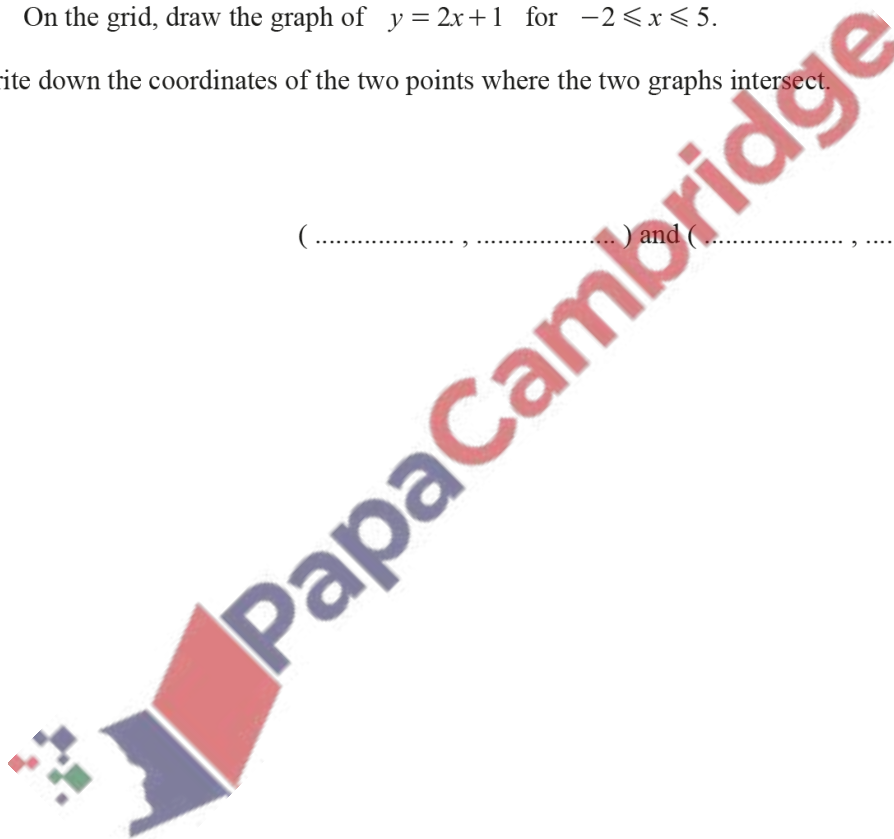
Two friends, Diego and Javier, meet at a swimming pool.
The travel graph shows Diego's journey by bicycle from his home to the swimming pool.
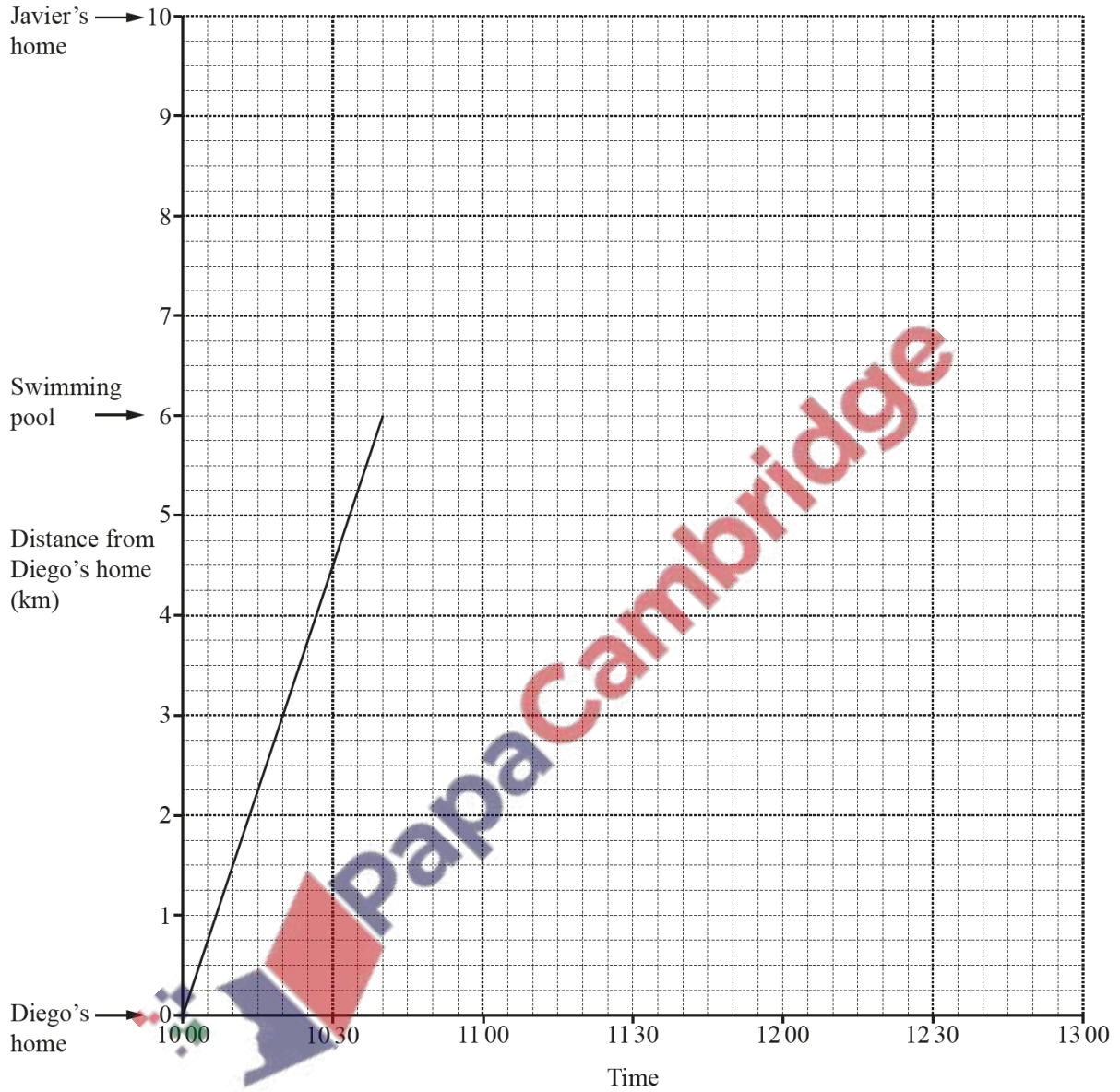
(a) Calculate Diego's speed for his journey from his home to the swimming pool.
Give your answer in kilometres per hour.

.... km/h **[2]**

(b) Diego stays at the swimming pool until 12 20.

   (i) On the grid, draw the line representing the time he stays at the swimming pool. [1]

   (ii) Work out how long, in hours and minutes, he is at the swimming pool.

   .... h .... min [1]

(c) Javier leaves his home 15 minutes later than Diego.
He walks to the swimming pool at a constant speed of 6 km/h.

On the grid, show Javier's journey from his home to the swimming pool.

[3]

(d) They both leave the swimming pool at 12 20 and return to their own homes, each at a constant speed.
Diego arrives home at 12 45.
Javier arrives home 5 minutes later than Diego.

Complete the travel graph. [2]

45
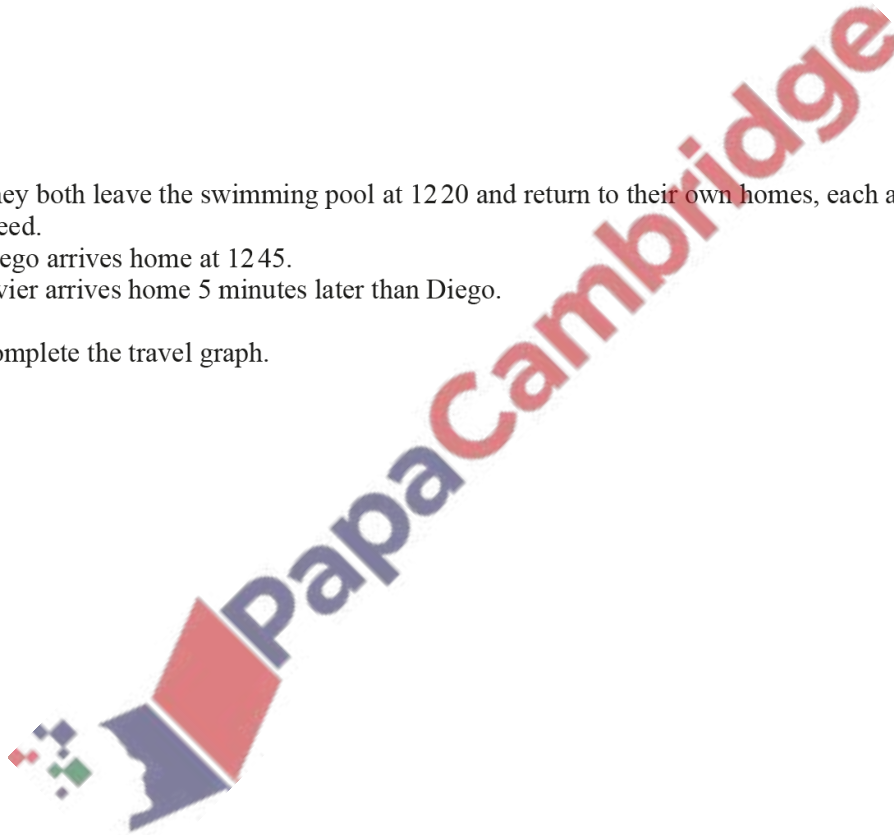
(a)

50 Australian dollars = 540 South African rands
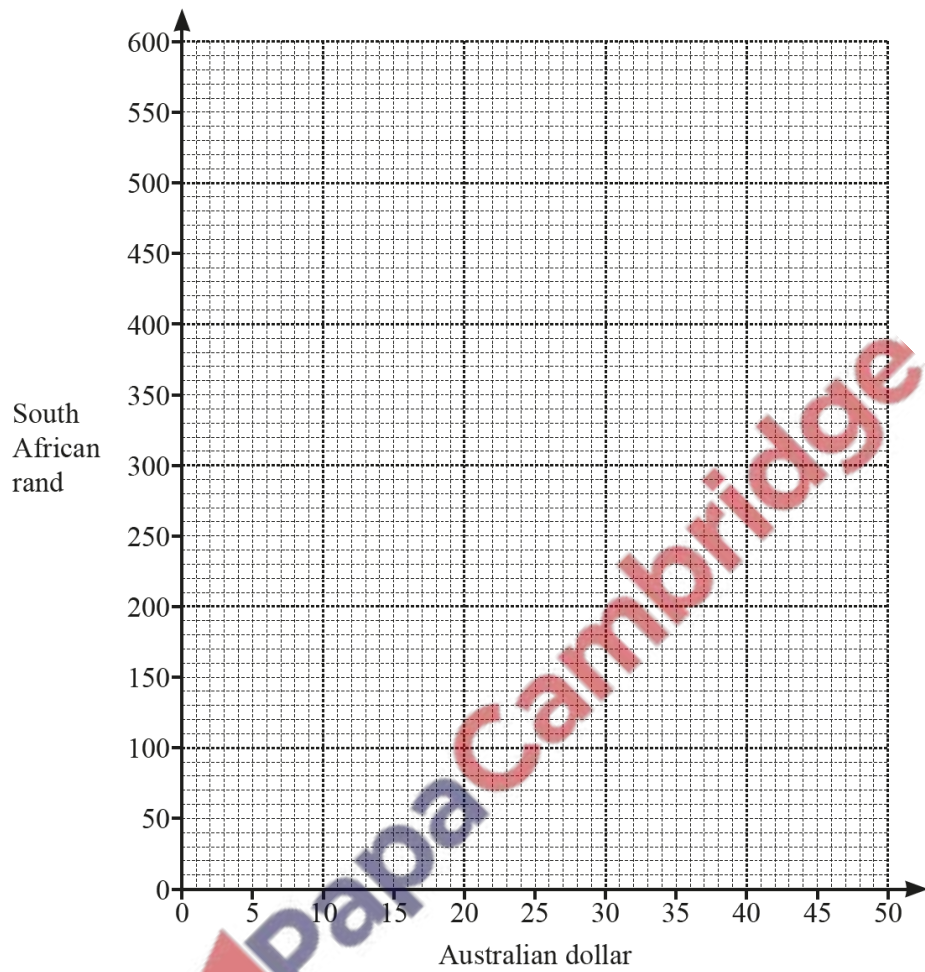
(i) On the grid, draw a conversion graph between Australian dollars and South African rands.

[2]

(ii) A watch costs 1350 South African rands.

Find the cost of this watch in Australian dollars.

.... Australian dollars [2]

(b) (i) A plane leaves Sydney at 21 48 local time to fly to Johannesburg.
The flight takes 14 hours 15 minutes.
The local time in Sydney is 8 hours ahead of the local time in Johannesburg.

Find the local time in Johannesburg when the plane arrives.

.... [3]

(ii) On the plane there are 315 people.
The ratio of children : adults = 7 : 8.

Work out the number of adults on the plane.

.... [2]

(iii) Another plane has 420 seats.
90% of the seats are occupied.

Work out the number of seats that are occupied.

.... [2]

Fidel gives different amounts of water to some plants.
The scatter diagram shows the height (cm) and the amount of water (ml) for each of 15 plants.
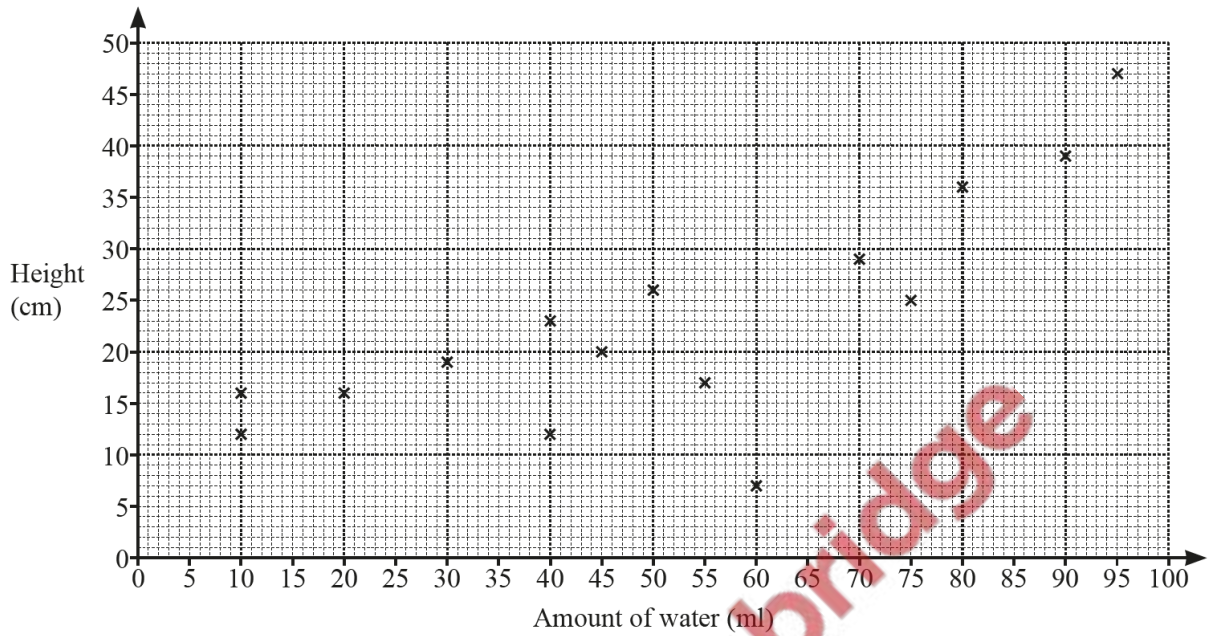
(a) Plot these two results on the scatter diagram.

Amount of water (ml)	60	85
Height (cm)	27	41

[1]

(b) What type of correlation is shown in the scatter diagram?

.... [1]

(c) One of the plants had a lower height than expected for the amount of water given.

On the scatter diagram, put a ring around the point for this plant. [1]

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(d) (i) On the scatter diagram, draw a line of best fit. [1]

(ii) Another plant is given 65 ml of water.

Use your line of best fit to estimate the height of this plant.

.... cm [1]

(e) Find the percentage of these 17 plants that have a height of more than 24 cm.
Give your answer correct to 1 decimal place.

.... % [3]
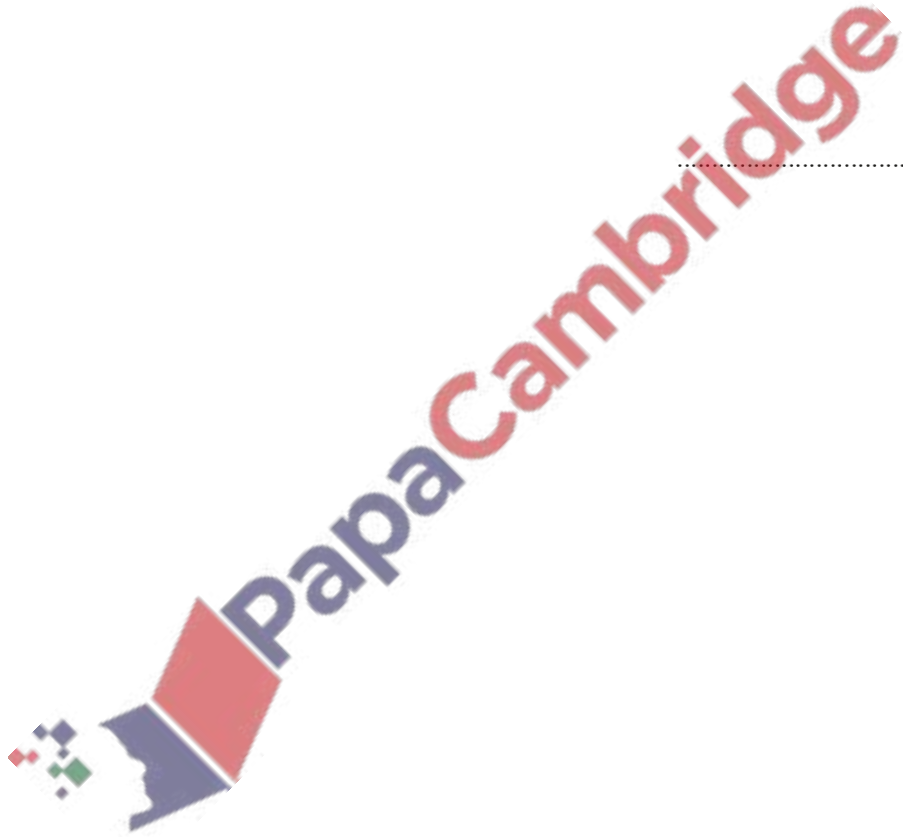
(a) $T = 5P + 3Q$

Find the value of T when $P = 6$ and $Q = 8$.

$T =$ .... [2]

(b) Simplify.
$$3a - 7b + 2a + 4b$$

.... [2]

(c) Multiply out.
$$5(2x - 3y)$$

.... [1]

(d) Solve.
$$5x - 1 = 3x + 19$$

$x =$ .... [2]

(e) Make t the subject of the formula $p = 5t - 3$.

$t =$ .... [2]
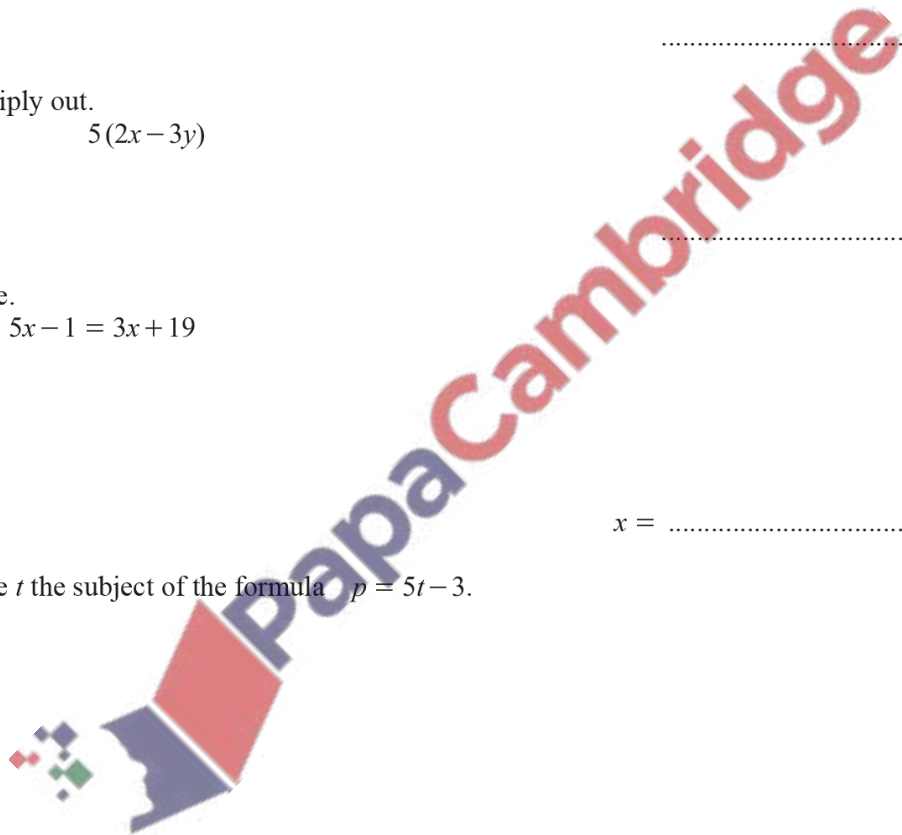
(f) Entry to a castle costs $x for an adult and $y for a child.

Entry for 2 adults and 3 children costs $15.00 .
Entry for 3 adults and 5 children costs $23.50 .

Write down a pair of simultaneous equations to show this information and solve them to find the value of x and the value of y.
You must show all your working.
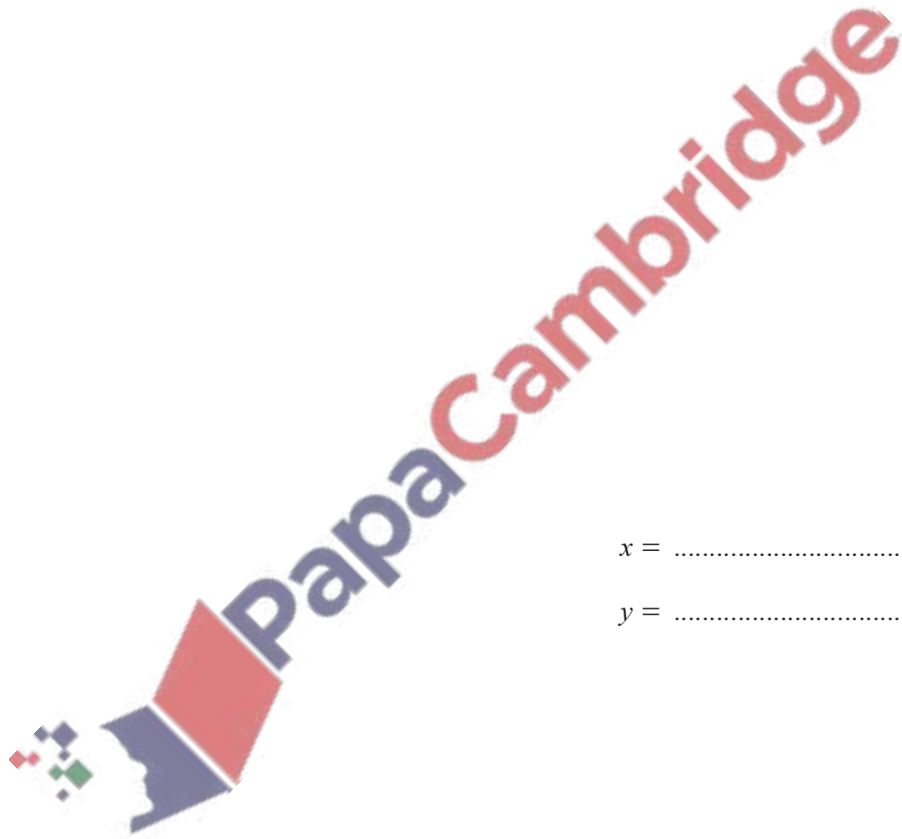
$x = $ ....

$y = $ .... [6]

(a) A boat sails from A to B.
The travel graph shows this journey.
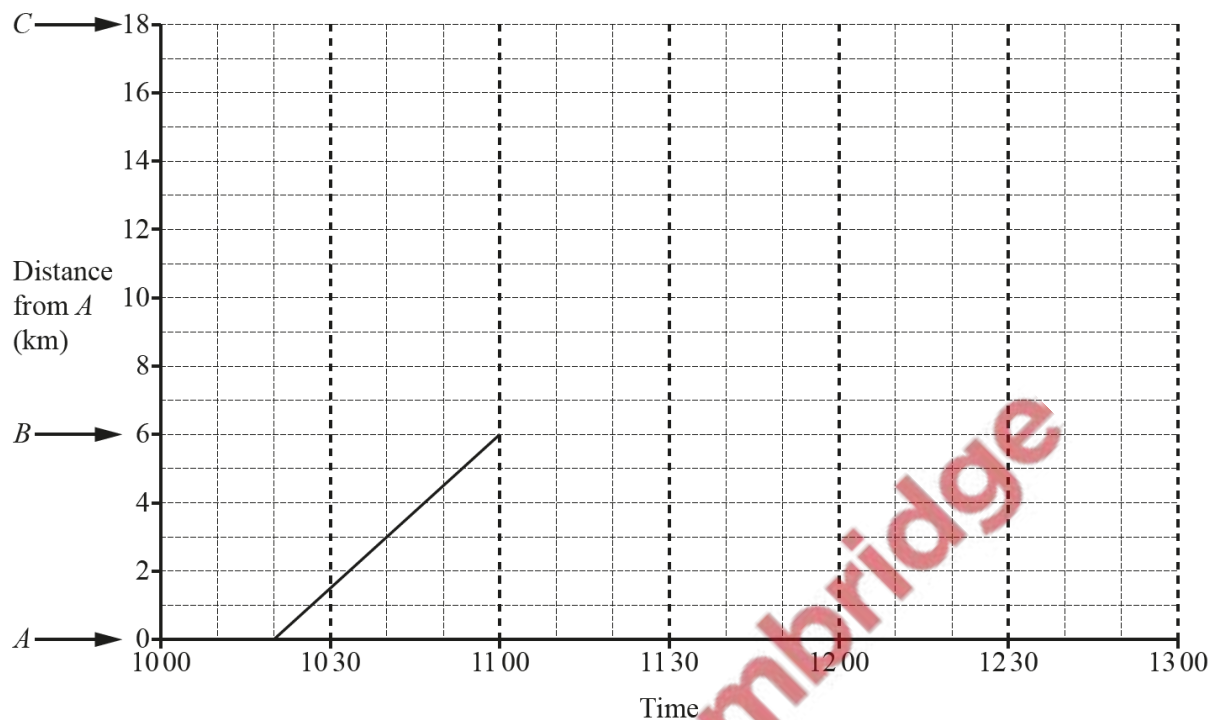
(i) Write down the time that the boat leaves A. .... [1]

(ii) Work out how long, in minutes, it takes the boat to sail from A to B.

.... min [1]

(iii) The boat stays at B for 20 minutes.
The boat then sails to C at a constant speed of 8 km/h.

Complete the travel graph.

[2]

(iv) Work out the average speed, in km/h, for the whole journey from A to C.

.... km/h [3]

(b) The scale drawing shows the positions of two ports, *X* and *Y*.
The scale is 1 cm represents 8 km.

North

North

sea

Y

land

X

Scale: 1 cm to 8 km
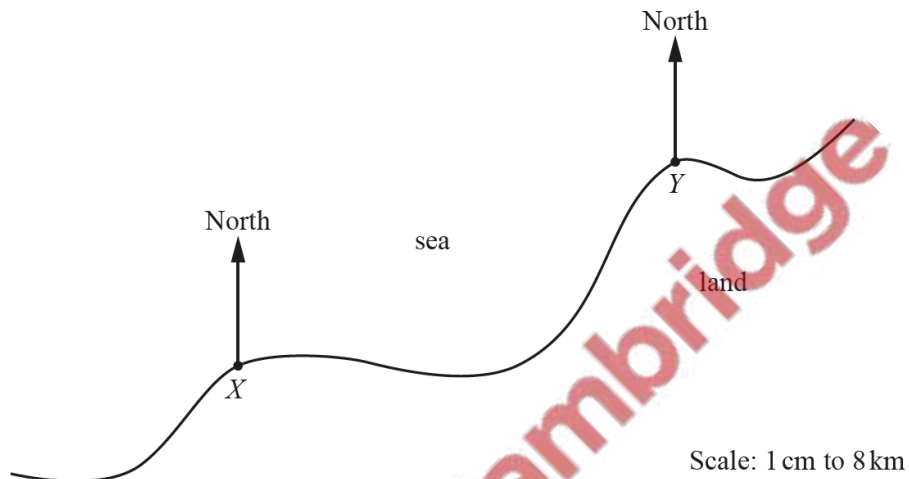
(i) Measure the bearing of *Y* from *X*.

.... [1]

(ii) A boat, *B*, is 52 km from *X* and 80 km from *Y*.

On the scale drawing, mark the position of *B*. [2]

(iii) A ship, *S*, is on a bearing of 284° from *X*.

Work out the bearing of *X* from *S*.

.... [2]

(a) Write down the equation of line L in the form $y = mx + c$.
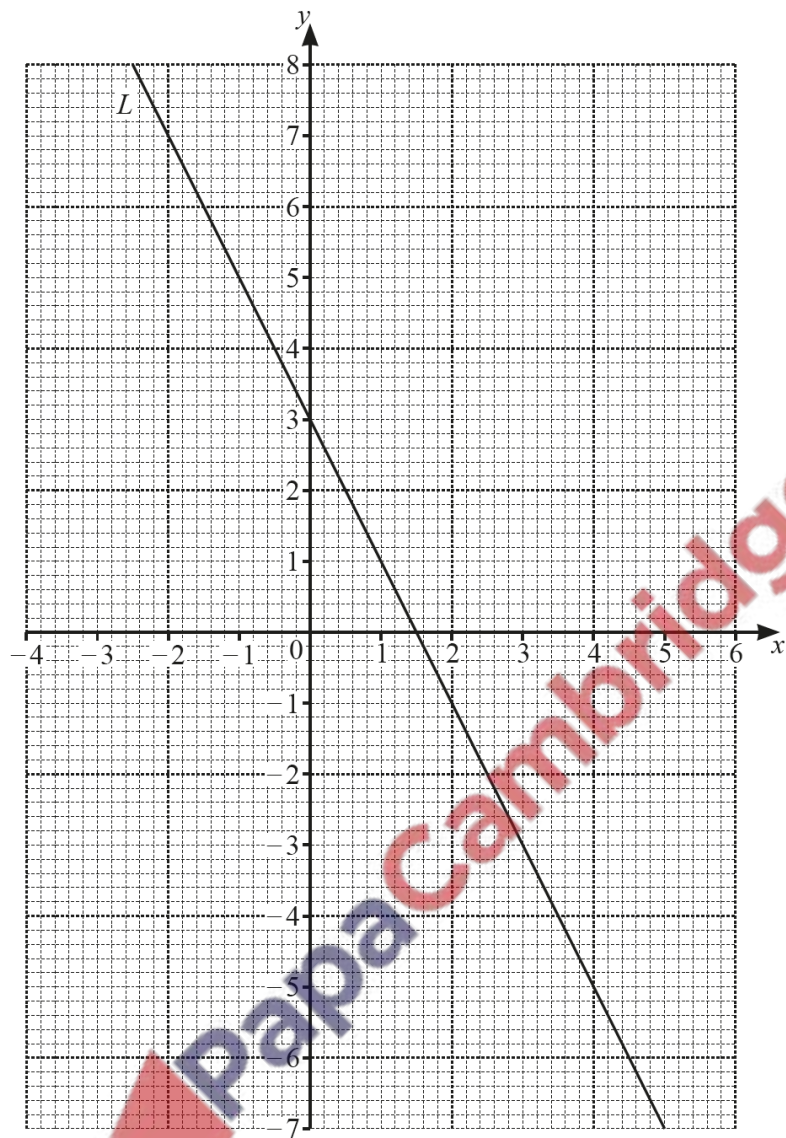
$y =$ .... [2]

(b) (i) Complete the table of values for $y = x^2 - 3x - 3$.

x	-2	-1	0	1	2	3	4	5
y		1		-5	-5		1	

[2]

(ii) On the grid, draw the graph of $y = x^2 - 3x - 3$ for $-2 \leqslant x \leqslant 5$. [4]

(c) (i) Write down the coordinates of the lowest point of the graph of $y = x^2 - 3x - 3$.

(....................... , ...) [1]

(ii) On the grid, draw the line of symmetry of the graph of $y = x^2 - 3x - 3$. [1]

(iii) Write down the equation of the line of symmetry.

.... [1]

(d) Write down the coordinates of the point where line L intersects the graph of $y = x^2 - 3x - 3$ for $x > 0$.

(....................... , ...) [1]
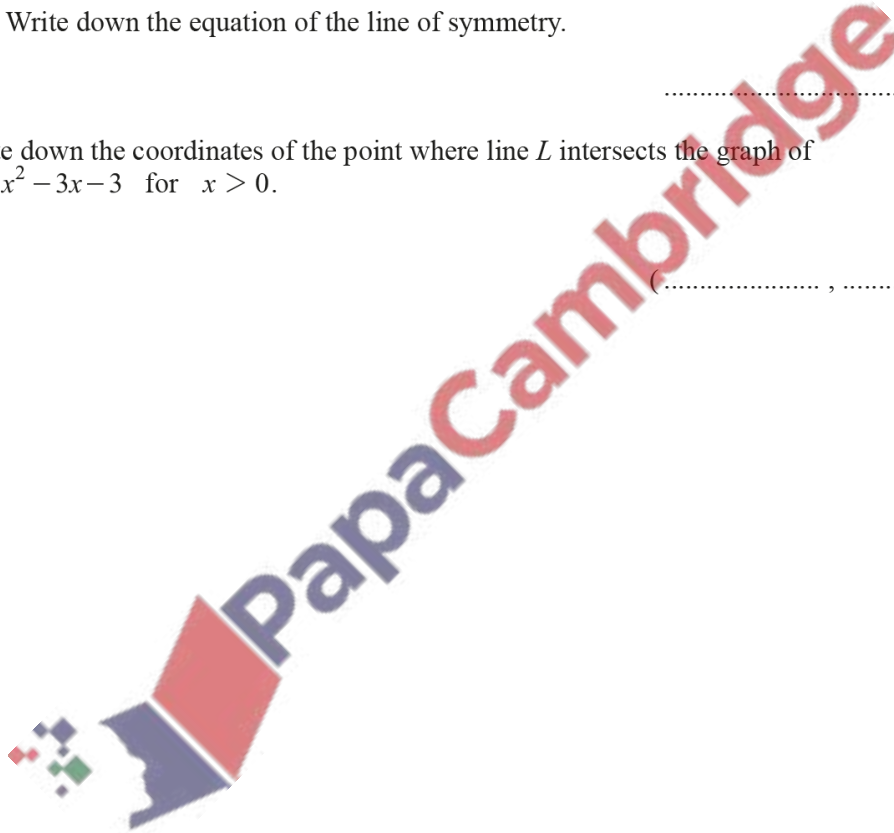
(a) Factorise fully.

   (i) $27y^2 - 3$

                           .... [3]

   (ii) $2m - pk + 2k - pm$

                           .... [2]

(b) Solve $\dfrac{x-1}{x+1} - \dfrac{6}{x-1} = 1$.

                         $x = $ .... [5]

(c) Solve $4x^2 - 3x - 2 = 0$.
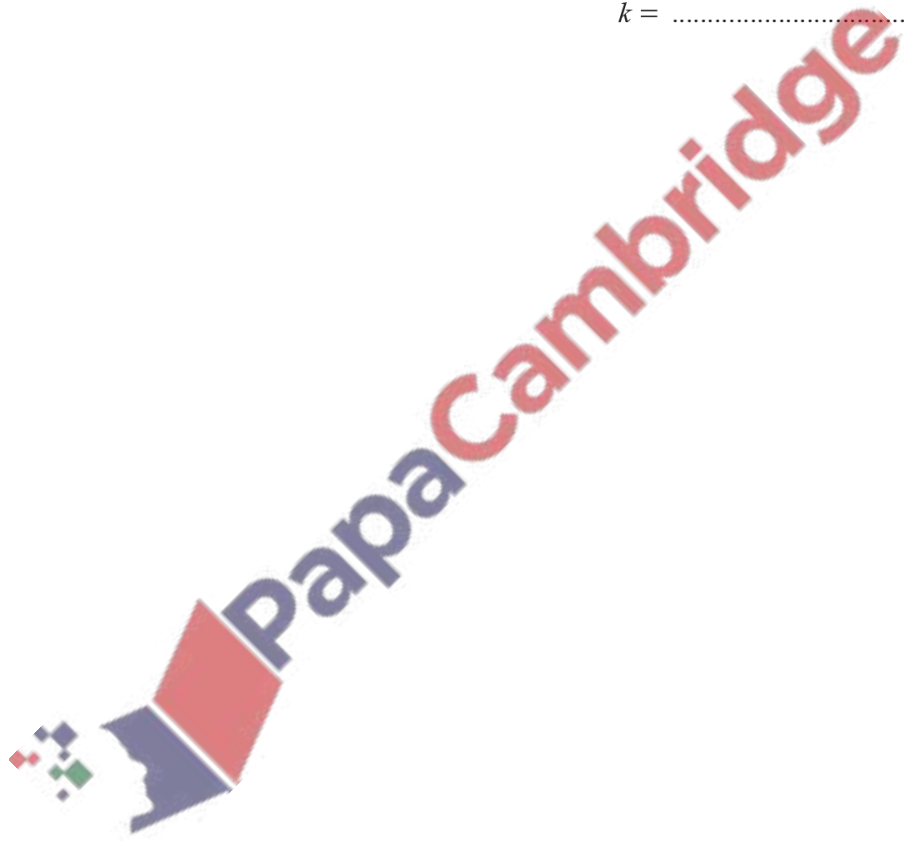You must show all your working and give your answers correct to 2 decimal places.

                $x = $ .... or $x = $ .... [4]
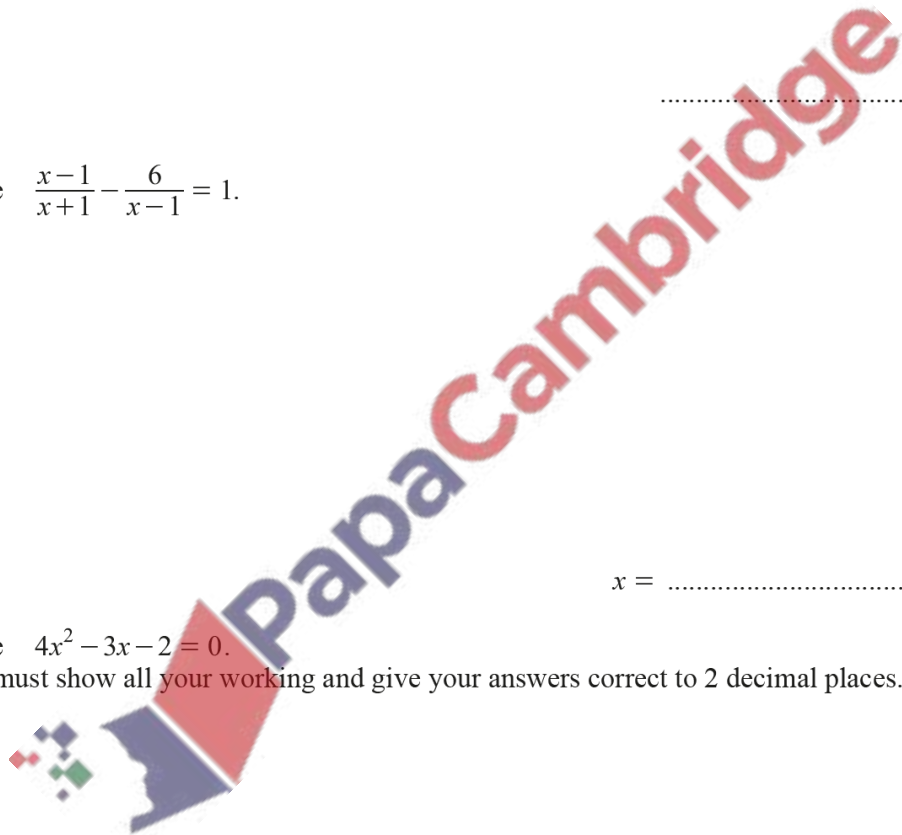
(d) Make k the subject of the formula.

$$\frac{k}{m} = 4 + kp$$

$k =$ .... [4]

A tailor makes x dresses and y shirts in one week.
In one week

- he makes at least 4 dresses
- he makes no more than 7 shirts
- he makes less than 14 dresses and shirts altogether
- the number of shirts he makes is more than $\frac{2}{3}$ of the number of dresses.
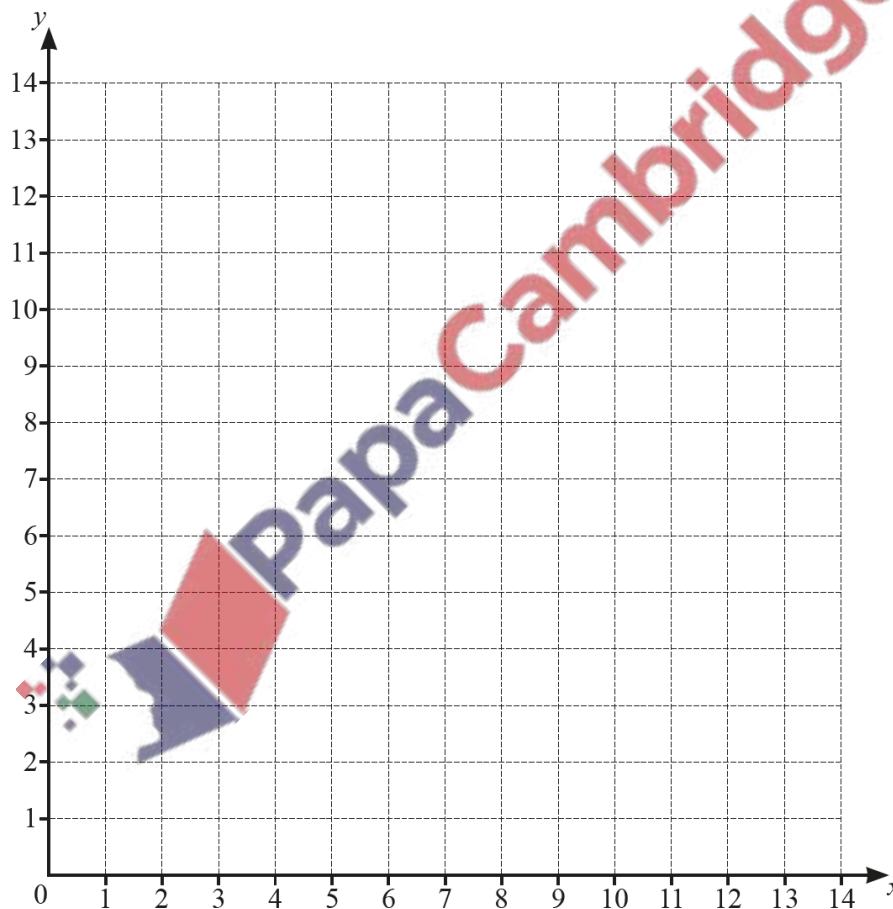
One of the inequalities that shows this information is $x \geqslant 4$.

(a) Write down the other three inequalities in x and/or y.

....   [3]

(b)

On the grid, draw 4 straight lines and shade the unwanted regions to show these inequalities.
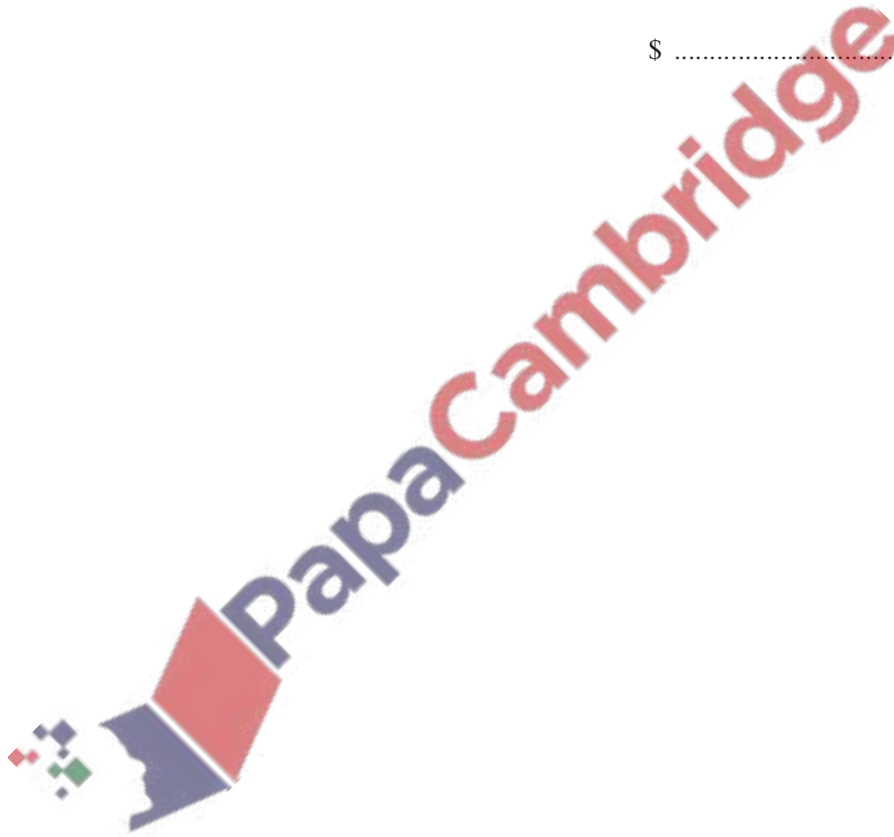Label the region R that satisfies the 4 inequalities. [6]

(c) Use your diagram to find the smallest number of dresses and the smallest number of shirts the tailor makes in one week.

.... dresses and .... shirts [1]

(d) The profit the tailor makes on one dress is $10 and the profit on one shirt is $6.

Use your diagram to find the largest profit the tailor can make in one week.

$ .... [2]

$$f(x) = x - 4 \qquad g(x) = 2x + 5 \qquad h(x) = 3^x$$

(a) Find

(i) $f(-3)$

.... [1]

(ii) $g^{-1}(x)$

$g^{-1}(x) =$ .... [2]

(iii) $f(x) \times g(x) \times f(x)$.

.... [4]

(b) Find x when $h(x) = g(f(2))$.

$x =$ .... [2]

C

NOT TO
SCALE

$(x+3)$ cm

A $(2x+5)$ cm   B
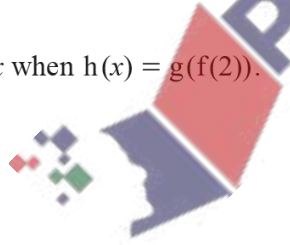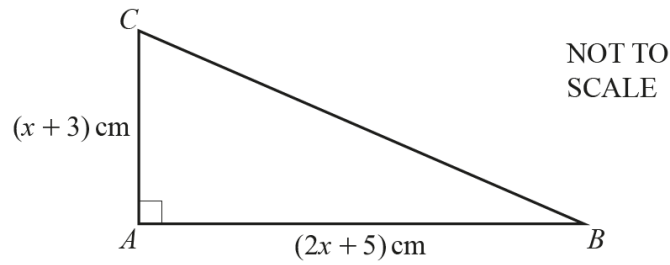
The diagram shows a right-angled triangle ABC.

(a) **(i)** The area of the triangle is $60\,\text{cm}^2$.

Show that $2x^2+11x-105=0$.

[3]

(ii) Solve by factorisation.

$$2x^2+11x-105=0$$

$x=$ .... or $x=$ .... [3]

(iii) Calculate angle ACB.

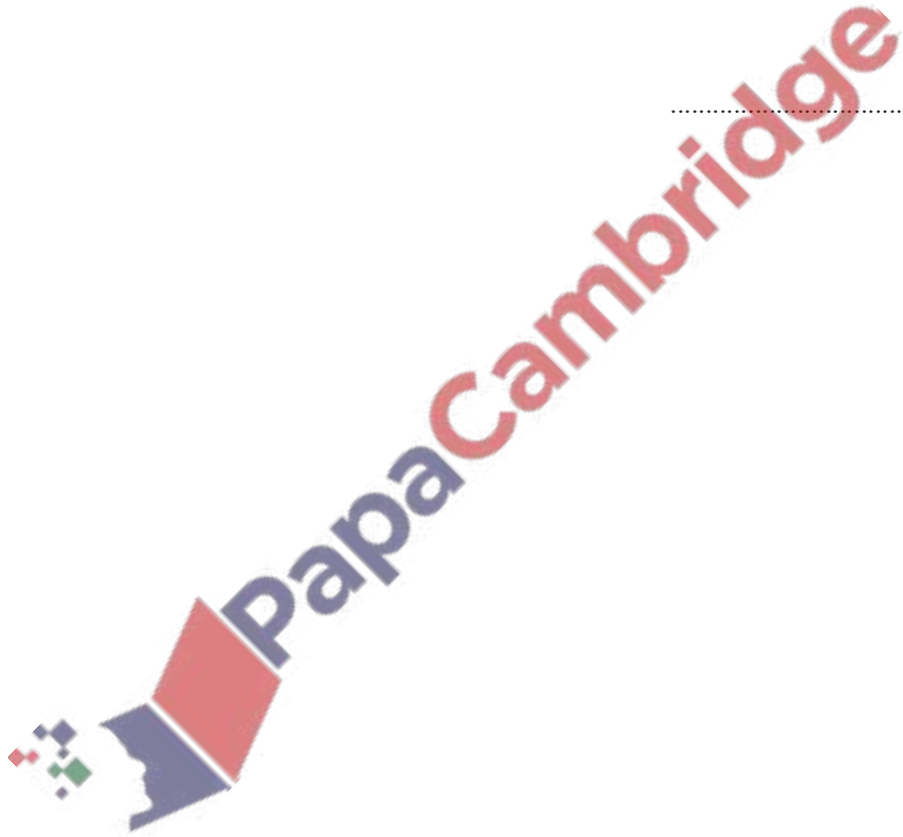.. [3]

(b) Triangle *ABC* is similar to triangle *DEF*.
Triangle *DEF* has an area of $93.75\,\text{cm}^2$.

(i) Find the size of the smallest angle of triangle *DEF*.

.... [1]

(ii) Find the length of the shortest side of triangle *DEF*.

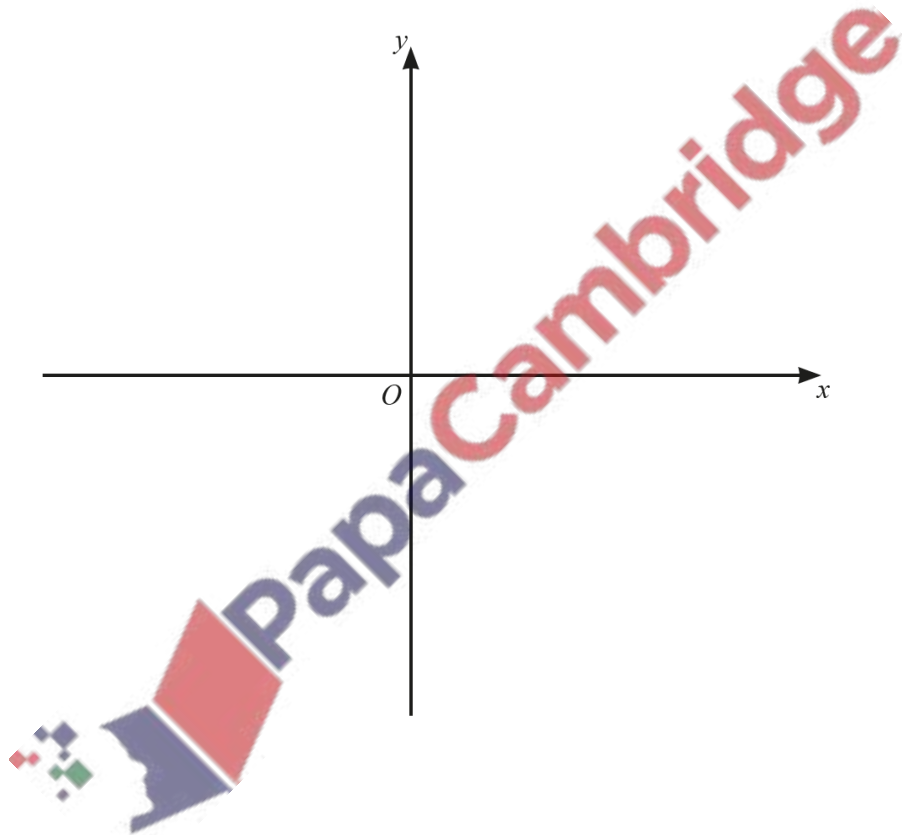... cm [3]

(a) **(i)** Show that the equation $y = (x-4)(x+1)(x-2)$ can be written as $y = x^3 - 5x^2 + 2x + 8$.

[2]

(ii) On the diagram, sketch the graph of $y = x^3 - 5x^2 + 2x + 8$, indicating the values where the graph crosses the axes.
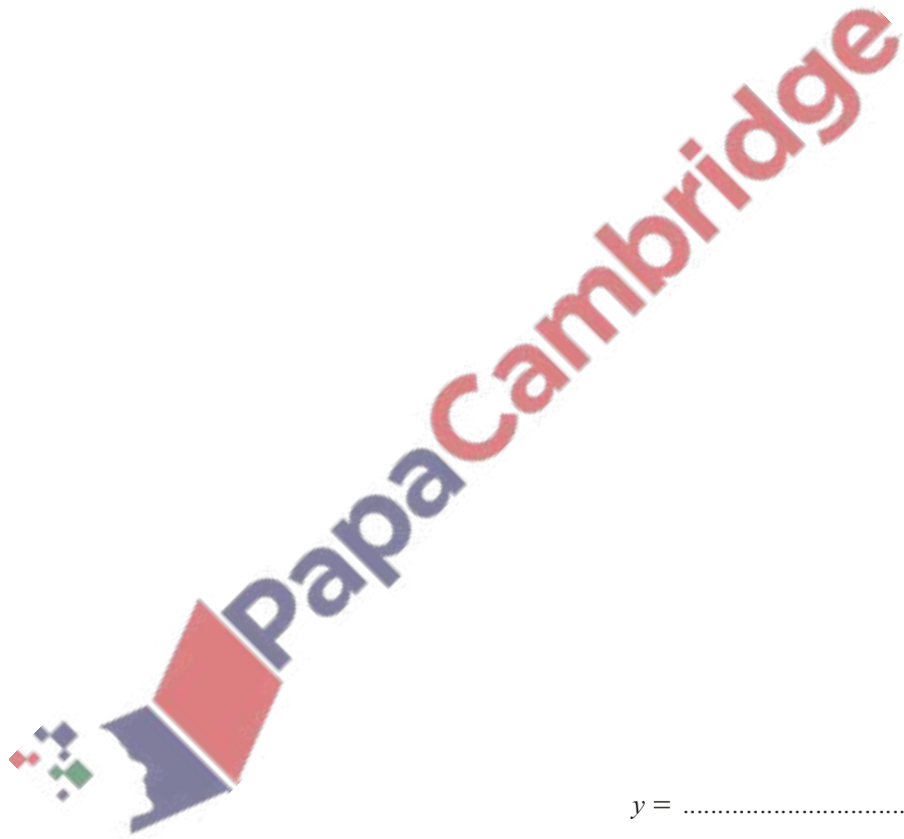
[4]

(b) The graph of $y = x^3 - 5x^2 + 2x + 8$ has two tangents with a gradient of 10.

Find the equations of these two tangents.
You must show all your working and give your answers in the form $y = mx + c$.

$y =$ ....

$y =$ .... [7]

(a) Simplify.

(i) $(3x^2y^4)^3$

.... [2]

(ii) $\left(\dfrac{16}{x^{16}y^8}\right)^{-\frac{3}{2}}$

.... [3]

(b) (i) Factorise.

$$x^2 - 9$$

.... [1]

(ii) Simplify.

$$\frac{x^2 - 9}{2xy - 6y + 5x - 15}$$

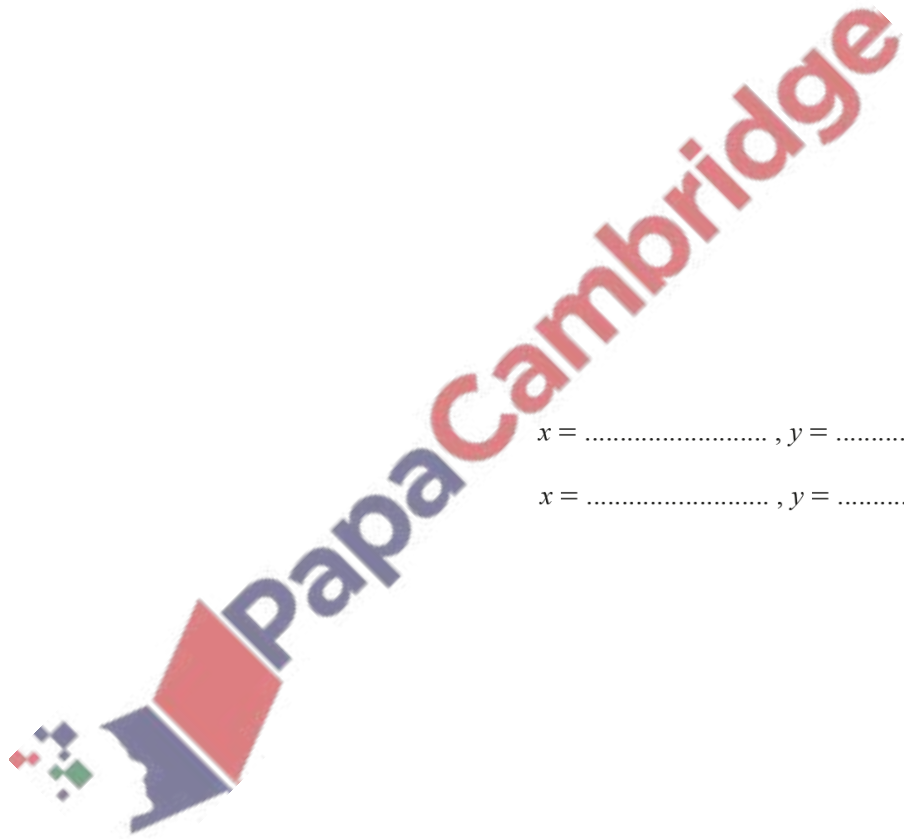.. [3]
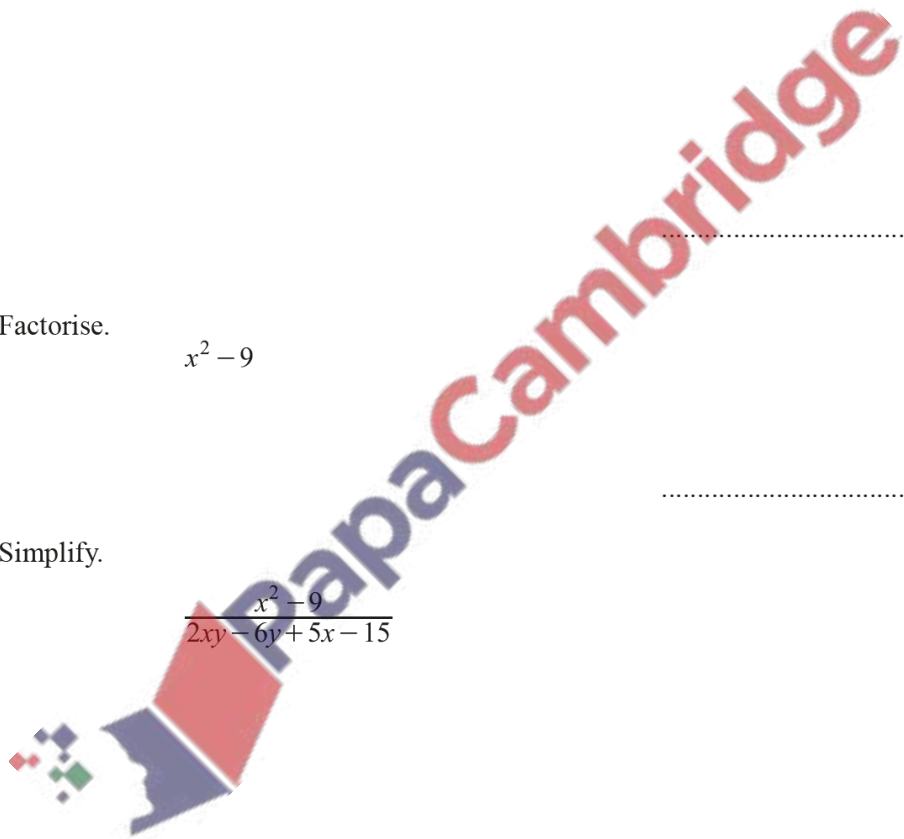
(c) Solve the simultaneous equations.

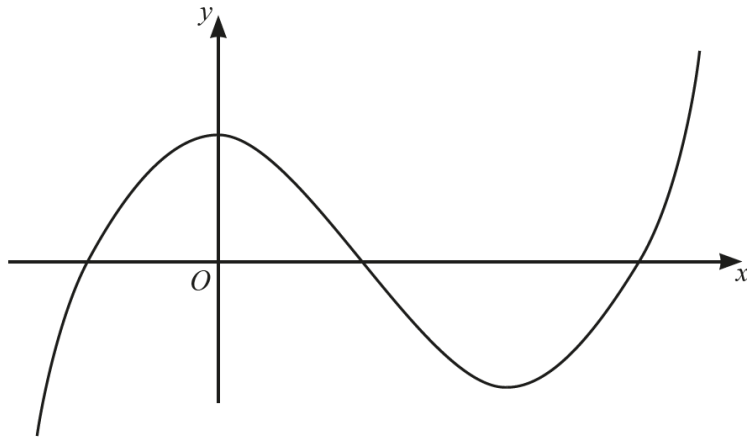You must show all your working and give your answers correct to 2 decimal places.

$$2x + y = 7$$
$$y = 5x^2 + 2x - 13$$

$x =$ .... , $y =$ ....

$x =$ .... , $y =$ .... [6]

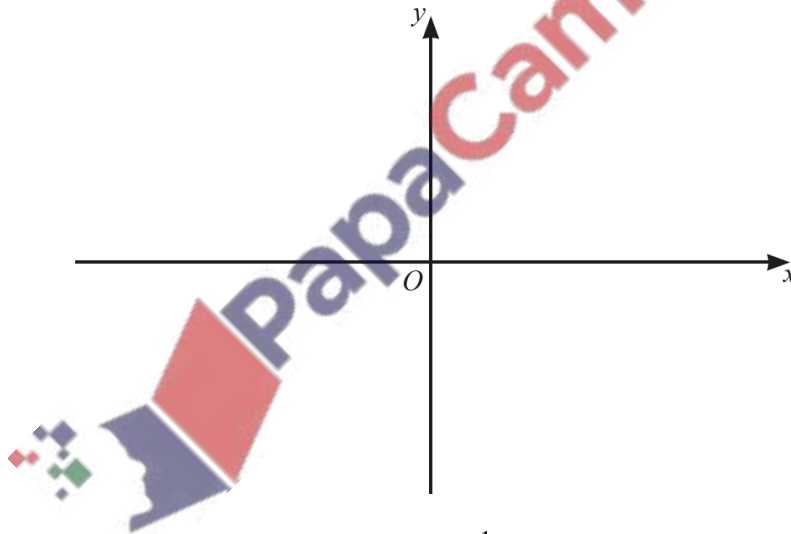**(a)** The diagram shows the graph of a function.

Put a ring around the word which correctly identifies the type of function.

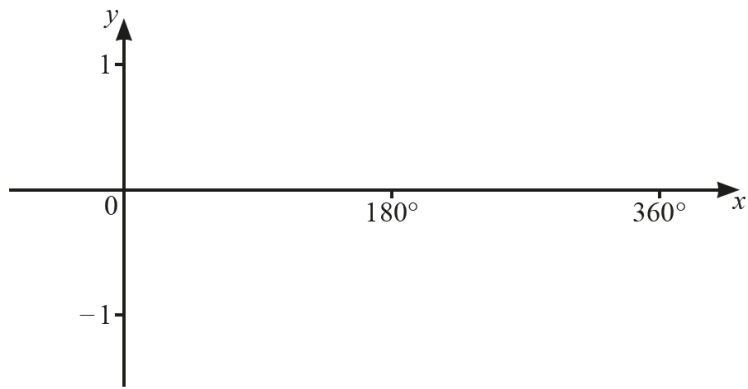reciprocal   quadratic   cubic   exponential   linear

[1]

(b) **(i)**

On the diagram, sketch the graph of $y = \dfrac{1}{2x}$, $x \neq 0$.   [2]

(ii) Solve the equation $\dfrac{1}{2x} = 2x$.

$x =$ .... and $x =$ .... [2]
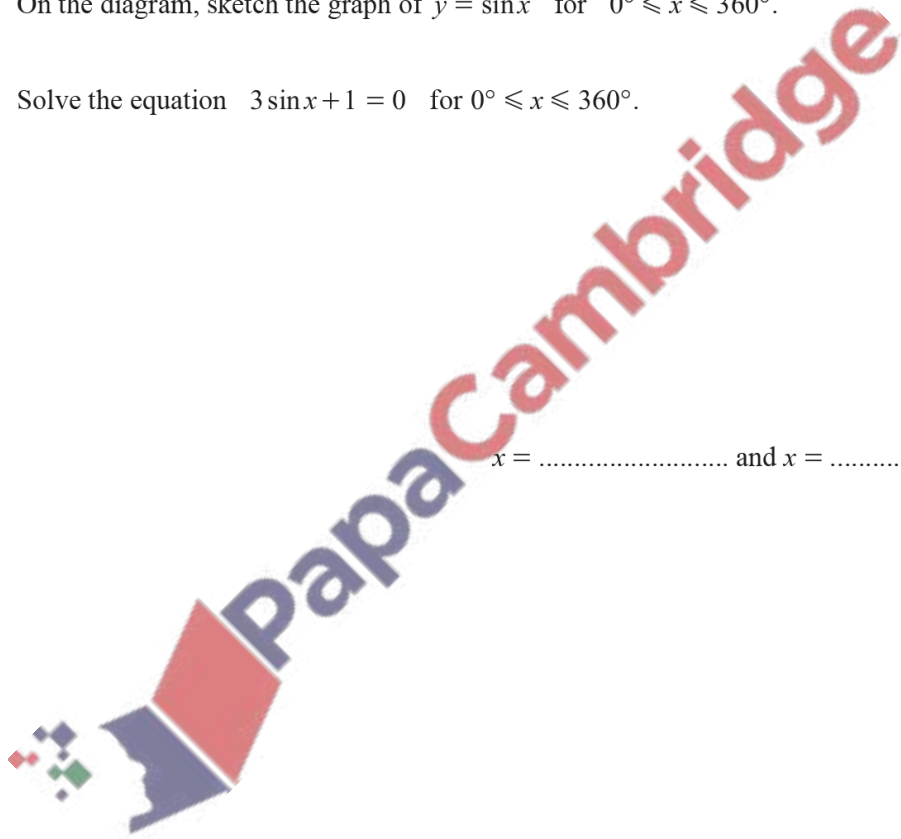
(c) (i)

On the diagram, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$. [2]

(ii) Solve the equation $3\sin x + 1 = 0$ for $0° \leqslant x \leqslant 360°$.

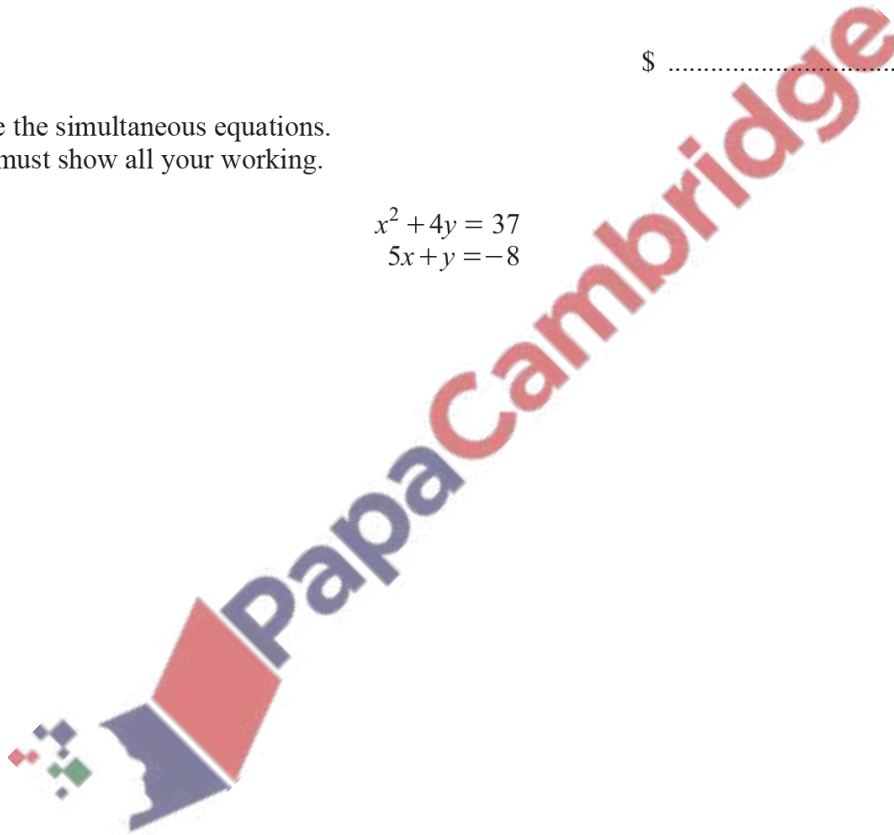$x =$ .... and $x =$ .... [3]

(a) A shop sells shirts for $\$x$ and jackets for $\$(x+27)$.
The shop sells 4 shirts and 3 jackets for a total of $\$194.75$.

Write down and solve an equation to find the cost of one shirt.

$ .... [3]

(b) Solve the simultaneous equations.
You must show all your working.

$$x^2 + 4y = 37$$
$$5x + y = -8$$

$x = ... , y = ...$

$x = ... , y = ...$ [5]

(c) A solid cylinder has radius x and height $6x$.
A sphere of radius r has the same surface area as the total surface area of the cylinder.

Show that $r^2 = \dfrac{7}{2}x^2$.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

[4]
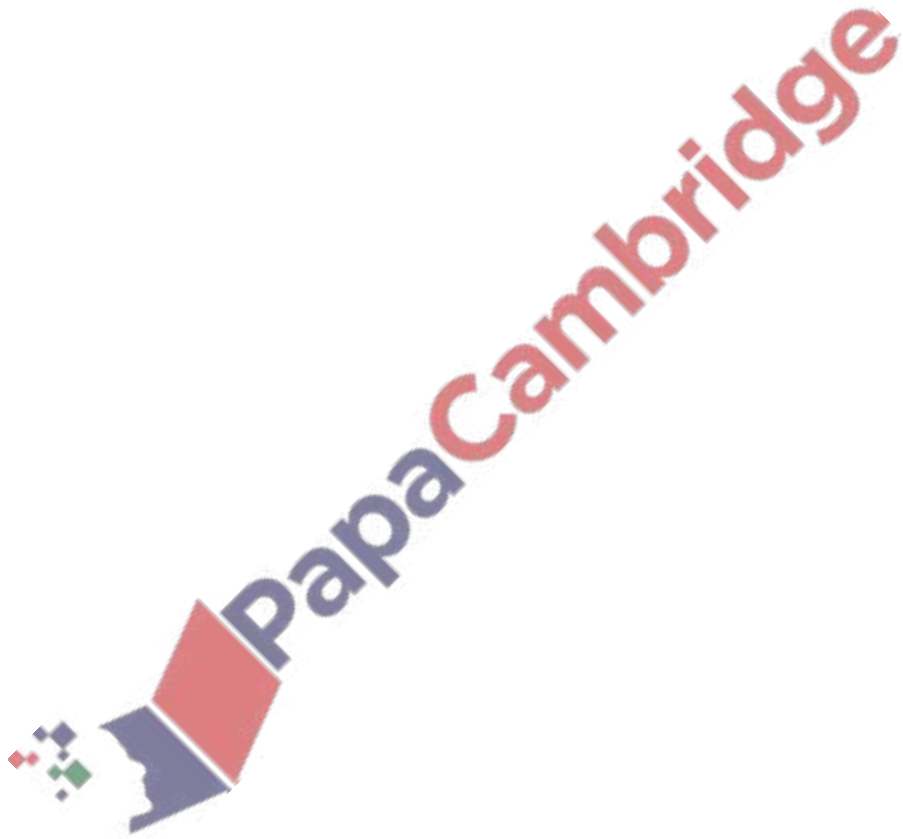
The table shows some values for $y = 2^x - 3$.
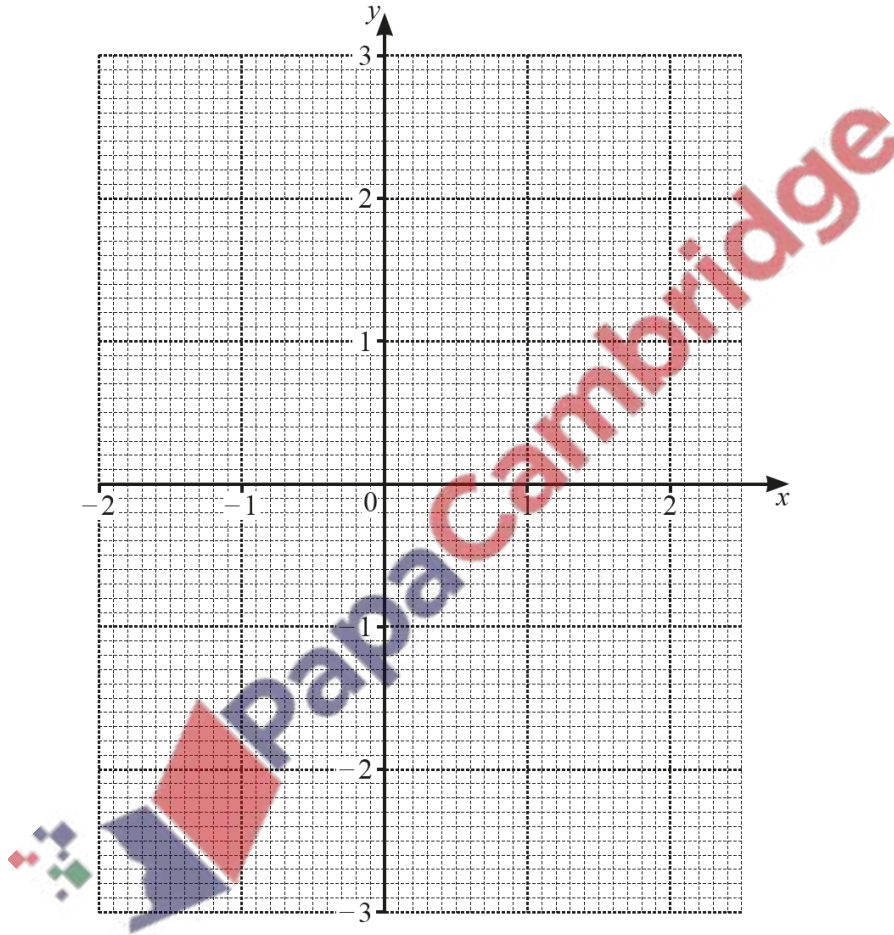
x	-2	-1	0	0.5	1	1.5	2	2.5
y	-2.75			-1.58		-0.17	1	2.66

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2^x - 3$ for $-2 \leqslant x \leqslant 2.5$.

[4]

(c) Use your graph to solve the equation $2^x - 3 = 2$.

$x =$ .... [1]

(d) By drawing a suitable straight line, solve the equation $2^x - x - 1.5 = 0$.

$x =$ .... or $x =$ .... [4]

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The equation of a curve is $y = x^4 - 8x^2 + 5$.

(a) Find the derivative, $\left(\dfrac{dy}{dx}\right)$, of $y = x^4 - 8x^2 + 5$.

.... [2]

(b) Find the coordinates of the three turning points.
You must show all your working.

(.......... , ...) and (.......... , ...) and (.......... , ...) [4]

(c) Determine which one of these turning points is a maximum.
Justify your answer.

[2]