ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates are reminded of the importance of reading questions carefully and ensuring that they are giving their answer in the required form and to the appropriate level of accuracy. An exact answer will not require the use of a calculator and giving an additional decimal answer alongside an exact answer is not necessary. It is essential that clear and sufficient working is shown to support the answer a candidate gives for a question. Candidates must also understand that if a question starts with the instruction 'Hence', it is an indication that work from the previous question part must be used.

General comments

It was clear that many candidates had prepared for the examination as evidenced by the comprehensive solutions that were produced, showing a good coverage of the syllabus. It was also pleasing to see that candidates made use of additional/supplementary sheets when necessary, making solutions easier to follow. It is essential that these sheets are labelled with the appropriate question number.

Comments on specific questions

Question 1

Most candidates recognised the given equation as a quadratic equation in terms of $\ln 5x$. Solutions using factorisation were most common with the majority of candidates obtaining the solutions $\ln 5x = \frac{1}{3}$ and $\ln 5x = -1$. It was essential that candidates recognised the importance of the demand for exact solutions. Some candidates were unable to obtain the final two accuracy marks as they went straight from the equations $\ln 5x = \frac{1}{3}$ and $\ln 5x = -1$ to decimal solutions for x. Many candidates gave their final solutions in both exact form and decimal form, rounded to 3 significant figures. When an exact solution is required, it is not necessary to give a decimal equivalent as well.

Question 2

Most candidates recognised the relationships between the amplitude, period and vertical translation of the given graph to the positive constants in the trigonometric equation for the given graph. Most errors concerned the relating the period of $a\sin bx + c$ to the value of b, with 2, 2π and $\frac{1}{2}\pi$ being common errors.

Question 3

(a) This part of the question was solved very well by the great majority of candidates, who showed clearly the processes of finding the mid-point of the line and the gradient of the perpendicular to the straight line AB, before finding the value of a. Any errors tended to be arithmetic slips rather than errors in method.

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(b) Fewer candidates were able to determine the coordinates of the point *D*. A simple diagram would have helped candidates visualise the situation so that a relevant application of displacements parallel to both axes could be made. Methods attempted using the equation of the perpendicular bisector were very often lengthy and unsuccessful. In this particular case, candidates should have been guided by the mark allocation and the amount of space given for the answer.

Question 4

(a) Correct 'completions of the square' were obtained by many candidates. Errors usually involved the calculation of the value of the constant term *c*.

(b) It was important that candidates realised that they were to use their answer from **part (a)** to write down the coordinates of the stationary point on the curve. This was indicated by the use of the word 'Hence' and implied by the instruction to 'write down'. Solutions that were obtained by calculus were not given credit. The intention of this question was to check the knowledge of the properties of quadratic functions.

(c) Many correct sketches were seen. In some cases, the mark for the shape of the curve was not awarded as some candidates assumed a maximum point on the *y*-axis, in spite of the work needed for the previous part of the question. In other cases, the parts of the graph for $x < -3$ and $x > \frac{1}{2}$ were incorrect. It was also important that the graph had cusps on the x-axis rather than minima. Some candidates omitted the intercepts on the axes.

(d) Many candidates were able to recognise the connection with the value of *k* and the *y*-coordinate of the maximum point.

Question 5

(a) It was evident that some candidates calculated the displacement of the boat over one hour and then found the displacement over *t* hours and gave this in terms of a position vector.

(b) Most candidates were able to write down the position vector of the boat *B*. Simplification to a single column vector was not essential.

(c) A subtraction of the relevant vectors was attempted by most candidates. The correct subtraction $\overrightarrow{OQ} - \overrightarrow{OP}$ was attempted by the majority of candidates and was followed by a correct attempt to find $\left|\overrightarrow{PQ}\right|^2$.

(d) The key words in this part of the question are 'Hence' and 'show'. Candidates were expected to use the result from **part (c)**, so attempts to compare times were not accepted. Most candidates made use of the discriminant of the quadratic from **part (c)**, but omitted to mention that they were considering $\left|\overrightarrow{PQ}\right|^2 = 0$, for which there are no real roots. The statement $\left|\overrightarrow{PQ}\right|^2$ has no real roots, or similar, does not make sense as it needs to be equated to zero, which is part of the process of showing no collision.

Question 6

(a) This topic is a relatively new addition to the syllabus, but candidates performed well in it and there were very few incorrect solutions seen in either **part (i)** or **part (ii)**. Errors tended to be due to incorrect calculation from a correct statement of the sum to 7 terms.

(b) (i) Very few incorrect common differences were seen.

(ii) Most candidates were able to write down the correct sum to *n* terms, but were unable to simplify it either sufficiently or correctly to obtain the accuracy mark available. It was essential that a single factor of $\log_x 3$ was seen in the final answer and also that any powers of 3 were also simplified as far as possible. There were several acceptable answers.

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(iii) Most candidates were able to find the value of n even if they had not simplified their sum to n terms sufficiently. It was essential that a quadratic equation in n be obtained correctly in order to obtain the method mark.

(iv) Candidates were able to use either their answer to **part (iii)** or equate 1027 and $3081\log_x 3$ as both were associated with **part (iii)**. More success was had by candidates who chose to equate 1027 and $3081\log_x 3$.

Question 7

It was evident that some candidates made use of a calculator in this question even though they were instructed not to. Some candidates did not provide enough evidence to convince examiners that they were not using a calculator. In general, candidates would be expected to give at least three terms in an expansion of two binomial factors containing surds, unless these factors are the difference of two squares.

(a) Most solutions seen were correct. Errors were mostly due to omission of working and occasionally incorrect addition of the lengths of the sides of the trapezium.

(b) Again, many correct solutions were seen with errors mostly due to omission of working.

(c) A correct trigonometric ratio was used by most candidates, with correct rationalisation following to obtain the answer in the given form. A simplification from $\frac{18+2\sqrt{17}}{16}$ was needed to gain full marks.

(d) The word 'Hence' implied that the result for $\tan AED$ from **part (c)** needed to be used. Candidates were expected to use the trigonometric identity $\sec^2 AED = \tan^2 AED + 1$ with their expression for $\tan AED$. Many correct solutions were seen, with candidates being able to check their working in order to obtain the given answer. Again, errors were often due to omission of working. Candidates who chose to calculate $\cos AED$ and then square and find the inverse of this result were not given credit as this part of the question was intended to test the knowledge of the trigonometric identities.

Question 8

(a) (i) Most candidates were able to show the required result clearly and with sufficient detail.

(ii) Many candidates used the result **part (i)** to attempt to solve the equation $\cos\frac{\theta}{2} = \frac{1}{4}$. Most dealt with the half angle correctly but were unable to obtain the accuracy marks usually due to either or both of the following reasons. Answers in radians must be given correct to 3 significant figures, but many candidates gave their final answers correct to 1 decimal place. When working with decimals, candidates should be working with more than 3 significant figures, before giving their final answer correct to 3 significant figures, but many candidates were not doing this and due to premature approximation, were obtaining answers such as 2.63 radians or 9.94 radians.

(b) Most candidates were able to obtain at least one correct solution. Some candidates chose, incorrectly, to consider solutions for $\tan\left(y + 38^\circ\right) = -\frac{1}{\sqrt{3}}$ as well. To obtain the final accuracy mark, candidates had to obtain a second correct solution with no extra solutions in the given range.

Question 9

(a) There were very few incorrect solutions seen to this part. It was expected that candidates give their final answer as the product of two factors and not just state the quadratic factor. Candidates who chose to use synthetic division often did not take into account the additional factor of 2 obtained when using this method. It was expected that either algebraic long division or observation be used in this part.

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(b) Candidates were expected to equate the equations of the two curves, rearrange the resulting cubic equation and realise that this cubic equation could be solved by using the factors obtained in **part (a)**. It was evident that many candidates used their calculator to solve this cubic equation. To obtain full marks in this part it was essential for candidates to show that they were trying to solve the quadratic factor obtained in **part (a)** equated to zero together with the linear factor equated to zero. Some candidates did not give their *x*-coordinate for the point *B* in exact form as was required. Other errors included incorrect identification of the *x*-coordinate for *A* and for *B*. However, most candidates were able to gain marks in this part.

(c) Most candidates were able to gain credit for the integration of $-2x^2+3x+1$ and for the integration of $\frac{1}{x}$. However many candidates were unable to correctly identify the areas to be added which resulted in incorrect limits often being applied. Again, the word 'exact' was used in the question, but too many candidates resorted to the use of their calculator and decimals. For those candidates that did obtain a correct exact area, it was not necessary to add the decimal equivalent.

Question 10

(a) Many correct unsimplified derivatives were seen. Problems arose when candidates attempted to simplify their derivative to the required form, displaying a lack of higher algebraic skills.

(b) For candidates with a correct quadratic factor from **part (a)**, solution of this quadratic factor equated to zero with the solution $x=-1$ being discounted was essential. It was also essential that the equation $\left(2x^2+10\right)^{\frac{1}{2}}=0$ also be considered and discounted as having no real solutions. This meant that there was only the solution of $x=\frac{5}{2}$. For candidates with an incorrect quadratic factor, an attempt to equate to zero and solve was given credit. All candidates were able to obtain a mark for correct consideration of $\left(2x^2+10\right)^{\frac{1}{2}}=0$. Unfortunately, too many candidates did not consider $\left(2x^2+10\right)^{\frac{1}{2}}=0$ at all.

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ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to succeed in this examination, candidates need to demonstrate that they are able to solve problems by applying techniques. Candidates need to show full method so that marks can be awarded. Attention should be given to the instructions on the front page of the examination paper. Candidates who do not show full method because they have used their calculator to perform key operations, such as finding the value of the integral of a function for a particular set of limits, without evidence of integration having been performed, will not be credited. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is asked for, candidates should ensure that they follow the instructions on the front page. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with angles.

General comments

The majority of candidates seemed to be well prepared for this examination. The recall and application of skills and techniques in order to solve problems, when necessary, was very good. Some questions required the application of several techniques to answer them correctly. Very many candidates were able to do this successfully.

Many candidates offered complete solutions and showed enough accurate method to be awarded full credit. Occasionally, candidates rounded working values to three significant figures. This resulted in a premature approximation error and a loss of final accuracy. In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. In this examination, this was evident in **Question 6**, **Question 11(b)(ii)** and **Question 12**.

The instructions on the front page of the examination paper indicate that non-exact numerical answers should be given correct to 3 significant figures. This instruction does not apply to exact values. Values that are integers are not inexact and should not be rounded in this way. This was seen on occasion in **Question 8** in this paper. Similarly, if an answer is required in exact form, it is very unlikely that this will be a decimal and it will certainly not be a rounded value. This was commonly seen in **Question 4** in this examination.

Algebraic expressions should be correctly formed and notation used should be correct. For example, the domain of a function with argument *x* must be stated in terms of *x*. The range of a function must be stated in terms of the function name. This was required in **Question 10(a)(i)** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. This was good as their work remained well presented and could be marked without difficulty. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful to examiners.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

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Comments on specific questions

Question 1

The majority of candidates chose to square the linear expressions and equate them to form a quadratic equation. Most candidates who did this were accurate and earned full marks for a correct solution. Occasionally, candidates made sign errors when collecting terms. Those candidates who formed and solved a pair of linear equations were equally successful although, again, sign errors were sometimes made. Very few candidates had extra solutions. Those who did were generally those who formed four linear equations and made a slip when solving one or more of them. On occasion, candidates rejected the negative solution, indicating that it was invalid.

Question 2

Many excellent and fully correct solutions were seen to this question. Some candidates made slips in their method, usually when interpreting the expression for b from the equation. These candidates did not always show the method they used to solve their quadratic equation in k. Candidates should understand that when their equation is incorrect, the method of solving it needs to be seen for further marks to be awarded. Most understood the requirement to work with $b^2 - 4ac$ and most equated this to 0, as required for equal roots. Some candidates incorrectly used $b^2 - 4ac \geqslant 0$, misinterpreting 'equal' roots as 'real' roots.

Question 3

(a) A good proportion of candidates were able to find the correct values of a, b and c from the graph. Other candidates were able to state b and c correctly, but gave a as -2 or gave the negative of each value. This was sometimes a misinterpretation but sometimes a result of algebraic errors.

(b) A reasonable number of candidates found the correct three critical values using algebra and were able to form a correct pair of inequalities from them, using the graph to guide them. Candidates who did not find the correct values might have improved if they had looked at the diagram and read the critical values from that, which was acceptable. A few candidates who had found the correct critical values either only formed one correct inequality or solved $f(x) \geqslant -1$. Some candidates made slips in writing the critical values as part of their inequalities. These candidates may have improved if they had written the correct critical values down before they tried to combine them into inequalities.

Question 4

This question was very well answered and a very good number of candidates earned 5 or 6 marks. Most candidates made one of the simple substitutions expected and were able to simplify correctly to find the values of x or y. A few candidates omitted the negative square root and generally had one point of intersection only. A few other candidates formed quartic equations and determined that (0, 0) was a point of intersection. These candidates sometimes discarded this point as being extraneous, but not always.

A small number of candidates used complex methods to try to eliminate one of the variables, rearranging the equation of the curve to make x or y the subject. This was unnecessary and introduced a greater chance of making an error. The question demanded an exact distance to be found. It is a necessary skill to understand the difference between an exact and a rounded value. Candidates who gave their final answer as a rounded decimal were not able to gain full credit as it needed to be clear that candidates understood what was required.

Question 5

(a) Again, a good number of fully correct solutions, with correct notation, were seen. Some candidates used, for example, δx rather than $\frac{dx}{dt}$ and so on. This was condoned for full marks if the correct calculation for $\frac{dx}{dt}$ was seen. A few candidates were unable to find $\frac{dV}{dx}$ and $x = 8$ or $\frac{dx}{dV}$ correctly. Some candidates were credited only for the statement of a correct and relevant chain rule. Weaker candidates tended to solve $3x^2 = 480$ or $3x^2 = 512$ and offer the positive solution as their answer or to simply divided 512 by 480.

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(b) It was necessary for candidates to interpret S as $6x^2$ and differentiate. Many candidates did this and were able to use their value of x and $\frac{dx}{dt}$ in a correct solution. A small number of candidates used $S = 6\left(V^{\frac{1}{3}}\right)^2$ and then $\frac{dS}{dt} = \frac{dS}{dV} \times \frac{dV}{dt}$ successfully. A few candidates incorrectly used $S = x^2$. A few other candidates found $12 \times 8 = 96$ and offered no further working or found the surface area to be 384 when $x = 8$ and tried to combine this with 480 in some way.

Question 6

(a) A high proportion of candidates formed a correct sum of three values for the perimeter. A few candidates rounded their working values prematurely and the final answer offered was often slightly inaccurate. Some candidates assumed that triangle *AOC* was right-angled. These candidates incorrectly used basic trigonometry or Pythagoras' theorem to find the length of *AC*. These candidates may have improved if they had not made assumptions that were not justified. A few of the candidates who did use the cosine rule to find *AC* made an order of operations error and calculated $\sqrt{72.25\cos\frac{2\pi}{7}}$. A small number of candidates omitted to include the length of *BC* in their perimeter calculation or had their calculator in degree mode rather than radian mode. Other candidates used more complex methods to find *AC*, finding the length of the perpendicular from *C* to *OA* and using that. This often resulted in unnecessary errors in both parts of the question, as candidates confused themselves over which lengths they needed to work with.

(b) Many good and accurate solutions were seen. Most candidates offered a correct calculation for the area of the sector and tried a correct strategy. Fewer candidates made premature approximation errors in this part of the question. Those candidates who thought that triangle *AOC* was right-angled in **part (a)** usually did so again in this part, although some did recover to form a correct expression using $\frac{1}{2}bc\sin A$.

Question 7

(a) This part of the question was generally answered very well and very many candidates earned all the marks available. Most candidates differentiated and showed full and clear method. A few candidates were unable to earn all the marks as they had used their calculator to find $\left.\frac{d}{dx}(p(x))\right|_{x=3}$ without showing any differentiation. A few candidates found the coordinates of a second point on the curve and used that to find the gradient of a chord. This was not accepted.

(b) (i) This question required the combination of several skills. It was expected that candidates would appreciate that the gradient of the tangent to the inverse function would be the reciprocal of the gradient of the tangent to the original curve. Some candidates made simple sketches to help them visualise this. Others were able to think the problem through and write the answer down. A reasonable number of correct answers were seen. The most common incorrect answers offered were $-\frac{1}{9}$, -9 or 9.

(ii) The most efficient way to answer this part of the question was to understand that the tangents would intersect where $y = x$ and make a simple substitution into their answer to **part (a)**. This method was rarely seen. Most candidates found the equation of the second tangent and solved this simultaneously with their answer to **part (a)**. Whilst this was a valid method it introduced a greater likelihood of making an error. A fair number of candidates earned both marks here and a reasonable number earned a mark for finding a value of x using a valid method.

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Question 8

(a) (i) This part of the question was very well answered. A value was expected and 12! was not acceptable as an answer. Candidates who rounded to 3 significant figures without having first written down the correct unrounded answer were also penalised.

(ii) This part was also commonly correctly answered. Once again, a value was expected and, as it was an integer, the answer should not have been rounded to 3 significant figures. Occasionally, incorrect answers came from $3 \times {}^{12}P_{10} \times 4$, for example. Candidates should also be aware that writing, for example, $3\ {}^{10}P_{10}\ 4$ without any indication of multiplication is not sufficient for a method mark to be awarded should the final answer be incorrect.

(iii) Candidates were a little less successful in this part, although a good number of correct answers were seen. Commonly incorrect answers were from $7! \times 5!$ or $7 \times 7! \times 5!$ or $7! \times 5! \times 2$ or ${}^{5}P_{5} \times {}^{12}P_{7}$.

(b) (i) Most candidates found the correct answer from the most efficient method, ${}^{9}C_{3}$. A few candidates attempted the more complex method, ${}^{4}C_{0} \times {}^{5}C_{3} + {}^{4}C_{1} \times {}^{5}C_{2} + {}^{4}C_{2} \times {}^{5}C_{1} + {}^{4}C_{3} \times {}^{5}C_{0}$. Whilst this was perfectly acceptable, it did introduce a greater likelihood of an error being made. On occasion, part of the sum was omitted.

(ii) Again, this part of the question was well answered. The most common incorrect answer offered was 12 from the summation of the combinations, rather than the product.

Question 9

(a) Most candidates were able to identify the correct terms. Some candidates would have improved if they had taken more care with bracketing and signs. A few slips in simplification were made and again, some candidates would have done better should they have taken a little more care with this.

(b) (i) This part of the question was very well answered with few incorrect answers seen.

(ii) Candidates found this part of the question more challenging, although a good number of fully correct solutions were seen. Some candidates made algebraic errors, resulting in an incorrect solution. A common incorrect answer from weaker candidates was $a = 2$ and $b = 613$. Presentation of solutions was variable and candidates whose work was disorganised were more likely to make slips in their solution.

Question 10

(a) (i) This was well answered by many candidates. Those who did not gain full credit were sometimes confused about the correct notation for domain and range with the domain indicated as the range and vice versa. A few candidates did not take note of the domain of the function f and occasionally the domain was given as $0 \leqslant x < 1$, as candidates considered what happened to f as x tended to infinity. Some candidates were unnecessarily finding the inverse function in this part. Candidates who prefer to use interval notation need to understand the necessity to write $x \in [0, 0.943]$ or $y \in [0.5, 1.5]$ as giving only [0, 0.943] or [0.5, 1.5] was not awarded full marks.

(ii) Again, this was often well answered. Some candidates made sign errors or made an order of operations error, for example, square-rooting $4x^2 - 1$ term-by-term. Some candidates were not awarded the final mark as their square root did not extend down to the denominator of the algebraic fraction.

(b) Most candidates formed the correct composite function and simplified correctly. A small number of candidates used f^{-1} rather than f in their composition. These candidates may have improved if they had read the question more carefully.

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Question 11

(a) (i) Some good solutions were seen. A few candidates omitted to divide $(10x - 1)^{-5}$ by -5 and/or 10. The most common incorrect solutions involved natural logarithms, for example $\frac{\ln(10x-1)^6}{10}$.

(ii) Again, some good solutions were seen. A few candidates were unable to square the numerator successfully. Many weaker candidates offered solutions based on a product of integrals. Some candidates tried unsuccessfully to use integration by parts. Many candidates were penalised for omitting the constant of integration which was required for full credit in this question.

(b) (i) This was usually very well answered, although a few candidates used the quotient rule instead of using the standard result which they should have known. This was acceptable but introduced a greater likelihood of making an error.

(ii) Many good and fully correct solutions were seen. A few candidates made a premature approximation error and were not awarded the final accuracy mark. Weaker candidates were usually able to integrate the $\sin x$ term correctly. Most candidates realised how to use the answer to the previous part, although it was not uncommon for the multiple of $\tan(3x + 1)$ to be $\frac{1}{2}$ or $\frac{3}{2}$. The substitution of the limits was not always correctly carried out. Sign errors were common and, occasionally, substitution of $\frac{\pi}{10}$ and $\frac{\pi}{12}$, which should have been shown, was omitted. A correct answer that did not follow complete and correct work was not accepted.

Question 12

There were two approaches to solving this problem and candidates used them in about equal measure. Candidates who tried to find expressions for s and evaluate them at $t = 1$ and $t = 3$ often had an incorrect constant for the second expression. This was commonly stated as $-2e^{-\frac{t}{2}} + 2$ or $-2e^{-\frac{t}{2}}$ rather than $-2e^{-\frac{t}{2}} + \frac{3}{e}$. Many candidates were confused about exactly what they were integrating with respect to. It was common for weaker candidates to offer $\int \frac{t}{2e}\,dt = t\ln 2e$ for example. Those candidates applying the more sophisticated technique of definite integration of the expressions for v between the limits 1, 2 and 2, 3 found the method easier to carry out and were usually more successful.

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