ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key Messages

Candidates should be reminded to read the instructions on the front of the examination paper carefully. In particular, attention should be paid to the accuracy required especially when dealing with angles in degrees and angles in radians. Candidates should be encouraged to check that they have actually completed the question by answering in full. The use of a calculator for solution of quadratic equations is recommended as a check only; solution by either factorisation or use of the quadratic formula is expected to be shown as part of the solution of a question. A similar approach may be used for questions involving surds.

If a response to a question is written elsewhere in the question paper, or on additional paper, candidates should be advised to make an appropriate comment by the original question as to where the solution may now be found.

General Comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues.

Comments on Specific Questions

Question 1

(i) (ii) A number of candidates were unsure of the meaning of "period" and "amplitude" and many gave the answer 2 to both parts of the question.

(iii) Most candidates obtained the mark for sketching the graph of $y = \sin 2x$.

Many sketches of $y = 1 + \cos 2x$ were incorrectly drawn as, although most started at a maximum of 3 and ended at a minimum of –1, the intermediate stationary points were not also at those values.

(iv) The majority appreciated that the answer to this question was related to the number of intersections of the graphs in part **(iii)**.

Answers: **(i)** 180° **(ii)** 2 **(iv)** 3

Question 2

(i) Although candidates were instructed not to use calculators and to show all working in this question, many answers were offered showing no working at all. These were given no credit.

(ii) These were a reasonably easy 3 marks for those who used $\sec^2\theta = 1 + \tan^2\theta$. Those who worked out the length of the hypotenuse and $\sec^2\theta = \dfrac{1}{\cos^2\theta}$ rarely got far enough to earn any marks.

Answers: **(i)** $-1 + 2\sqrt{2}$ **(ii)** $10 - 4\sqrt{2}$

Question 3

(i) This part of the syllabus has clearly been taught well by most Centres. The majority of candidates were able to gain full marks here; however a common mistake was to work out $\left(x^2\right)^3$ as x^5.

(ii) Again, this was generally done well, although many candidates only saw two, rather than three, terms independent of *x*.

Answers: **(i)** $64 + 192x^2 + 240x^4 + 160x^6$ **(ii)** 1072

Question 4

(a) Nearly all candidates made a reasonable attempt here. A few left the elements of their matrix in unsimplified forms, such as –8 + 0. Only a few attempted to find $\mathbf{X}^2$ by squaring each of the elements of **X**.

(b) A number of different ways of solving this problem were offered by candidates. Some found the inverse of **A** in terms of *a* and *b*, others inverted $\mathbf{A}^{-1}$, and only a few equated the product $\mathbf{AA}^{-1}$ to the identity matrix. Most candidates gained full marks.

Answers: **(a)** $\begin{pmatrix} 4-4k & -8 \\ 2k & -4k \end{pmatrix}$ **(b)** $a = 2$, $b = 4$

Question 5

Most candidates were able to find the coordinates of the points *A* and *B*, and this question was a good source of marks for many, although a large number appeared not to understand the meanings of either "perpendicular" and/or "bisector". Consequently, many candidates found the equations of lines perpendicular to *AB* through either *A* or *B* rather than through the mid-point, or even lines of gradient *3* rather than $-\frac{1}{3}$.

Answer: $3y + x - 2 = 0$

Question 6

(i) The remainder theorem was well-known by most candidates who were able to form and solve the two simultaneous equations in *a* and *b*. Those who found only one equation and used the given value of *b* to find *a* were given little credit.

(ii) The division of the cubic expression by the linear factor was generally done well. Those who chose to do this by synthetic division frequently arrived at an incorrect quadratic which was twice the correct answer.

(iii) It was hoped that candidates would evaluate the discriminant and deduce that there were no further real roots because it was negative. Some candidates suggested, incorrectly, that as the quadratic would not factorise there were no further roots. Others ignored the negative and gave two further roots in terms of $\sqrt{40}$, presumably misinterpreting "real" as "rational".

Answers: **(i)** $a = 14$ **(ii)** $(2x-1)\left(7x^2 - 4x + 2\right)$

Question 7

(i) Most candidates quoted and/or used the quotient rule correctly. It was common to see the differential of $\ln\left(4x^2+3\right)$ being given as $\dfrac{1}{4x^2+3}$. Otherwise the question was generally done well. Approximation of $-\ln 3$ to –1 lost accuracy marks.

(ii) Most candidates attempted to use $\dfrac{1}{2}bh$ on the right-angled triangle *AOB*. Finding the coordinates of *B* from their answer to part **(i)** was part of the solution which a few candidates performed incorrectly by assuming that $x = 0$, rather than $y = 0$ on the *x*-axis.

Answers: **(i)** $y = \dfrac{x}{\ln 3} - \ln 3$ **(ii)** $\dfrac{1}{2}(\ln 3)^3$

Question 8

(i) The concept of range was generally well understood.

(ii) Most candidates could find the inverse of g, but the domain was often wrongly stated as $x \geqslant 5$.

(iii) The majority of candidates knew the correct order of operations for gf and how to solve the equation, but many lost the final mark by giving the answer as a decimal, rather than the exact answer required by the question.

(iv) f' was misinterpreted by many as f^{-1}.

Answers: **(i)** $\mathrm{f}(x) \geqslant 3$, $\mathrm{g}(x) \geqslant 9$ **(ii)** $-2+\sqrt{x-5}$, $x \geqslant 9$ **(iii)** $\dfrac{1}{2}\ln\left(\dfrac{4}{3}\right)$ **(iv)** 96

Question 9

(i) It was necessary either to show that the gradient of both the line and the curve at *A* was 3, or to show that the equation of the tangent to the curve at *A* was indeed $y = 3x + 10$. It was not sufficient, as many candidates tried, to solve the equations of the line and the curve, leading to $x = 0$ or $x = 5$, without any further explanation as to why the line was a tangent.

(ii) Most candidates successfully differentiated, equated to 0 and solved. As only the *x*-coordinates were asked for, many wasted time by also finding the *y*-coordinates.

(iii) Most candidates knew to integrate to find the area under a curve. The most common error was to use a lower limit of $\dfrac{1}{3}$ rather than 0.

Answers: **(ii)** 3 and $\dfrac{1}{3}$ **(iii)** $\dfrac{99}{4}$

Question 10

(i) Most candidates were able to reach $\sin x = \frac{1}{2}$ leading to 30° and 150° but many did not use the negative square root to obtain the two other solutions.

(ii) Those who used $\tan^2\theta = \sec^2\theta - 1$ to obtain a quadratic equation in $\sec^3 y$ were generally successful, more so than those who attempted to convert the equation to one in $\cos^3 y$ using three separate identities. The solutions 143.5° and 180° were frequently omitted.

(iii) Many completely correct solutions to this part were seen. Most candidates knew the correct order of operations to solve trigonometric equations of this type.

Answers: **(i)** 30°, 150°, 210°, 330° **(ii)** 60°, 180°, 23.5°, 96.5°, 143.5° **(iii)** $\frac{2\pi}{3}$, $\frac{5\pi}{3}$

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key Messages

Candidates should be reminded to read the instructions on the front of the examination paper carefully. In particular, attention should be paid to the accuracy required especially when dealing with angles in degrees and angles in radians. Candidates should be encouraged to check that they have actually completed the question by answering in full. The use of a calculator for solution of quadratic equations is recommended as a check only; solution by either factorisation or use of the quadratic formula is expected to be shown as part of the solution of a question. A similar approach may be used for questions involving surds.

Candidates should be reminded of the implication of the word 'Hence' in a mathematical context. It is very often there to help them with the next step in a question, but is also often there as a particular method of solution is required, for example, solving simultaneous equations using matrices.

If a response to a question is written elsewhere in the question paper, or on additional paper, candidates should be advised to make an appropriate comment by the original question as to where the solution may now be found.

General Comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues.

Comments on Specific Questions

Question 1

Most candidates made use of the discriminant '$b^2 - 4ac$' correctly in order to find the critical values. Some candidates then stopped without continuing to find the range. Those who used a sketch once they had found the critical values were generally successful in determining the correct range. There were some candidates who got the wrong region and also a number who used two inequalities without making a clear indication that the intersection of the two sets of values was what was required. Very few alternative solutions were seen.

Answer: $-2 < k < 10$

Question 2

There were many different but correct ways to complete the proof. Some candidates seemed to understand the process but unfortunately missed out vital brackets, losing a mark. Showing working correctly is of paramount importance in questions of this type. Knowledge of identities and dealing with fractions within fractions was good in general. It was pleasing to see that the great majority of candidates started with the left hand side of the expression and re-arranged it to obtain the right hand side of the expression. Very few candidates chose to take terms from 'one side to the other'.

Question 3

The great majority of candidates were able to find the inverse matrix correctly and most proceeded to earn the final three marks of the question by using it correctly. Very few errors were seen. However, there were a significant number who ignored the instruction 'hence' and proceeded to solve the equations by a method of elimination which caused a loss of three marks for a very careless decision. A small number attempted to post multiply by the inverse matrix. Candidates should be aware of the importance of writing down matrices in the correct order when performing matrix multiplication.

Answer: $x = 3$, $y = -2$

Question 4

(i) Many candidates are very proficient at questions involving circular measure and this question was no exception. There were very few candidates who were unable to make a reasonable attempt at both parts. The key to this particular question was finding the angle BOC. Even if this was done incorrectly and the ensuing incorrect angle was subsequently used in both parts of the question, candidates were able to gain credit for correct methods used. Because of the nature of the question, answers of differing accuracy were obtained during the various calculations but all these slightly different lengths/angles all gave an answer which rounded to 181. Candidates should still be encouraged to work to an appropriate level of accuracy. Most candidates were able to obtain a correct value for the required area.

(ii) Most candidates were able to correctly find the appropriate arc length and chord length. Unfortunately, some candidates chose to add on an 'extra radius' and in some cases an 'extra 2 radii'. For those candidates who had been working at less than the required level of accuracy throughout, their work would yield an answer of 65.8, which was penalised by withholding the last accuracy mark.

Answer: **(i)** 181 **(ii)** 65.7

Question 5

This seemed to be the question that was met with the least success. Candidates tried many different methods to gain some marks here, few were able to obtain correct answers without using the standard responses that were to be expected. The important decision to be made in each part was whether to consider permutations or combinations, a concept that some candidates do find difficult to deal with.

(a) (i) Candidates were successful if they knew to use ${}^8P_6$.

(ii) Candidates were given credit if they realised that they had to use ${}^6P_4$ as part of a product.

(iii) Most commonly the correct answer came from $5 \times 2 \times {}^6P_4$ (alternatives were rarely used), with credit again being given if candidates realised they had to use ${}^6P_4$ as part of a product.

(b) (i) Many candidates tried to use ${}^{14}C_4$ and/or ${}^{14}C_8$ but not in a product, but rather as part of a sum.

(ii) Many candidates made a similar error to that in part **(i)** by choosing to add ${}^8C_4$ and ${}^6C_4$ instead of multiplying ${}^8C_4$ and ${}^6C_4$.

Answer: **(a)(i)** 20 160 **(ii)** 2160 **(iii)** 3600 **(b)(i)** 210 210 **(ii)** 1050

Question 6

(i) This was well done by most candidates with the most common error being to truncate the answer to 13.8.

(ii) Most candidates were able to differentiate well and subsequently solve the resulting equation correctly. Errors that were seen on a regular basis were $\frac{10}{t^2+4}$, $\frac{20}{t^2+4}$ and $\frac{2t}{t^2+4}$, but credit was given for a correct approach. There were a small number of candidates who somehow retained $\ln(t^2 + 4)$ or forgot to differentiate $-4t$.

(iii) Many candidates made this part more difficult than it was by not using the quotient rule. Combining the two terms before differentiating and using the product rule invariably caused errors which resulted in an incorrect expression as well as far more laborious algebra. Those who used the quotient rule generally did it well with a minus sign between two correct terms in the numerator. A significant minority did not discard $t = -2$.

Answer: **(i)** 13.9 **(ii)** 1 and 4 **(iii)** 2

Question 7

(i) Well done by most candidates. The most common error was finding $\overrightarrow{AD}$ rather than $\overrightarrow{DA}$.

(ii) Well done by most candidates. The most common error was finding $\overrightarrow{BD}$ rather than $\overrightarrow{DB}$.

(iii) Again, well done by most candidates, however, from a correct expression of $\lambda(4\mathbf{a}+\mathbf{b})$ the result on opening out the brackets often became $4\mathbf{a}\lambda + 4\mathbf{b}\lambda$.

(iv) This was generally well done, but a number of candidates started again and did not make use of the previous parts.

(v) Most candidates did get the first method mark but then many did not know how to subsequently proceed, with a number attempting to divide vectors. Of those who were aware of how to obtain a pair of simultaneous equations, errors in signs (usually $-\mu$ changing to μ in the second equation) often prevented full marks. Although this part of the question had been intended to be solved using vector methods, some candidates successfully made use of the properties of similar triangles to obtain a correct result.

Answer: **(i)** $3\mathbf{a}-\mathbf{b}$ **(ii)** $7\mathbf{a}-\mathbf{b}$ **(iii)** $\lambda(4\mathbf{a}+\mathbf{b})$ **(iv)** $3\mathbf{a}-\mathbf{b}+\lambda(4\mathbf{a}+\mathbf{b})$ **(v)** $\lambda=\frac{4}{11}$, $\mu=\frac{7}{11}$

Question 8

(i) Candidates were often successful in part **(i)**. Errors came from a lack of knowledge of how to integrate rather than actual numerical errors; most commonly the incorrect responses were differentiated. Occasionally the minus sign in the second term was omitted.

(ii) Errors in part **(ii)** were usually from incorrect signs. A completely incorrect part **(i)** often meant no marks in part **(ii)** if the integral being used was not in the form $a\mathrm{e}^{2x}+b\mathrm{e}^{-2x}$.

(iii) If the candidate was correct in part **(ii)** it usually followed that they would gain the two marks in part **(iii)**, The given answer was intended to enable candidates to check and thus correct incorrect work done in previous parts as well as help with the next part of the question.

(iv) This part of **Question 8** was generally more successful. Candidates were often able to form a quadratic equation and attempt to solve it. The negative log was often rejected. More commonly the candidate lost the final mark as they did not give the constants *a* and *b* but a decimal answer. This highlights the need for candidates to make sure that they have their answer in the required form.

Answer: **(i)** $5e^{2x} - \frac{1}{2}e^{-2x}$ **(ii)** $\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) - \left(5e^{-2k} - \frac{1}{2}e^{2k}\right)$ **(iv)** $k = \frac{1}{2}\ln\frac{1}{11}$

Question 9

This was a completely unstructured question which was designed to test the candidates' knowledge of several syllabus objectives and form a logical approach to the solution of the problem. Most candidates performed very well and most were able to get 7 out of the 8 marks available. Common errors included the inability to differentiate $\cos 2x$ correctly, not finding the value of y when $x = \frac{\pi}{4}$ and use of a tangent rather than a normal. Many candidates produced perfectly correct solutions until the final step when they resorted to the use of a calculator to work out the required area, rather than give the exact answer as required. Again this highlights the need of candidates to check that they are giving their answer in the correct form required.

Answer: $\frac{49\pi^2}{64}$

Question 10

(a) Most candidates were aware of the correct order of operations needed to provide correct solutions. However, some candidates forgot to deal with the square root and others forgot that a multiple angle was involved. The most common error however, was not giving all the solutions, forgetting to deal with solutions obtained from use of the negative square root.

(b) Usually done well by most, with the use of the correct identity and correct use of the relationship between $\tan y$ and $\cot y$. Some candidates did not give all the appropriate solutions. Candidates who adopted the approach of dealing with everything in terms of $\sin y$ and $\cos y$ were usually less successful, due to errors in simplification and subsequent factorising. Accuracy of answers is an important aspect of solving trigonometric equations and some candidates lost marks as they chose to give their solutions to 3 significant figures rather than the required one decimal place.

(c) Candidates are now showing good skills at the solution of equations of this type, especially involving radians. Many completely correct solutions were seen and many solutions of just one correct angle were also seen. Candidates do still find it difficult to solve equations where their first solution is not in the range required.

Answer: **(a)** $15°$, $45°$, $75°$, $105°$ **(b)** $71.6°$, $251.6°$ $153.4°$, $333.4°$ **(c)** $\frac{\pi}{2}$, $\frac{11\pi}{6}$

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

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(i) (ii) A number of candidates were unsure of the meaning of "period" and "amplitude" and many gave the answer 2 to both parts of the question.

(iii) Most candidates obtained the mark for sketching the graph of $y = \sin 2x$.

Many sketches of $y = 1 + \cos 2x$ were incorrectly drawn as, although most started at a maximum of 3 and ended at a minimum of –1, the intermediate stationary points were not also at those values.

(iv) The majority appreciated that the answer to this question was related to the number of intersections of the graphs in part **(iii)**.

Answers: **(i)** 180° **(ii)** 2 **(iv)** 3

Question 2

(i) Although candidates were instructed not to use calculators and to show all working in this question, many answers were offered showing no working at all. These were given no credit.

(ii) These were a reasonably easy 3 marks for those who used $\sec^2\theta = 1 + \tan^2\theta$. Those who worked out the length of the hypotenuse and $\sec^2\theta = \dfrac{1}{\cos^2\theta}$ rarely got far enough to earn any marks.

Answers: **(i)** $-1 + 2\sqrt{2}$ **(ii)** $10 - 4\sqrt{2}$

Question 3

(i) This part of the syllabus has clearly been taught well by most Centres. The majority of candidates were able to gain full marks here; however a common mistake was to work out $\left(x^2\right)^3$ as x^5.

(ii) Again, this was generally done well, although many candidates only saw two, rather than three, terms independent of x.

Answers: **(i)** $64 + 192x^2 + 240x^4 + 160x^6$ **(ii)** 1072

Question 4

(a) Nearly all candidates made a reasonable attempt here. A few left the elements of their matrix in unsimplified forms, such as –8 + 0. Only a few attempted to find $\mathbf{X}^2$ by squaring each of the elements of **X**.

(b) A number of different ways of solving this problem were offered by candidates. Some found the inverse of **A** in terms of a and b, others inverted $\mathbf{A}^{-1}$, and only a few equated the product $\mathbf{AA}^{-1}$ to the identity matrix. Most candidates gained full marks.

Answers: **(a)** $\begin{pmatrix} 4-4k & -8 \\ 2k & -4k \end{pmatrix}$ **(b)** $a = 2$, $b = 4$

Question 5

Most candidates were able to find the coordinates of the points A and B, and this question was a good source of marks for many, although a large number appeared not to understand the meanings of either "perpendicular" and/or "bisector". Consequently, many candidates found the equations of lines perpendicular to AB through either A or B rather than through the mid-point, or even lines of gradient *3* rather than $-\frac{1}{3}$.

Answer: $3y + x - 2 = 0$

Question 6

(i) The remainder theorem was well-known by most candidates who were able to form and solve the two simultaneous equations in a and b. Those who found only one equation and used the given value of b to find a were given little credit.

(ii) The division of the cubic expression by the linear factor was generally done well. Those who chose to do this by synthetic division frequently arrived at an incorrect quadratic which was twice the correct answer.

(iii) It was hoped that candidates would evaluate the discriminant and deduce that there were no further real roots because it was negative. Some candidates suggested, incorrectly, that as the quadratic would not factorise there were no further roots. Others ignored the negative and gave two further roots in terms of $\sqrt{40}$, presumably misinterpreting "real" as "rational".

Answers: **(i)** $a = 14$ **(ii)** $(2x-1)\left(7x^2 - 4x + 2\right)$

Question 7

(i) Most candidates quoted and/or used the quotient rule correctly. It was common to see the differential of $\ln\left(4x^2+3\right)$ being given as $\dfrac{1}{4x^2+3}$. Otherwise the question was generally done well. Approximation of $-\ln 3$ to –1 lost accuracy marks.

(ii) Most candidates attempted to use $\dfrac{1}{2}bh$ on the right-angled triangle *AOB*. Finding the coordinates of *B* from their answer to part **(i)** was part of the solution which a few candidates performed incorrectly by assuming that $x = 0$, rather than $y = 0$ on the *x*-axis.

Answers: **(i)** $y = \dfrac{x}{\ln 3} - \ln 3$ **(ii)** $\dfrac{1}{2}(\ln 3)^3$

Question 8

(i) The concept of range was generally well understood.

(ii) Most candidates could find the inverse of g, but the domain was often wrongly stated as $x \geq 5$.

(iii) The majority of candidates knew the correct order of operations for gf and how to solve the equation, but many lost the final mark by giving the answer as a decimal, rather than the exact answer required by the question.

(iv) f' was misinterpreted by many as f^{-1}.

Answers: **(i)** $f(x) \geq 3$, $g(x) \geq 9$ **(ii)** $-2+\sqrt{x-5}$, $x \geq 9$ **(iii)** $\dfrac{1}{2}\ln\left(\dfrac{4}{3}\right)$ **(iv)** 96

Question 9

(i) It was necessary either to show that the gradient of both the line and the curve at *A* was 3, or to show that the equation of the tangent to the curve at *A* was indeed $y = 3x + 10$. It was not sufficient, as many candidates tried, to solve the equations of the line and the curve, leading to $x = 0$ or $x = 5$, without any further explanation as to why the line was a tangent.

(ii) Most candidates successfully differentiated, equated to 0 and solved. As only the *x*-coordinates were asked for, many wasted time by also finding the *y*-coordinates.

(iii) Most candidates knew to integrate to find the area under a curve. The most common error was to use a lower limit of $\dfrac{1}{3}$ rather than 0.

Answers: **(ii)** 3 and $\dfrac{1}{3}$ **(iii)** $\dfrac{99}{4}$

Question 10

(i) Most candidates were able to reach $\sin x = \frac{1}{2}$ leading to 30° and 150° but many did not use the negative square root to obtain the two other solutions.

(ii) Those who used $\tan^2\theta = \sec^2\theta - 1$ to obtain a quadratic equation in $\sec^3 y$ were generally successful, more so than those who attempted to convert the equation to one in $\cos^3 y$ using three separate identities. The solutions 143.5° and 180° were frequently omitted.

(iii) Many completely correct solutions to this part were seen. Most candidates knew the correct order of operations to solve trigonometric equations of this type.

Answers: **(i)** 30°, 150°, 210°, 330° **(ii)** 60°, 180°, 23.5°, 96.5°, 143.5° **(iii)** $\frac{2\pi}{3}$, $\frac{5\pi}{3}$

ADDITIONAL MATHEMATICS

Paper 0606/21

Paper 21

Key Messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. On occasion, drawing or marking information on a diagram is helpful, and candidates should be encouraged to do this. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Omitting method steps through using a calculator often results in full credit not being given for a solution. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions.

General Comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, if a question requests the answer in a particular form, they give the answers in that form. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question.

Where an answer was given and a proof was required, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

When asked for a sketch, many candidates plotted and joined coordinates, rather than making a sketch showing the key features. Candidates would improve if they realised that this often resulted in diagrams that were of the wrong shape or incomplete.

Comments on Specific Questions

Question 1

(a) Candidates used a variety of methods to answer this part of the question. The most successful approach was to use the change of base rule. This immediately gave a term in base 3, as required. Alternative methods in other bases, for example $\log_{3^3} x$ were only given credit when converted into the required base. Candidates would improve by avoiding using these, less direct, methods to answer the question. Methods which involve an increased number of steps increase the opportunity for errors to be made. This was also true of those solutions which began by making use of the reciprocal. On occasion, the base of the logarithm looked like a multiple.

(b) There were some elegant solutions where the required result was obtained in three or four lines. Some candidates who knew some of the rules of logarithms were unable to apply them to the question. These candidates would do better if they attempted to apply their laws one at a time, rather than attempting to apply many together. Many would also have made more progress if they had recalled that 1 could be written as $\log_a a$. Some candidates anti-logged the given equation and then gave that as their final answer. These candidates would have done better if they had appreciated that they should continue to simplify their answers where it is clear that further simplification can be done, such as in this case.

Answer: **(a)** $\dfrac{\log_3 x}{3}$ **(b)** $y = 125a$

Question 2

(a) Many candidates gave only one answer and this was usually for the part of the function with a negative gradient. These candidates may have done better if they had extended the lines in the diagram and considered them separately. While there were some correct second equations given, it was common for candidates to assume that the line with positive gradient was perpendicular to the line they had already found, resulting in lines with gradients of ±0.5. Candidates also sometimes incorrectly gave their answer in the form of modulus functions. This resulted in the loss of a solution as they had, in effect, made their answers the same.

(b) Candidates should know the shape of the graph of an exponential function. The curve demanded was a simple translation of $y = e^{-x}$. Candidates would have done better if they had thought of it in this way. Many candidates attempted to plot points rather than sketch a curve. This takes time and should not be necessary for a question of this type. Poor choice of 'scale' at times led to curves approaching the *x*-axis. Some candidates concentrated their efforts on the part of the curve in the first quadrant with no indication of what was in the second. Candidates drew a variety of graphs, with straight lines, perhaps the most popular. Many candidates identified the intercept with the *y*-axis as the point (0, 4). Candidates should be aware that marks are awarded for the correct shape and positioning of graphs and that coordinates of intercepts alone, do not gain credit unless the graph is attempted also.

Answer: **(a)** $f(x) = 2x - 4$ and $f(x) = -2x + 4$

Question 3

(a) This question was well done by the majority of candidates. Occasionally, one or two elements were miscalculated. A few candidates attempted to find a 2 x 2 matrix. A small number of candidates would have improved if they had taken more care with the order of operations required or had read the question more carefully.

(b) Candidates were asked to state what was represented by certain matrix products, rather than to evaluate them. This was challenging for many. Many candidates gave clear and concise statements that were easy to credit. Other candidates misinterpreted what was required, sometimes thinking that identifying the order of the resultant matrix was sufficient. While much leniency was given in the use of terms to indicate value, candidates needed to be clear regarding whether the matrix they were describing represented the total value of the stock in **each** shop (as in part **(b)(i)**) or the total value of the stock in **all** the shops (as in part **(b)(ii)**). Simple statements are generally the best solutions to such questions, as answers which offered more description often became ambiguous or contradictory. Some candidates found evaluating the matrix product useful. Others became confused and related the values to the 4 different types of television rather than shops. It was not uncommon for candidates to reverse the answers to part **(b)(i)** and part **(b)(ii)**.

Answer: **(a)** $\begin{pmatrix} 8 & -2 & 6 \\ 4 & 3 & 10 \end{pmatrix}$

Question 4

(i) This was generally well attempted. Many candidates recalled that the angle between radius and tangent was a right-angle and used the tangent ratio. Others found other relevant angles correctly and used the sine rule. Some candidates gave lengthy solutions involving calculating numerous interim lengths. These methods were not always successful as the extra steps needed introduced a greater risk of making an error. A few candidates selected incorrect trigonometric ratios.

(ii) Most candidates made a good start to this question. They realised that the area of a sector was relevant and applied the formula correctly. There was no uniform method favoured for calculating the required area. “Kite minus sector” was most common, with the area of the kite being found by various methods. Some candidates calculated the lengths of the diagonals of the kite, most found the areas of two triangles and summed. Calculating twice the area of triangle *OPT* was most common, as this made use of the value already found. Candidates who found the area of the segment were often unsure what to do with it. Other candidates realised it was possible to use this successfully by subtracting it from the area of triangle *PQT*. When candidates used longer, multi-step methods, it was sometimes difficult to follow their intended logic. Calculations appeared randomly and sometimes, due to the resulting lack of space, alongside the diagram. This resulted in marks being difficult to award as the candidate’s intention was not clear. Also, with the extra calculations involved, there was often a loss in accuracy in the final answer.

(iii) The vast majority of candidates found the length of the arc *PQ* correctly. Most of these candidates were able to apply the correct method of adding this to twice the length of *PT*. Full credit depended on having found *PT* correctly in part **(i)**.

Answer: **(i)** 19.3 **(ii)** 79.1 **(iii)** 57.5

Question 5

(a) A majority of candidates realised the correct response was a permutation of the digits. A good number of these gave clear and precise answers about the order of the digits being important and earned the mark. Some candidates needed to explain their reasoning more clearly. It was not sufficient to comment on arrangements without mentioning the need for a specific order. Stating that this was the only number that would work, did not add to the information given in the question. Candidates need to be aware that, when explaining something, using some phrases from the question may be helpful, but alone, these phrases will not be enough to gain credit. A few candidates chose ‘combination’ but then gave an explanation that matched ‘permutation’, so had mixed up the terminology. Others thought that it was a ‘combination’ as ‘order did not matter’.

(b) A few candidates understood this question well and scored full marks. There were a good number of answers given that gain some credit. Some candidates needed to improve their understanding of when they should be multiplying – when the process involves “and” – and when they should be adding – when the process involves “or”. It is a common misconception by candidates that “and” always means they should “add”. Permutations were commonly used in this part of the question. Candidates who used them needed to read the question more carefully to gain a better understanding of what was required.

(i) Many candidates understood that they were selecting 4 from 6, 4 from 5, 4 from 7. The interpretation of “4 from 6 or 4 from 5 or 4 from 7” was often not made correctly. Candidates demonstrated this by multiplying their combinations, rather than adding them.

(ii) Again, many candidates understood which selections needed to be made. In this part the interpretation needed to be of “ 1 from 2 and 1 from 6 and 1 from 5 and 1 from 7”. Candidates often misinterpreted the “and” as “add” rather than correctly multiplying their combinations.

(iii) Some candidates listed the possible outcomes – this was good practice. Some of these included impossible outcomes for the context such as 0, 0, 0, 3. A few candidates gave one product only – listing outcomes may have avoided this. Reversing addition/multiplication was again seen in solutions offered for this part of the question.

Answer: **(b)(i)** 55 **(ii)** 420 **(iii)** 70

Question 6

This question was generally well answered, with many candidates obtaining most of the marks available. There was very little confusion over when to integrate and when to differentiate.

(i) The majority of candidates made a good start to the question, setting $v = 0$ and solving the quadratic correctly. The method of solution was mostly omitted, with the correct values simply being stated. Candidates should be encouraged to show their method and check their solutions using their calculator, rather than relying on their answers being accurate enough to earn credit. Some candidates need to read the question more carefully. The question asked for the time at which the particle first came to rest and some did not choose the first occurrence (at $t = 1$). Others seemed to assume that time began at 1, rather than 0, and therefore stated that $t = 6$ was the first occurrence. A small number of candidates misinterpreted the question and either substituted $t = 0$ or used calculus.

(ii) Again, this question was well done by the majority of candidates. Integration was usually attempted and accurately carried out. Sometimes, candidates omitted to evaluate the constant of integration which, given the initial conditions, was zero. Weaker candidates need to take care with their variables as, occasionally, some terms were correctly given in t and others incorrectly in x.

(iii) All candidates answered this part very well. Very occasionally, candidates changed the negative answer to a positive one without reference to deceleration.

Answer: **(i)** 1 **(ii)** $\frac{2t^3}{3} - \frac{14t^2}{2} + 12t$ **(iii)** -2

Question 7

(a) Many candidates were able to make a positive start, usually by finding $\overrightarrow{AC}$. A large number of these candidates progressed further by finding two relevant vectors and showing that they were parallel. Very few candidates made the key statement that their vectors had a common point. The question required a given result to be shown and candidates who omitted to state this did not fully complete the argument. A common misunderstanding was to assume that showing $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ would be sufficient. It was also common for candidates to incorrectly rewrite the given vectors as column vectors in **a** and **b**. This question proved to be a good discriminator.

(b) (i) The best approach was to find and state $\overrightarrow{PQ}$ before finding the modulus. Many candidates attempted to do this. Some candidates added the position vectors given – a simple diagram may have helped these candidates correct this error. Candidates who omitted to state their $\overrightarrow{PQ}$ before calculating the modulus were penalised if their values were incorrect as method steps that were not explicitly stated could not be credited.

(ii) Many candidates showed that they fully understood what was required to answer the question. Some candidates demonstrated that they had not understood the concept of a ‘unit vector’ by restating, or even finding again, the vector found in part **(i)**. Conversely some candidates did not find the modulus in part **(i)**, but then found it as part of finding the unit vector correctly in part **(ii)**. These candidates needed a better understanding of the notation, the general concepts or the connections between them.

(iii) The required correct method was shown by a good number of candidates. In this part of the question in particular, candidates would have perhaps benefitted from drawing a simple diagram. Some candidates did this although they were very rarely seen. Candidates often halved their answer to part **(i)** to find the midpoint. If they had incorrectly added and found 4**i**+3**j** in part **(i)** this resulted in a seemingly correct answer from a wrong method.

Answer: **(b)(i)** $\sqrt{125}$ **(ii)** $\frac{1}{5\sqrt{5}}$(2**i** + 11**j**) **(iii)** 2**i** + 1.5**j**

Question 8

In both part **(a)** and part **(b)**, candidates were usually able to give a correct form for the integral if not always the correct coefficient. As a result most candidates gained marks on all parts. Candidates were expected to show the substitution of limits before evaluating their final answer. Some candidates simply used their calculator to evaluate the integral, without carrying out the required integration. These candidates did not take note of the hence in part **(a)(ii)** and part **(b)(ii)**. Candidates who did not show the method of using limits were penalised. This was most notable when a correct evaluation was stated following incorrect integration.

In part **(c)** candidates who thought about the manipulation needed were usually successful. The brackets needed to be expanded before integrating. Those who realised this were usually accurate in their expansion and in the subsequent integration. Some candidates omitted the cross terms when squaring and then integrated two terms only. Many candidates would have done better if they had appreciated that the integration of such complex functions is not required in this syllabus. This should have directed them towards the simplification to three terms that was required. Commonly, candidates attempted an invalid "chain rule for integration method". As the integral was indefinite, it was appropriate to include the constant of integration.

Answer: **(a)(i)** $\frac{1}{4}e^{4x+3}+c$ **(ii)** 707 000 **(b)(i)** $3\sin\frac{x}{3}+c$ **(ii)** 0.521 **(c)** $\frac{x^{-1}}{-1}+2x+\frac{x^3}{3}+c$

Question 9

(a) This was a very well done question. Most candidates correctly identified the critical values and sign errors were rarely made. Some candidates did not state a range of values as their answer. Those who did were frequently correct. Some candidates would have found a sketch helpful, although they were rarely drawn. Some candidates gave their answer as two separate inequalities. While some of these candidates correctly connected their inequalities using "and", many were unable to gain full credit as they omitted the word "and" or used "or" or a comma, for example. Most candidates used the correct inequality signs and, incorrect, strict inequalities were rarely seen.

(b) (i) This part was also generally well done with many correct solutions. Some candidates omitted to subtract 16 when finding *b*.

(ii) Candidates were expected to use their previous answer to find the greatest value. Credit was given to those who started again or differentiated. There were some good, neat answers, using part **(b)(i)**, as directed in the question. Weaker candidates often carried out algebraic manipulation which had little to do with what was required. Many candidates clearly identified which of their two values was which. Other candidates would do well to realise that this was also good practice, as some solutions were quite vague. Stating the coordinates of the maximum point only did not score as the candidate had not interpreted the information given by each coordinate.

(iii) Candidates drew some very good sketches in this part. Many identified the three points of contact with the axes correctly. Some candidates omitted to indicate where their curve met the *y*-axis in their, otherwise, very good sketches. Weaker candidates would improve if they paid more attention to the overall shape of the curve generated by a quadratic function. There was a tendency to assume that the maximum must lie on the *y*-axis leading to some distortion of the quadratic shape.

Answer: **(a)** $-5 \leqslant x \leqslant \frac{1}{4}$ **(b)(i)** $(x+4)^2-25$ **(ii)** greatest value 25 at $x = -4$

Question 10

(i) This part was generally well done. Some candidates linked their equations explicitly to $y = mx + c$ and this is good practice in questions of this type. The most common error was to use logs to base 10 rather than logs to base e. Candidates needed to read the question more carefully to avoid this error as this was clearly stated in both part **(i)** and part **(ii)**. Most candidates were able to apply at least one of the laws of logarithms.

(ii) While a good many candidates were able to find the intercept and gradient, very few managed to gain full credit. The reasons for this were numerous. Candidates need to be aware that, when finding the gradient and intercept of a line of best fit, they must use points on the line, not the original data points. Using the data points rather than the given line was the most common error. Others equated A and b to the intercept and gradient or used logs to base 10. In finding the gradient, the differences were sometimes inverted or inconsistent leading to a positive gradient. Candidates needed to read the question carefully as some did not give their, otherwise correct, answer to the required accuracy.

(iii) The best and simplest approach was to read the value for $\ln y$ directly from the line in the given diagram. This gave an accurate solution if read and anti-logged correctly. Some of these candidates gave their answer as 9 or $\ln 9$. Many candidates chose to use their values from part **(ii)** and substituted them into one of the forms of the equation. While this was a possible method it usually lacked accuracy especially as candidates tended to use an already rounded value. Candidates would do well to realise that, using values given in the question where possible, rather than those they have calculated, should produce more accurate answers.

Answer: **(i)** $\ln y = \ln A + x\ln b$ **(ii)** $A = 90\,000$, $b = 0.4$ **(iii)** $y = e^9$ or 8000 to 1 sf

Question 11

Candidates found this question challenging. If they had identified the regions of the Venn diagram correctly and had started to complete it from “the centre outwards”, they would have achieved greater success. The few candidates who followed a logical strategy were successful. Often, it was not possible to identify a candidate’s method. Many candidates would improve by ordering their method steps and thinking the problem through using the diagram to help them. Often, candidates omitted to use x and y in their method. Occasionally, correct answers appeared with no working at all. Candidates were likely to be successful in the first two parts of the question if part **(i)** was answered correctly with x, $7 - x$, and $6 - x$ being correctly marked on the diagram. Candidates marking the 7 and 6 on the diagram incorrectly usually obtained no marks. Candidates did not always identify the correct area – so they may have had 4 and 9 on the diagram, but then gave x and y as different values. This was more common in part **(ii)** than in part **(i)**. The last part of the question was not always ‘shown’ as thoroughly as it needed to be. Some candidates did not use numbers in their answer or omitted to state “= 0”. This part was rarely correct if part **(ii)** was incorrect. The exception to this was where the candidate had the correct numbers on the diagram, but had identified y incorrectly as a value other than 9.

Answer: **(i)** 4 **(ii)** 9

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key Messages

In order to do well in this examination, candidates need to give clear answers to questions, with sufficient method being shown that marks can be awarded. Candidates need to take note of instructions such as "Do not use a calculator" or "You must show all your working" in a question. These instructions mean that omitted method will result in a significant loss of marks. Candidates should ensure that their answers are given to no less than the accuracy stated on the paper in order to be credited.

General Comments

Many candidates gave clearly presented answers. Others need to appreciate that poorly presented work is often difficult to credit. If answers are written in alternate locations within their script, candidates should indicate where they have written the continuation of their solution. Some candidates seem to write their solution in pencil first and then write over it in pen – this should be avoided. Some candidates could improve by understanding that their working must be detailed enough to show their method clearly, with each key step being shown as this can allow method marks to be awarded; it is essential if a question asks candidates to "Show that…". This indicates that the answer has been given to the candidates and that the marks will be awarded for showing how that answer has been found. The need for this was highlighted in **Question 10(b)(i)** and **Question 11(i)**. Showing clear and full method is also very important if the use of a calculator is not allowed, such as in **Question 3**. Occasionally it was evident that candidates needed to read the question more carefully. Trial and improvement is not a technique that is assessed in this syllabus and candidates who employ this as a method generally are not given full credit for their solution. When a question has a context, such as **Question 4**, answers should be integer values or decimals rounded as appropriate – answers in surd form are not generally acceptable for full credit in questions of this type.

Comments on Specific Questions

Question 1

This question proved to be an accessible start to the paper for almost all candidates. In part **(i)**, some candidates could have improved if they had listed the elements of the universal set and deleted them as they were placed in the Venn diagram. It was common for candidates to have omitted the elements 5 and 11. Some candidates did not use their Venn diagram to answer part **(ii)** and part **(iii)**. These candidates started again, working with the separate sets stated in the question. Whilst this was not wrong, it did waste some time. The majority of answers offered for part **(ii)** were correct, although some candidates were penalised for poor use of notation – such as giving their answer as {3}. Otherwise, a good start to the paper.

Answer: **(ii)** 3 **(iii)** {4, 6}

Question 2

(i) Many candidates found this question challenging. The question assessed their ability to form a pair of matrices suitable for matrix multiplication in a particular order. Many candidates reversed the order, giving matrices suitable to find the matrix product **PQ**, rather than **QP**. Those candidates who wrote down and showed clear consideration of the orders 1 by 2 and 2 by 3 were usually successful. A small number of candidates either summed the number of passengers in each class or separated the data out so that they had 4 matrices rather than 2. Candidates who did this may have improved if they had understood that the presentation of data in a matrix does not usually involve combining or separating the elements in a category before the matrix is formed.

(ii) Most candidates earned a mark for multiplying their matrices correctly. A few needed to give more attention to their arithmetic. The majority of those who found the correct matrices in part **(i)** went on to multiply them correctly and give their answer in the correct form to earn both marks here. A small number of these presented their figures as a 3 by 1 matrix rather than a 1 by 3 matrix. These candidates may have improved if they had again considered the order of the matrices:

(iii) Although candidates were not asked to evaluate **QPR**, many did, finding it helpful in understanding exactly what information it gave. Some candidates suggested that **QPR** gave *the total revenue from each flight* – which is a correct interpretation of **QP** rather than **QPR**. Some candidates commented that **QPR** could not be found. This should have been an indicator of an error in an earlier part. A good proportion of comments made were clear and accurate enough to be credited.

Answer: **(i)** $\mathbf{P} = \begin{pmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 120 & 300 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 22,200 & 24,000 & 17,160 \end{pmatrix}$

Question 3

(i) A high proportion of candidates formed a correct equation, such as $BC = \dfrac{36 + 15\sqrt{5}}{6 + 3\sqrt{5}}$. A few candidates clearly used their calculator and so did not score any marks. Some candidates attempted some spurious cancelling, arriving at an answer of $6 + 5\sqrt{5}$. There is a possibility that these candidates were confused between the binomial presentation of irrational numbers and numbers presented in standard form. Most candidates indicated that the next step in their method was to multiply the numerator and denominator by the square root conjugate of their denominator. Some candidates went from this step directly to the final answer and therefore earned no further marks as key method steps had been omitted. Candidates needed to show clear evidence that a calculator had not been used. Multiplying out was very well done and most candidates who showed this step in the method also found the correct answer. A small number of candidates formed and solved simultaneous equations and this was generally well done.

(ii) Most candidates applied Pythagoras' theorem correctly and a high proportion showed sufficient evidence of squaring their values to earn the first mark. A few candidates made unfortunate arithmetic slips. A very good number of fully correct answers were seen.

Answer: **(i)** $1 + 2\sqrt{5}$ **(ii)** $102 + 40\sqrt{5}$

Question 4

(i) The majority of candidates drew a diagram and many of these candidates scored both marks. Those who drew right-angled triangles with the shorter legs labelled 2 and 3 showed that they had misinterpreted the question. Indicating which angle had been found was necessary to earn both marks. Some candidates would have improved if they had made this clear, either in their diagram or by making a reasonable comment. Some assumed the position of North and incorrectly described the direction as a bearing. Other candidates gave an angle and no reference axis, some simply said "to the other side". These candidates would do better should they understand that they simply needed to give a clear and unambiguous indication of which angle they were finding.

(ii) Most candidates attempted to calculate a resultant velocity and divide 80 by that velocity. A few candidates calculated the distance rowed and divided that by 3. The simplest approach was to apply Pythagoras and find the resultant using the figures given in the question – this reduced the likelihood of a premature approximation error being made, using trigonometry with their angle from part **(i)**. Candidates should be aware that they are likely to be more accurate in their solutions if they use figures given in the question, where possible, rather than ones they have calculated. Some candidates simply added or subtracted the two speeds given in the question, showing a lack of understanding of their vector triangle. A resultant velocity of $\sqrt{13}$ was not uncommon from those candidates who had misinterpreted the question in part **(i).** Weaker candidates tended to try to find the relevant distance or resultant velocity in this part using a mixture of 80 with 2 or 3, not taking into account the units given. Some neat approaches were seen from more able candidates, using Pythagoras and a correct right-angled triangle with sides 80, 2*t* and hypotenuse 3*t*.

Answer: **(i)** 48.2° to the bank **(ii)** 35.8 seconds

Question 5

This was a standard simultaneous equations question which was answered very well by almost all candidates. Although a few candidates were careless with their initial substitution or expansion of brackets, errors were rare and fully correct answers common. Whilst most solutions were well presented, occasionally solutions were poorly presented with scribbled out sections or alterations where candidates had over-written figures making it difficult to determine whether the work was actually correct. Some candidates, having found all the correct values, then incorrectly rejected some of them as being incorrect – usually the fractional solutions.

Answer: $x = \frac{4}{3}$, $x = 3$, $y = \frac{8}{3}$, $y = 1$

Question 6

(a) Well answered, although many answers were given as 1.22, with no greater accuracy seen. Candidates who did truncate their answer in this way were penalised and they would have done better if they had written down the longer decimal answer first, then rounded. Almost all candidates realised that, generally, when the unknown is an exponent, logarithms need to be taken at some point. Some very good algebraic manipulation was displayed by candidates here, with many rearranging the given equation to $6^x = 9$. Many took logs to base 6 at this point, although some candidates incorrectly attempted to convert both sides to powers of 3 in order to compare exponents. Occasionally bracketing errors were made with a first step of $x - 2\log 6 = \log\frac{1}{4}$ being evaluated as $x = \log\frac{1}{4} + 2\log 6$.

(b) The simplest methods of solution were to either combine all the logarithms or separate them all. Combining them was, by far, the most popular method. Candidates achieved most success using this method if they kept their solutions simple, neat and were careful with their cancelling of terms. Those separating the logs often made errors when bringing down powers – for example, $\log_a 2y^2$ became $2\log_a 2y$ or $\log_a 16y = \log_a 2^4 y$ became $4\log_a 2y$ and were less successful. A good number of correct solutions were seen, although some neglected to discard the negative two. Some, weak candidates simply "cancelled" the logs and incorrectly formed a quadratic in y from the sums and differences of the arguments, which they then solved. Some seemingly correct answers were from wrong working. Some candidates decided to let $a = 2$, simplifying the question greatly and these were not credited. A small number of candidates did correctly change the base of each logarithm from a to 2; this was unnecessary, resulted in extra work to arrive at the answer and often was incomplete or unsuccessful.

Answer: **(a)** 1.23 **(b)** $y = 2$

Question 7

A good number of candidates presented neat, well thought out and fully correct solutions which showed good understanding of what was required. Other candidates would have done better if they had paid more attention to presentation, as factorials were lost or figures misread in their deletions and amendments.

Most candidates were able to start the question, with many writing their coefficients in terms of ${}^nC_2$ and ${}^nC_4$. Of these candidates, a few did not deal with 1^{n-2} and 1^{n-4} and made no progress. Those who did, often correctly arrived at $40\,{}^nC_2 = 16\,{}^nC_4$ and found the answer by trial and improvement techniques on their calculator, rather than showing the full method. Some candidates made good use of the given Binomial Theorem on the formulae page and immediately converted ${}^nC_2$ and ${}^nC_4$ into factorial form. Some slips were made when handling the factorials, $4!$ becoming 4 was not uncommon, as was $(n-4)!$ becoming $(n-4)$, for example. Some candidates simplified $\frac{(n-2)!}{(n-4)!}$ correctly showing good understanding of what the notation meant. Other candidates thought that $(n-4)! = (n-4)(n-3)(n-2)!$, showing a clear misunderstanding. Candidates who avoided the factorial form, and who seemed to have learned the expansion for $(1+x)^n$, made the simplest immediate progress, writing the coefficients directly in terms of n. These candidates were more likely to omit the powers of 2, however. Some candidates misplaced the factor of 10 and would have benefitted from reading the question more carefully. Candidates who realised that, as n was positive and also needed to be greater than 1 for the expansion to exist, they could safely divide both sides by n and $n-1$ arrived at the correct answer much more easily than those who multiplied out and then had to factorise a quartic or cubic expression, which often contained an error.

Answer: 8

Question 8

Candidates were instructed to show all their working in this question. and those who omitted to do so were not awarded full marks. This question, assessing the application of integration to evaluate a plane area, was well answered. Most candidates found the area of the trapezium *OABC* and subtracted the necessary area between the curve and the *x*-axis from it. Many candidates correctly found the coordinates of *A*, *B* and *C*, although occasionally 7 was calculated as 5. The most popular method of finding the area of the trapezium, *OABC*, was to use $\frac{OC(OA+BC)}{2}$. Others integrated the equation of the line from 0 to 7. Most went on to correctly calculate the required area between the curve and the *x*-axis by integrating. Some candidates omitted to show the substitution of the limits into their integral or integrals and were penalised for omitting a key part of the method, as all working needed to be shown in full to gain full marks. Some candidates integrated the difference of the equations. This more sophisticated approach was often very successful, though sign errors were sometimes made in the process. Occasionally, candidates erroneously mixed methods and, after having found the correct answer, went on to subtract it from the trapezium, for example.

Answer: $\frac{343}{6}$

Question 9

This whole question was well-answered. Few errors were seen and candidates displayed very good understanding of what was required.

(i)(ii) Almost universally correct.

(iii) Again, well-answered. Candidates either used a gradient calculation with the coordinates of *P* or formed the equation of *PR* to correctly find the coordinates of *R*. Those who chose to find the equation of *PR* sometimes gave their answer as (0, 8).

(iv) Most candidates correctly stated the perpendicular gradient using the value of *m* they had found in part **(i)**. Some candidates unnecessarily attempted to find the gradient of *PQ* again, and were not always successful in their attempts. Many candidates omitted to rearrange the equation into the form required in the question, with *a* and *c* often being quoted as fractions. These candidates may have benefitted from rereading the question once they had completed their solution.

(v) This was very well answered. A few candidates calculated $\left(\frac{x_P - x_Q}{2}, \frac{y_P - y_Q}{2}\right)$.

(vi) The simplest method of solution was to use half base times height, as all the lengths could easily be calculated from the coordinates found. Very few candidates observed this so various methods of solution were offered – some of which took considerable time. Candidates would do well to appreciate that the more complex the solution offered, the more opportunity there is for error.

Answer: **(i)** 3 **(ii)** 1 **(iii)** (–1, 7) **(iv)** $x+3y=32$ **(v)** $\left(\frac{1}{2}, \frac{11}{2}\right)$ **(vi)** 4.5

Question 10

(a) Some candidates drew neat and accurate sketches, with all key features clear and present and were rewarded for their attention to detail. Other candidates would improve if they paid more attention to the key features stated here. Many candidates realised that the curve drawn needed to be all above the *x*-axis, though some candidates sketched f : $x \to \sin x$. The quality of drawing was variable. Although it was often omitted, many candidates marked 1 on the vertical axis. These candidates then often neglected to take any notice of where they had marked 1 and had maxima well above or below their indicated position. Careful attention was not always paid to the maxima being of equal height. Some candidates drew a turning point at (180, 0) rather than a cusp. Most candidates attempted to draw a curve, although on occasion candidates seemed to have used a pair of compasses as their curves were semi-circular.

(b) (i) As this was a question where the answer had been given, all steps relevant to arriving at that answer needed to be clearly indicated. This was not always the case and the question proved to be a good discriminator. Many candidates used the fact that h and g were inverse functions to answer this part of the question, starting with $y = \ln(4x - 3)$ and rearranging to find the inverse. These candidates often justified this by stating h = g^{-1} although this information was sometimes omitted. Some candidates justified their solution by stating hgg^{-1} = h only. This was insufficient as, although this is generally true, candidates needed to provide further evidence to link this statement to the question given. An alternative approach used by many candidates was to form hg and show that the result was *x*. This was the simplest way to arrive at a complete and correct answer.

Candidates who started by forming an identity by putting $\frac{e^{\ln(4x-3)}+3}{4} = x$ which they then went on to manipulate as if it were an equation, stating results such as $4x - 3 = 4x - 3$ would do better if they understood that this is not good practice. Weaker candidates started from $x = \frac{e^x+3}{4}$ or $x = \ln(4x-3)$ or $\frac{hg}{g} = \frac{x}{\ln(4x-3)}$ and made no progress.

(ii) A good proportion of candidates drew curves in the first quadrant only with the *y*-intercept indicated, as required, and these were sufficiently accurate to gain credit. Some candidates omitted to mark the *y*-intercept or state its coordinates as required in the question. These candidates may have improved by rereading the question. Some candidates thought the *y*-intercept was $\frac{3}{4}$ through misevaluating $\frac{e^0+3}{4}$ rather than using the relationship between the graph of a function and its inverse. Candidates who used a square scale and drew $y = x$ approximately at 45° to the *x*-axis produced the most accurate sketches. Some candidates drew sketches that curved back or that had turning points close to the *y*-axis. These candidates may have improved if they had considered the functions given earlier in the question – g being logarithmic and h being exponential in form.

(iii) Many candidates stated the domain of g rather than h here. Some are still seemingly unclear about which variable to use in their statements. Candidates should realise that domains are always described using the independent variable, in this case *x*. Some candidates needed to be more careful with their choice of inequality sign or words, as $x > 0$ was a common answer.

(iv) There were more correct answers given to this part of the question. Candidates should realise that ranges are always described using the dependent variable, in this case *y* or h(*x*). Candidates may improve in this part and in part **(iii)** if they are able to link the domain and range of a function to its graphical presentation.

Answer: **(iii)** $x \geqslant 0$ **(iv)** $y \geqslant 1$

Question 11

(i) Many candidates used the simplest approach and formed a proportional relationship using the heights of the pyramids and the sides *AD* and *PS*. Some candidates adopted much more complex approaches using other proportional relationships, such as the volumes of the two pyramids. Whilst this is not incorrect, the extra manipulation required to find an expression for *AD* in terms of *h*, introduced a greater probability of making an error. Some candidates found other forms of the expression for *AD* using Pythagoras' theorem repeatedly. Weaker candidates tended to state $\frac{AD}{4} = \frac{h}{8}$ and then were unable to make progress with finding the given expression for the volume. Other candidates would have done better if they had realised that, working back from the expression given for the volume to find *AD* and then using that expression to find the volume given, was a circular argument that gained no credit. Candidates who correctly found *AD* almost universally scored all 4 marks in this part. A small number of candidates omitted to find the volume, rereading the question might have helped.

(ii) A good number of fully correct solutions were presented for this part. Most candidates were able to differentiate the given volume correctly, having realised that differentiation was appropriate. A few slips were seen in handling the fractional coefficient of the h^2 term, with some candidates multiplying by 4 prior to differentiating. Many candidates showed good skills in this question, equating their derivative to zero and solving successfully. Most candidates found the correct values, though some selected 8 rather than $\frac{8}{3}$ as giving the maximum volume. Many candidates related their values to the question and stated that 8 was not possible. This was the simplest way to select the correct answer.

Answer: **(i)** $AD = \frac{8-h}{2}$ **(ii)** $\frac{8}{3}$

Question 12

(i)(ii) Candidates found both parts straightforward. Most substituted the appropriate values into the given cubic equation/expression and evaluated successfully. Very few errors were seen. Those who chose to use longer, division methods were generally also successful, though some omitted to indicate their answer in part **(ii)** and some sign errors were made using these methods.

(iii) Only a few candidates observed that the simplest method of finding *p* and *q* was to identify, by inspection, the linear factor as (x – 1) and multiply out. Those who solved the problem using this method were almost always correct and arrived at the answer very quickly. Some candidates resorted to long division to find the linear factor and this resulted in quite some work. The resulting complexity of the method often led to sign errors being made in the working or in candidates not completing the division and not knowing what to do with what they had found. The most popular method of solution was to use two of the roots of the cubic equation and form a pair of simultaneous equations to solve. This needed great care and attention to signs and powers and this method often resulted in arithmetic errors. Weaker candidates correctly substituted –2 and equated the expression found to 0 then incorrectly substituted 3 and equated the expression found to 600, copying the processes in part **(i)** and part **(ii)**. These candidates did not fully consider that the remainder when the quartic expression is divided by $x - 3$ is not the same as when the cubic is divided by $x - 3$ since there is an extra factor to take into account.

Answer: **(ii)** 600 **(iii)** $p = 11$, $q = 5$

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key Messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. On occasion, drawing or marking information on a diagram is helpful, and candidates should be encouraged to do this. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Omitting method steps through using a calculator often results in full credit not being given for a solution. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions.

General Comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, if a question requests the answer in a particular form, they give the answers in that form. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question.

Where an answer was given and a proof was required, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

When asked for a sketch, many candidates plotted and joined coordinates, rather than making a sketch showing the key features. Candidates would improve if they realised that this often resulted in diagrams that were of the wrong shape or incomplete.

Comments on Specific Questions

Question 1

(a) Candidates used a variety of methods to answer this part of the question. The most successful approach was to use the change of base rule. This immediately gave a term in base 3, as required. Alternative methods in other bases, for example $\log_{3^3} x$ were only given credit when converted into the required base. Candidates would improve by avoiding using these, less direct, methods to answer the question. Methods which involve an increased number of steps increase the opportunity for errors to be made. This was also true of those solutions which began by making use of the reciprocal. On occasion, the base of the logarithm looked like a multiple.

(b) There were some elegant solutions where the required result was obtained in three or four lines. Some candidates who knew some of the rules of logarithms were unable to apply them to the question. These candidates would do better if they attempted to apply their laws one at a time, rather than attempting to apply many together. Many would also have made more progress if they had recalled that 1 could be written as $\log_a a$. Some candidates anti-logged the given equation and then gave that as their final answer. These candidates would have done better if they had appreciated that they should continue to simplify their answers where it is clear that further simplification can be done, such as in this case.

Answer: **(a)** $\frac{\log_3 x}{3}$ **(b)** $y = 125a$

Question 2

(a) Many candidates gave only one answer and this was usually for the part of the function with a negative gradient. These candidates may have done better if they had extended the lines in the diagram and considered them separately. While there were some correct second equations given, it was common for candidates to assume that the line with positive gradient was perpendicular to the line they had already found, resulting in lines with gradients of ±0.5. Candidates also sometimes incorrectly gave their answer in the form of modulus functions. This resulted in the loss of a solution as they had, in effect, made their answers the same.

(b) Candidates should know the shape of the graph of an exponential function. The curve demanded was a simple translation of $y = e^{-x}$. Candidates would have done better if they had thought of it in this way. Many candidates attempted to plot points rather than sketch a curve. This takes time and should not be necessary for a question of this type. Poor choice of 'scale' at times led to curves approaching the *x*-axis. Some candidates concentrated their efforts on the part of the curve in the first quadrant with no indication of what was in the second. Candidates drew a variety of graphs, with straight lines, perhaps the most popular. Many candidates identified the intercept with the *y*-axis as the point (0, 4). Candidates should be aware that marks are awarded for the correct shape and positioning of graphs and that coordinates of intercepts alone, do not gain credit unless the graph is attempted also.

Answer: **(a)** $f(x) = 2x - 4$ and $f(x) = -2x + 4$

Question 3

(a) This question was well done by the majority of candidates. Occasionally, one or two elements were miscalculated. A few candidates attempted to find a 2 x 2 matrix. A small number of candidates would have improved if they had taken more care with the order of operations required or had read the question more carefully.

(b) Candidates were asked to state what was represented by certain matrix products, rather than to evaluate them. This was challenging for many. Many candidates gave clear and concise statements that were easy to credit. Other candidates misinterpreted what was required, sometimes thinking that identifying the order of the resultant matrix was sufficient. While much leniency was given in the use of terms to indicate value, candidates needed to be clear regarding whether the matrix they were describing represented the total value of the stock in **each** shop (as in part **(b)(i)**) or the total value of the stock in **all** the shops (as in part **(b)(ii)**). Simple statements are generally the best solutions to such questions, as answers which offered more description often became ambiguous or contradictory. Some candidates found evaluating the matrix product useful. Others became confused and related the values to the 4 different types of television rather than shops. It was not uncommon for candidates to reverse the answers to part **(b)(i)** and part **(b)(ii)**.

Answer: **(a)** $\begin{pmatrix} 8 & -2 & 6 \\ 4 & 3 & 10 \end{pmatrix}$

Question 4

(i) This was generally well attempted. Many candidates recalled that the angle between radius and tangent was a right-angle and used the tangent ratio. Others found other relevant angles correctly and used the sine rule. Some candidates gave lengthy solutions involving calculating numerous interim lengths. These methods were not always successful as the extra steps needed introduced a greater risk of making an error. A few candidates selected incorrect trigonometric ratios.

(ii) Most candidates made a good start to this question. They realised that the area of a sector was relevant and applied the formula correctly. There was no uniform method favoured for calculating the required area. "Kite minus sector" was most common, with the area of the kite being found by various methods. Some candidates calculated the lengths of the diagonals of the kite, most found the areas of two triangles and summed. Calculating twice the area of triangle *OPT* was most common, as this made use of the value already found. Candidates who found the area of the segment were often unsure what to do with it. Other candidates realised it was possible to use this successfully by subtracting it from the area of triangle *PQT*. When candidates used longer, multi-step methods, it was sometimes difficult to follow their intended logic. Calculations appeared randomly and sometimes, due to the resulting lack of space, alongside the diagram. This resulted in marks being difficult to award as the candidate's intention was not clear. Also, with the extra calculations involved, there was often a loss in accuracy in the final answer.

(iii) The vast majority of candidates found the length of the arc *PQ* correctly. Most of these candidates were able to apply the correct method of adding this to twice the length of *PT*. Full credit depended on having found *PT* correctly in part **(i)**.

Answer: **(i)** 19.3 **(ii)** 79.1 **(iii)** 57.5

Question 5

(a) A majority of candidates realised the correct response was a permutation of the digits. A good number of these gave clear and precise answers about the order of the digits being important and earned the mark. Some candidates needed to explain their reasoning more clearly. It was not sufficient to comment on arrangements without mentioning the need for a specific order. Stating that this was the only number that would work, did not add to the information given in the question. Candidates need to be aware that, when explaining something, using some phrases from the question may be helpful, but alone, these phrases will not be enough to gain credit. A few candidates chose 'combination' but then gave an explanation that matched 'permutation', so had mixed up the terminology. Others thought that it was a 'combination' as 'order did not matter'.

(b) A few candidates understood this question well and scored full marks. There were a good number of answers given that gain some credit. Some candidates needed to improve their understanding of when they should be multiplying – when the process involves "and" – and when they should be adding – when the process involves "or". It is a common misconception by candidates that "and" always means they should "add". Permutations were commonly used in this part of the question. Candidates who used them needed to read the question more carefully to gain a better understanding of what was required.

(i) Many candidates understood that they were selecting 4 from 6, 4 from 5, 4 from 7. The interpretation of "4 from 6 or 4 from 5 or 4 from 7" was often not made correctly. Candidates demonstrated this by multiplying their combinations, rather than adding them.

(ii) Again, many candidates understood which selections needed to be made. In this part the interpretation needed to be of " 1 from 2 and 1 from 6 and 1 from 5 and 1 from 7". Candidates often misinterpreted the "and" as "add" rather than correctly multiplying their combinations.

(iii) Some candidates listed the possible outcomes – this was good practice. Some of these included impossible outcomes for the context such as 0, 0, 0, 3. A few candidates gave one product only – listing outcomes may have avoided this. Reversing addition/multiplication was again seen in solutions offered for this part of the question.

Answer: **(b)(i)** 55 **(ii)** 420 **(iii)** 70

Question 6

This question was generally well answered, with many candidates obtaining most of the marks available. There was very little confusion over when to integrate and when to differentiate.

(i) The majority of candidates made a good start to the question, setting $v = 0$ and solving the quadratic correctly. The method of solution was mostly omitted, with the correct values simply being stated. Candidates should be encouraged to show their method and check their solutions using their calculator, rather than relying on their answers being accurate enough to earn credit. Some candidates need to read the question more carefully. The question asked for the time at which the particle first came to rest and some did not choose the first occurrence (at $t = 1$). Others seemed to assume that time began at 1, rather than 0, and therefore stated that $t = 6$ was the first occurrence. A small number of candidates misinterpreted the question and either substituted $t = 0$ or used calculus.

(ii) Again, this question was well done by the majority of candidates. Integration was usually attempted and accurately carried out. Sometimes, candidates omitted to evaluate the constant of integration which, given the initial conditions, was zero. Weaker candidates need to take care with their variables as, occasionally, some terms were correctly given in t and others incorrectly in x.

(iii) All candidates answered this part very well. Very occasionally, candidates changed the negative answer to a positive one without reference to deceleration.

Answer: **(i)** 1 **(ii)** $\frac{2t^3}{3} - \frac{14t^2}{2} + 12t$ **(iii)** -2

Question 7

(a) Many candidates were able to make a positive start, usually by finding $\overrightarrow{AC}$. A large number of these candidates progressed further by finding two relevant vectors and showing that they were parallel. Very few candidates made the key statement that their vectors had a common point. The question required a given result to be shown and candidates who omitted to state this did not fully complete the argument. A common misunderstanding was to assume that showing $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ would be sufficient. It was also common for candidates to incorrectly rewrite the given vectors as column vectors in **a** and **b**. This question proved to be a good discriminator.

(b) (i) The best approach was to find and state $\overrightarrow{PQ}$ before finding the modulus. Many candidates attempted to do this. Some candidates added the position vectors given – a simple diagram may have helped these candidates correct this error. Candidates who omitted to state their $\overrightarrow{PQ}$ before calculating the modulus were penalised if their values were incorrect as method steps that were not explicitly stated could not be credited.

(ii) Many candidates showed that they fully understood what was required to answer the question. Some candidates demonstrated that they had not understood the concept of a 'unit vector' by restating, or even finding again, the vector found in part **(i)**. Conversely some candidates did not find the modulus in part **(i)**, but then found it as part of finding the unit vector correctly in part **(ii)**. These candidates needed a better understanding of the notation, the general concepts or the connections between them.

(iii) The required correct method was shown by a good number of candidates. In this part of the question in particular, candidates would have perhaps benefitted from drawing a simple diagram. Some candidates did this although they were very rarely seen. Candidates often halved their answer to part **(i)** to find the midpoint. If they had incorrectly added and found 4**i**+3**j** in part **(i)** this resulted in a seemingly correct answer from a wrong method.

Answer: **(b)(i)** $\sqrt{125}$ **(ii)** $\frac{1}{5\sqrt{5}}$(2**i** + 11**j**) **(iii)** 2**i** + 1.5**j**

Question 8

In both part **(a)** and part **(b)**, candidates were usually able to give a correct form for the integral if not always the correct coefficient. As a result most candidates gained marks on all parts. Candidates were expected to show the substitution of limits before evaluating their final answer. Some candidates simply used their calculator to evaluate the integral, without carrying out the required integration. These candidates did not take note of the hence in part **(a)(ii)** and part **(b)(ii)**. Candidates who did not show the method of using limits were penalised. This was most notable when a correct evaluation was stated following incorrect integration.

In part **(c)** candidates who thought about the manipulation needed were usually successful. The brackets needed to be expanded before integrating. Those who realised this were usually accurate in their expansion and in the subsequent integration. Some candidates omitted the cross terms when squaring and then integrated two terms only. Many candidates would have done better if they had appreciated that the integration of such complex functions is not required in this syllabus. This should have directed them towards the simplification to three terms that was required. Commonly, candidates attempted an invalid "chain rule for integration method". As the integral was indefinite, it was appropriate to include the constant of integration.

Answer: **(a)(i)** $\frac{1}{4}e^{4x+3}+c$ **(ii)** 707 000 **(b)(i)** $3\sin\frac{x}{3}+c$ **(ii)** 0.521 **(c)** $\frac{x^{-1}}{-1}+2x+\frac{x^3}{3}+c$

Question 9

(a) This was a very well done question. Most candidates correctly identified the critical values and sign errors were rarely made. Some candidates did not state a range of values as their answer. Those who did were frequently correct. Some candidates would have found a sketch helpful, although they were rarely drawn. Some candidates gave their answer as two separate inequalities. While some of these candidates correctly connected their inequalities using "and", many were unable to gain full credit as they omitted the word "and" or used "or" or a comma, for example. Most candidates used the correct inequality signs and, incorrect, strict inequalities were rarely seen.

(b) (i) This part was also generally well done with many correct solutions. Some candidates omitted to subtract 16 when finding *b*.

(ii) Candidates were expected to use their previous answer to find the greatest value. Credit was given to those who started again or differentiated. There were some good, neat answers, using part **(b)(i)**, as directed in the question. Weaker candidates often carried out algebraic manipulation which had little to do with what was required. Many candidates clearly identified which of their two values was which. Other candidates would do well to realise that this was also good practice, as some solutions were quite vague. Stating the coordinates of the maximum point only did not score as the candidate had not interpreted the information given by each coordinate.

(iii) Candidates drew some very good sketches in this part. Many identified the three points of contact with the axes correctly. Some candidates omitted to indicate where their curve met the *y*-axis in their, otherwise, very good sketches. Weaker candidates would improve if they paid more attention to the overall shape of the curve generated by a quadratic function. There was a tendency to assume that the maximum must lie on the *y*-axis leading to some distortion of the quadratic shape.

Answer: **(a)** $-5 \leqslant x \leqslant \frac{1}{4}$ **(b)(i)** $(x+4)^2-25$ **(ii)** greatest value 25 at $x = -4$

Question 10

(i) This part was generally well done. Some candidates linked their equations explicitly to $y = mx + c$ and this is good practice in questions of this type. The most common error was to use logs to base 10 rather than logs to base e. Candidates needed to read the question more carefully to avoid this error as this was clearly stated in both part **(i)** and part **(ii)**. Most candidates were able to apply at least one of the laws of logarithms.

(ii) While a good many candidates were able to find the intercept and gradient, very few managed to gain full credit. The reasons for this were numerous. Candidates need to be aware that, when finding the gradient and intercept of a line of best fit, they must use points on the line, not the original data points. Using the data points rather than the given line was the most common error. Others equated *A* and *b* to the intercept and gradient or used logs to base 10. In finding the gradient, the differences were sometimes inverted or inconsistent leading to a positive gradient. Candidates needed to read the question carefully as some did not give their, otherwise correct, answer to the required accuracy.

(iii) The best and simplest approach was to read the value for $\ln y$ directly from the line in the given diagram. This gave an accurate solution if read and anti-logged correctly. Some of these candidates gave their answer as 9 or $\ln 9$. Many candidates chose to use their values from part **(ii)** and substituted them into one of the forms of the equation. While this was a possible method it usually lacked accuracy especially as candidates tended to use an already rounded value. Candidates would do well to realise that, using values given in the question where possible, rather than those they have calculated, should produce more accurate answers.

Answer: **(i)** $\ln y = \ln A + x\ln b$ **(ii)** $A = 90\,000$, $b = 0.4$ **(iii)** $y = e^9$ or 8000 to 1 sf

Question 11

Candidates found this question challenging. If they had identified the regions of the Venn diagram correctly and had started to complete it from “the centre outwards”, they would have achieved greater success. The few candidates who followed a logical strategy were successful. Often, it was not possible to identify a candidate’s method. Many candidates would improve by ordering their method steps and thinking the problem through using the diagram to help them. Often, candidates omitted to use x and y in their method. Occasionally, correct answers appeared with no working at all. Candidates were likely to be successful in the first two parts of the question if part **(i)** was answered correctly with x, $7 - x$, and $6 - x$ being correctly marked on the diagram. Candidates marking the 7 and 6 on the diagram incorrectly usually obtained no marks. Candidates did not always identify the correct area – so they may have had 4 and 9 on the diagram, but then gave x and y as different values. This was more common in part **(ii)** than in part **(i)**. The last part of the question was not always ‘shown’ as thoroughly as it needed to be. Some candidates did not use numbers in their answer or omitted to state “= 0”. This part was rarely correct if part **(ii)** was incorrect. The exception to this was where the candidate had the correct numbers on the diagram, but had identified y incorrectly as a value other than 9.

Answer: **(i)** 4 **(ii)** 9