Name:

Section:

Number Sequences Worksheet

1 The table below shows the number of ways John can pay when parking for various times.

Time (hours)	1	2	3	4	5	6	7		n	$n + 1$	$n + 2$
Number of ways	1	2	3	5	8	a	b		x	y	z

(i) Find the values of a and b. [3]

(ii) Write down an equation connecting x, y and z. [1]

2

The diagram shows the first four rows of a pattern of numbers.

Row 1   1   2   1

Row 2   2   3   2   3   2

Row 3   3   4   3   4   3   4   3

Row 4   4   5   4   5   4   5   4   5   4

The table shows some results obtained from this pattern.

Row number	1	2	3	4	5		n
Number of numbers in the row	3	5	7	9	p		x
Product of the first two numbers in the row	2	6	12	20	q		y
Sum of all the numbers in the row	4	12	24	40	r		z
Middle number in the row	2	2	4	4	s		

(a) Find the values of p, q, r and s. [2]

(b) Find expressions, in terms of n, for x, y and z. [3]

(c) Write down the middle number in Row 101. [1]

3 These are the first four patterns in a sequence made using counters.

Pattern 1   Pattern 2   Pattern 3   Pattern 4

(a) Complete the table for the patterns in this sequence.

Pattern number	1	2	3	4	5
Number of counters	3	8	15		

[1]

(b) Find an expression, in terms of n, for the number of counters in Pattern n.

.... [2]

(c) Ken has a bag containing 1358 counters.
He makes the largest possible pattern in the sequence, Pattern p, using these counters.
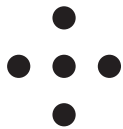
   (i) Find the value of p.

$p =$ .... [3]

   (ii) He uses all of the remaining counters to make another pattern in the sequence, Pattern q.

   Find the value of q.

$q =$ .... [2]

4 Shani makes a sequence of patterns using counters.

   Pattern 1   Pattern 2   Pattern 3

(a) Complete the table.

Pattern number	1	2	3	4	5
Number of counters	5	8	11		

[1]

(b) Find an expression, in terms of n, for the number of counters in Pattern n.

.... [2]
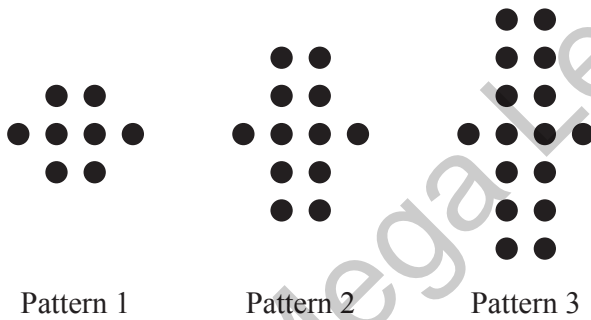
(c) Shani has 100 counters.
She uses some of the counters to make Pattern 20.
She uses all the remaining counters to make Pattern k.

Find the value of k.

$k = $ .... [3]

5 (a) Here are the first three patterns in a sequence made from counters.

Pattern 1   Pattern 2   Pattern 3

(i) Complete the table for the patterns in this sequence.

Pattern number	1	2	3	4	5
Number of counters	8	12	16		

[1]

(ii) Find an expression, in terms of n, for the number of counters in Pattern n.

.... [2]

(iii) Jamal has 150 counters.
He uses these counters to make the largest pattern possible, Pattern *p*.

Find the value of *p*.

p = ... [2]

(b) The 4th term in a different sequence is 26.
This sequence is linear and the 8th term is 2.

(i) Find the first term of this sequence.

.... [2]

(ii) Find an expression, in terms of *n*, for the *n*th term of this sequence.

.... [2]

6   Here are the first four terms of a sequence.

$$\frac{12}{16} \qquad \frac{17}{25} \qquad \frac{22}{36} \qquad \frac{27}{49}$$

Find an expression for the nth term of the sequence.

.... [4]

7   Here are the first four terms of a number sequence.

$T_1 = 1^2 + 3 \ = 4$

$T_2 = 2^2 + 8 \ = 12$

$T_3 = 3^2 + 13 = 22$

$T_4 = 4^2 + 18 = 34$

(a) Find T_5.

$T_5 = $ .... [1]

(b) Find an expression, in terms of n, for T_n.

$T_n = $ .... [3]
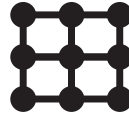
8 Here are the first three patterns in a sequence made using dots and lines.

Pattern 1   Pattern 2   Pattern 3

(a) Complete the table for the first five patterns in this sequence.

Pattern number	1	2	3	4	5
Number of dots	3	6			
Number of lines	2	7			

[2]

(b) Find an expression, in terms of *n*, for the number of lines in Pattern *n*.

.... [2]

(c) Anwar makes one of these patterns using 92 lines.

Find the number of dots in Anwar's pattern.

.... [2]

9 Here are the first five terms of a sequence.

$$\frac{3}{4} \qquad \frac{7}{8} \qquad \frac{11}{12} \qquad \frac{15}{16} \qquad \frac{19}{20}$$

(a) Write down the next two terms.

Answer .... , ... [1]

(b) The kth term is $\dfrac{1199}{1200}$.

Find k.

(c) Find an expression, in terms of n, for the nth term.

10 The first four terms, u_1, u_2, u_3 and u_4, in a sequence of numbers are given below.

$$u_1 \quad = \quad 1 \times 3 + 2^2 \quad = \quad 7$$

$$u_2 \quad = \quad 2 \times 4 + 3^2 \quad = \quad 17$$

$$u_3 \quad = \quad 3 \times 5 + 4^2 \quad = \quad 31$$

$$u_4 \quad = \quad 4 \times 6 + 5^2 \quad = \quad 49$$

(a) Evaluate u_5 .

(b) The nth term of the sequence, u_n, is of the form $n(n+p) + (n+q)^2$.

Write down the value of p and the value of q.

(c) u_n can also be written in the form $An^2 + Bn + C$.

Find the values of A, B and C.

Answer $A =$ ....

$B =$ ....

$C =$ .... [2]

11 The sequence of diagrams shows patterns made from some black beads and some white beads.
Each diagram has two rows more than the previous diagram.

Diagram 1   Diagram 2   Diagram 3   Diagram 4
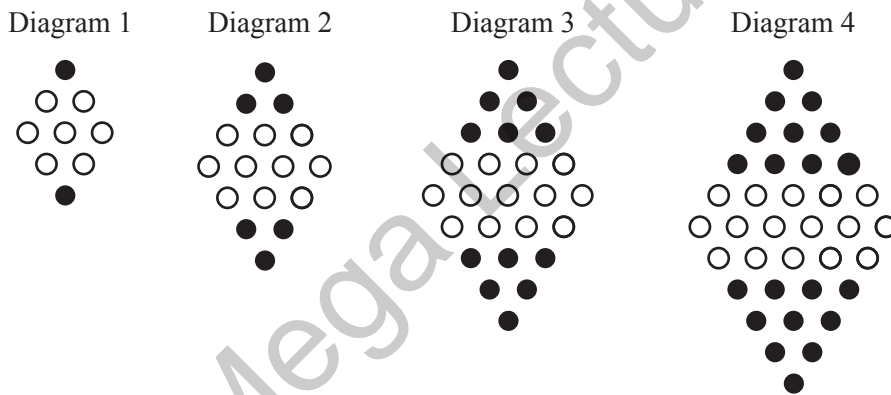
(a) Complete the table for Diagram 5.

Diagram number	1	2	3	4	5
Total number of beads	9	16	25	36	
Number of white beads	7	10	13	16	
Number of black beads	2	6	12	20	

[1]

(b) Write down an expression, in terms of *n*, for

(i) the number of white beads in Diagram *n*,

Answer .... [1]

(ii) the total number of beads in Diagram n.

Answer .... [1]

(c) Find an expression, in terms of n, for the number of black beads in Diagram n.
Give your answer in its simplest form.

Answer .... [2]

12

Pattern 1   Pattern 2   Pattern 3   Pattern 4   Pattern 5
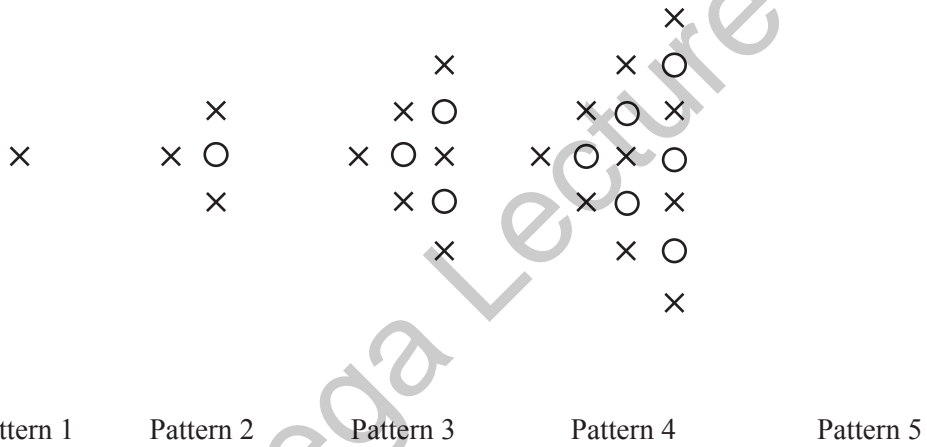
The diagrams show patterns made from crosses (✗) and circles (○).

(a) Draw pattern 5 above.   [1]

The table shows the number of crosses and circles in each pattern.

Pattern number (n)	1	2	3	4	5	6
Number of crosses	1	3	6	10		
Number of circles	0	1	3	6		
Total number of crosses and circles	1	4	9	16	25	36

(b) Complete the table.   [2]

(c) Find an expression, in terms of n, for the total number of crosses and circles in pattern n.

.... [1]

(d) An expression, in terms of n, for the number of crosses in pattern n is $\frac{1}{2}n^2 + \frac{1}{2}n$.

How many crosses are there in pattern 30?

.... [1]

(e) Show that the number of circles in pattern n is $\frac{1}{2}n^2 - \frac{1}{2}n$.

[1]

(f) The number of **crosses** in pattern m is equal to $5m$.

Find m.

Answer $m = $ .... [3]

13 Two sequences have $1, 3, 5$ as their first three terms.

(a) In the first sequence, each term is 2 more than the term before it.

(i) Find an expression, in terms of n, for the nth term.

Answer .... [1]

(ii) The kth term of this sequence is 841.

Find the value of k.

Answer $k =$ .... [1]

(b) The nth term of the second sequence is

$$2^{n-1} - \frac{(n-1)(n-4)}{2}.$$

(i) Find the fourth term of this sequence.

Answer   .... [1]

(ii) Find the fifth term of this sequence.

Answer   .... [1]

14 (a) The nth term of a sequence is given by $n^2 - 5n$.

(i) Find the 2nd term in the sequence.

Answer   .... [1]

(ii) The pth term in the sequence is 150.

Find the value of p.

Answer $p = $ .... [2]

(b) The nth term of another sequence is given by $3n^2 - kn$.
The 5th term in this sequence is 55.

Find the value of k.

Answer $k = $ .... [2]

15 The first term of a sequence is 13.
The following terms are found by alternately adding 4 and 6 to the previous term.
The first six terms are

$$13 \quad 17 \quad 23 \quad 27 \quad 33 \quad 37$$

(a) Write down the next two terms of the sequence.

Answer .... [1]

(b) Write down the **value** of the term that is closest to 999.

Answer .... [1]

(c) Write down the difference between the values of the 91st and 93rd terms.

Answer .... [1]

(d) Find the 80th term.

Answer .... [1]

(e) The nth term is 203.

Find n.

Answer $n = $ .... [1]

16 **(a)** The first four terms of a sequence, S, are 89, 83, 77, 71.

    (i) Find an expression for S_n, the nth term of this sequence.

Answer $S_n = $ .... [2]

    (ii) Find the smallest value of n for which $S_n < 0$.

Answer $n = $ .... [1]

(b) The nth term of a different sequence, T, is given by $T_n = n^2 - 4n$.

(i) Find and simplify an expression for $T_{n+1} - T_n$.

Answer .... [2]

(ii) The difference between T_{p+1} and T_p is 75.

Find the value of p.

Answer $p = $ .... [1]

17 The sequence of diagrams below shows small black and small white squares in an arrangement to form large squares.

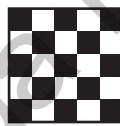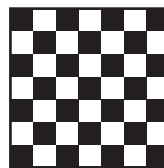Diagram 1   Diagram 2   Diagram 3

The table below shows the numbers of black and white squares in each diagram.

Diagram (n)	1	2	3	4
Black squares	5	13	25	
White squares	4	12	24	
Total number of black and white squares	9	25	49	

(a) For each diagram, how many more black squares are there than white squares?

Answer .... [1]

(b) On the table, complete the column for Diagram 4. [1]

(c) Write down an expression, in terms of n, for the total number of black and white squares in Diagram n.

Answer .... [1]

18 (a) The first five terms of a sequence are $17, 11, 5, -1, -7$.

Find, in terms of n, an expression for the nth term of this sequence.

Answer .... [2]

(b) The nth term, S_n, of a different sequence is found using the formula $S_n = n^2 + 3n$.

(i) Work out the first four terms of this sequence.

Answer .... , ... , ... , ... [2]

(ii) The nth term, T_n, of another sequence is found using the formula $T_n = 5n - 12$.
There are two values of n for which $\dfrac{S_n}{T_n} = 6$.

Form and solve an equation in n to find these two values.

Answer $n =$ .... and .... [4]

19 The first four lines of a pattern of numbers are shown below.

$$1\text{st line} \qquad 3^2 \quad - \quad 1^2 \quad = \quad 8 \times 1$$

$$2\text{nd line} \qquad 5^2 \quad - \quad 1^2 \quad = \quad 8 \times (1 + 2)$$

$$3\text{rd line} \qquad 7^2 \quad - \quad 1^2 \quad = \quad 8 \times (1 + 2 + 3)$$

$$4\text{th line} \qquad 9^2 \quad - \quad 1^2 \quad = \quad 8 \times (1 + 2 + 3 + 4)$$

(a) Write down the 7th line of the pattern.

Answer .... [1]

(b) Write down an expression, in terms of n, to complete the nth line of the pattern.

Answer .... $= 8 \times (1 + 2 + 3 + 4 + \ldots + n)$ [1]

(c) **Using the nth line of the pattern**, show that $1 + 2 + 3 + 4 + \ldots + n = \dfrac{n(n+1)}{2}$.

[2]

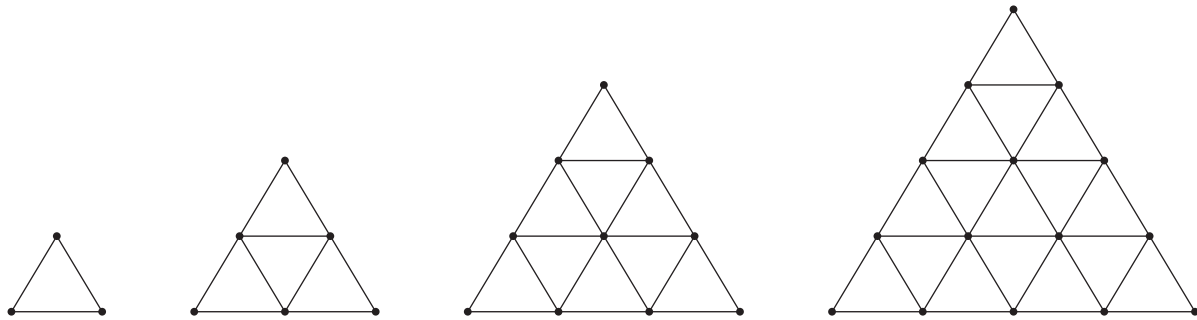**20** Small triangles are formed by placing rods between dots as shown in the diagrams.

Diagram 1   Diagram 2   Diagram 3   Diagram 4

(a) Complete the table.

Diagram n	1	2	3	4	5
Number of small triangles (T)	1	4	9	16	
Number of dots (D)	3	6	10	15	
Number of rods (R)	3	9	18	30	

[2]

(b) Find an expression, in terms of n, for the number of small triangles (T) formed in Diagram n.

Answer   ....[1]

(c) Given that $R = D + T - 1$,   find the value of n when $D = 561$ and $R = 1584$.

Answer $n =$ ....[2]

21 These are the first five terms of a sequence.

$$4 \quad 8 \quad 16 \quad 32 \quad 64$$

(a) Find the next number in the sequence.

.... [1]

(b) The nth term of the sequence above is 2^{n+1} .

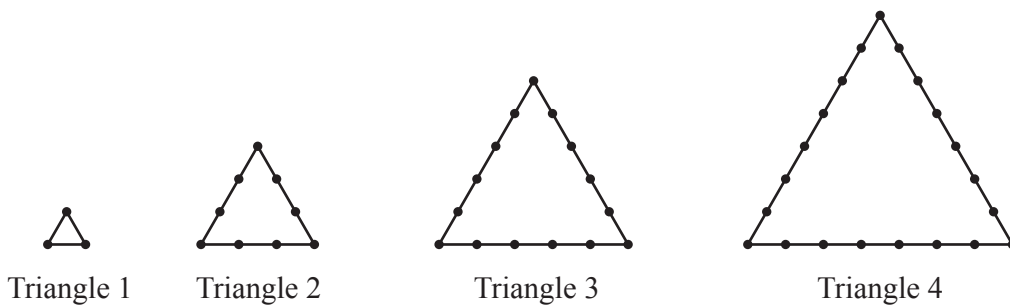Write down an expression, in terms of n, for the nth term of these sequences.

(i) 1   5   13   29   61   …

.... [1]

(ii) 10   19   32   53   90   …

.... [2]

22

Triangle 1   Triangle 2   Triangle 3   Triangle 4

The diagrams show a sequence of triangles made up of identical sticks.
Each triangle has two more sticks on each edge than its previous triangle.
The table shows information relating to this sequence.

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Each triangle has two more sticks on each edge than its previous triangle.
The table shows information relating to this sequence.

Triangle number	1	2	3	4		n
Number of sticks on each side	1	3	5			x
Number of sticks in the triangle	3	9	15			y

(a) Complete the column for triangle 4. [1]

(b) Find an expression, in terms of n, for x.

Answer $x =$ .... [1]

(c) Find an expression, in terms of n, for y.

Answer $y =$ .... [1]

(d) The total number of sticks in the first triangle   =   3
    The total number of sticks in the first two triangles   =   12
    The total number of sticks in the first three triangles   =   27

  (i) Write down the total number of sticks in the first four triangles.

Answer .... [1]

  (ii) Find an expression, in terms of n, for the total number of sticks in the first n triangles.

Answer .... [2]

23 The nth term of a sequence is $9n + 4$.

  (a) Calculate the value of the term that is closest to 2012.

Answer ....[2]

(b) Calculate the difference between the 10 th term and the 6 th term.

Answer ....[1]

(c) **(i)** Find an expression, in terms of x and y, for the difference between the x th term and the y th term.

Answer ....[1]

(ii) **Hence** explain why it is not possible for any two terms of this sequence to differ by 123.

Answer ....

....[1]

24 It is given that $N = 87 \times 132$.

(a) Complete the statements in the answer space.

Answer (a) $88 \times 132 = N +$ .... [1]

$87 \times 131 = N -$ .... [1]

(b) Hence evaluate $88 \times 132 - 87 \times 131$.

Answer (b) .... [1]