ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates must ensure that they are working to the correct level of accuracy when a solution is being worked through. Working with figures correct to 4 significant figures at least is essential to ensure a final answer correct to 3 significant figures is accurate. This is the accuracy required unless a different level of accuracy has been specified in the question, or an exact answer is requested.

Candidates should also ensure they have answered the demands of the question in full. Examples include stating intercepts with axes on graphs, taking into account that these may sometimes be required to be exact; a range of a function is stated if required after having found a particular function.

An awareness of the command words used in this syllabus and their meanings is essential. The words ‘Show that’ imply that each step of a solution needs to be written down so that an Examiner is convinced that the candidate understands the process they are working through. The word ‘Hence’ implies that work done in the previous part of the question needs to be used.

General comments

Many candidates had been well prepared for the examination, although it was evident that there were some candidates who had not. It appeared that some parts of the syllabus had not been covered to sufficient depth, if at all, for some candidates. It was also noticeable that the rules for algebraic manipulation by some candidates need to be more rigorously applied. Poor use of brackets and sign errors often resulting from poor use of brackets appeared all too often. However, there were some excellent scripts from well prepared candidates, showing a full understanding of the syllabus objectives and applying them appropriately and correctly.

Comments on specific questions

Question 1

Most candidates recognised that the graph was that of a sine curve which had been displaced by 4 parallel to the *y*-axis in the negative direction so $c = -4$. Fewer candidates were able to obtain the value of the constant *a*, by linking it to the amplitude of the curve. There were even fewer correct responses when the value of *b* was attempted, with many candidates giving an answer as an angle in radians. Few were able to make the correct link between the constant *b* and the period of the function.

Question 2

(a) There was a great variation between the responses to this part. Most candidates were able to attempt a sketch of the graph of the modulus of a quadratic function. Common errors included positioning the maximum point of the graph on the vertical axis at the point where $y = 10$, not considering symmetry at all. It is essential that there is an appreciation of the symmetry of the graphs of a quadratic function and its modulus. Some candidates omitted to state the intercepts with the coordinate axes, especially the *y*-axis, as required. It was also evident that there were some candidates who were not familiar with the concept of modulus functions.

(b) Very few correct responses were seen. It was intended that the graph in **part (a)** would help candidates with the determination of the value of the constant *k*. Most candidates considered the discriminant of the equation $3x^2 + 13x - 10 = 0$ and equated this value to k. Another common error was to state that $k = -5, \frac{2}{3}$, the intercepts with the x-axis.

There were different methods which would have been acceptable. Using calculus to find the coordinates of the stationary point on the curve $y = 3x^2 + 13x - 10$, or the axis of symmetry of the curve $y = 3x^2 + 13x - 10$, to find the coordinates of the stationary point. These methods were rarely seen and of those solutions that did gain marks, the most common method used was to consider the discriminant of the equation $3x^2 + 13x - 10 - k = 0$. This often resulted in sign errors and errors in the final inequality given as a solution. Again, referring back to the sketch in **part (a)** should have helped candidates with this. It should also be noted that the solution $k = 0$ was rarely seen.

Question 3

Most candidates were able to gain at least two marks, with many completely correct solutions being seen, showing that the majority of candidates are able to manipulate indices satisfactorily.

Question 4

Very few candidates obtained full marks as often a second solution to the given equation was not calculated. A number of candidates did not know how to progress with the solution of the given equation. It was expected that the equation be simplified to $\tan\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{3}$. Many candidates were able to obtain this equation, or similar, by letting, for example, $y = 2x + \frac{\pi}{4}$. A number of candidates were unable to evaluate $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ correctly. Credit was given if it was evident that the candidate was able to demonstrate a knowledge of the correct order of operations.

Question 5

(a) Many candidates have difficulty finding vectors when given a magnitude and direction. Many were able to obtain a mark for finding the magnitude of the direction vector, but far fewer were able to apply the magnitude correctly to obtain a final vector.

(b) There was a much better response to this part of the question, with candidates showing a reasonably good understanding of vector geometry. However, many candidates misinterpreted the ratio $AC : AB = 1 : 3$, ending with a final answer of $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ rather than the expected $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.
Credit was given for the understanding of the basic principles involved in the manipulation of vectors, but greater care is needed when interpreting any given ratios.

(c) Most candidates realised that they could obtain two simultaneous equations by comparing like vectors. However, many candidates were unable to obtain correct solutions to this question. Errors included incorrect simplification of brackets, incorrect signs and poor basic arithmetic.

Question 6

Many candidates were able to gain marks in this question by recognising that the condition that the 3 brothers could not be separated meant that only 12 people needed to be considered in the case where the brothers were included (3 positions left to be filled by any of 12 people) and the case where the brothers were not included (6 positions to be filled by 12 people). Candidates who did not gain any credit in the question were usually still considering 15 people or permutations were being used rather than combinations.

Question 7

(a) Most candidates were able to obtain a correct answer of 2.8, showing an understanding of the link between the arc length and the angle subtending it at the centre of the circle.

(b) Fewer correct solutions were seen for this part of the question. The length of *AC* or *BC* needed to be calculated. There were some misunderstandings about where the right angles were in either triangle *AOC* or BOC, which led to errors. Other errors occurred due to incorrect rounding. The length of *AC* or *BC* is 57.98 correct to 4 significant figures. Too many candidates truncated this length to 57.9 which resulted in an inaccurate value for the perimeter. A similar problem arose if the sine rule was used in the triangle *AOC* or *BOC*, with the angle *ACO* or *BCO* being calculated as 0.171 rather than 0.1708. It is essential that care is taken with the accuracy of figures used during solutions.

(c) Most candidates realised that they needed to calculate the area of the minor sector *AOB*, with most obtaining the correct area of 140 (cm^2). Some errors arose when attempting to calculate the area of the kite *AOBC*. Most candidates attempted to find the area of either triangle *AOC* or *BOC* and double it. Again, problems arose due to misunderstandings about where the right angles were in either triangle *AOC* or *BOC* which led to incorrect areas.

Question 8

(a) Many candidates did not appreciate the fact that they needed to consider the term $\ln(2x+3)$ only and the value for which this term would first become undefined. Correct answers of $a = -1.5$ were in the minority.

(b) Similarly for this part of the question, correct answers were in the minority, with candidates being unfamiliar with the properties of logarithmic graphs.

(c) Many candidates were unable to make a correct attempt at finding the inverse function. Many did not deal with the logarithmic terms correctly and subsequent algebraic manipulation was poor in most cases. Of those candidates that did also remember that the range of the inverse function needed to be stated, some did link this request to the value obtained in **part (a)** but all too often used incorrect notation e.g. $x > -1.5$.

(d) Very few correct sketches were seen. It was evident that few candidates were familiar with graphs of logarithmic functions and those of exponential functions. For those that did attempt curves, all too often the curves curved in the wrong direction or were not asymptotic. It was requested that the exact intercepts of each graph be stated, but for those with graphs, all too often a decimal equivalent was given for $\ln 12$. A high proportion of candidates did not attempt this part of the question, suggesting that this is a syllabus area that may need to be concentrated upon.

Question 9

(a) Many candidates were unable to show a convincing solution. It was expected that the expression $\frac{(2x+1)(4x-1)-(4x-1)+4(2x+1)^2}{(2x+1)^2(4x-1)}$ be seen, followed by an expansion and simplification of the numerator which could be seen to result in the given answer. Sometimes sign errors and expansion errors still led to the given answer. Candidates are advised to check their working when a specific result is not obtained as often it will be due to a simple sign slip. Candidates who simplified $\frac{1}{2x+1}-\frac{1}{(2x+1)^2}$ to $\frac{2x}{(2x+1)^2}$ and then added $\frac{4}{4x-1}$ were usually successful in gaining both marks provided sufficient detail was shown.

Some candidates ended up with cubic numerators, not having dealt correctly with the repeated factor of $(2x+1)$ in the denominator.

(b) The question uses the command word 'Hence' which implies that the work done in **part (a)** be used to help with **part (b)**. Most candidates attempted to make use of the fractions given in **part (a)**, but some candidates did attempt integration of the one fraction, which was impossible for them to do.

Most candidates who were dealing with the three separate fractions, recognised that there were some logarithmic terms to be obtained from integration. Occasionally the term $-\frac{1}{(2x+1)^2}$ was mistakenly integrated as a logarithmic term. Most errors were sign errors or coefficient errors. Very few candidates obtained a correct final result of $\frac{1}{2}\ln\frac{27}{2}$ in spite of having correct terms obtained from substitution of limits, seemingly being unable to simplify the terms to obtain the given form. It was more common for some candidates to be able to obtain a mark for obtaining $-\frac{1}{12}$ from correct working.

Question 10

(a) It was important that the common difference of $\lg x^4$, which many candidates obtained, be simplified to $4\lg x$ in order to be able to simplify the sum to *n* terms to the required form. Most were able to gain a mark for writing down the sum to *n* terms with a correct first term and common difference, but far fewer were able to simplify this sum correctly.

(b) For those candidates with a correct response to **part (a)**, a correct value of 50 was usually obtained from the resulting quadratic equation formed.

(c) Even if candidates had not been able to attempt **parts (a)** and **(b)**, there was enough information given in **part (b)** to be able to complete **part (c)** even if no correct work had been done in **part (a)**. Quite a few correct solutions were seen from candidates having realised this.

Question 11

(a) Most candidates recognised the need to differentiate a quotient, with most gaining three marks. There were some errors in the differentiation of $(2t+1)^{\frac{3}{2}}$ and the occasional slip with the order of terms or copying errors. Fewer candidates were able to simplify their result to the given form, being unable to extract a common term of $(2t+1)^{\frac{1}{2}}$ from the numerator of the derivative.

(b) Very few candidates obtained the mark for this part of the question. If the result for the velocity from **part (a)** was not in the required form, then it was almost impossible. For those candidates who had obtained a derivative in the required form, it was essential that they consider both factors in the numerator and state that when equated to zero, these factors gave negative values for *t* and as $t > 0$ after passing through the point *O*, the particle was never at rest.

Question 12

The most successful candidates were the ones who expanded $\left(ax+\frac{2}{5}\right)^5$ and $\left(1-\frac{b}{x}\right)^2$ separately, as it was easier for them to pick out the terms which would give them the values of the constants *a*, *b* and *c* more easily. Marks were not awarded when a correct unsimplified term was not seen. Many candidates had expansions of, for example, $ax^5+2ax^4+\frac{8}{5}ax^3$ or similar. These did not merit any marks unless previous unsimplified terms had been seen, for example, $(ax)^5+\left(5(ax)^4\times\frac{2}{5}\right)+\left(10(ax)^3\times\frac{4}{25}\right)$. Many candidates also made sign errors and errors involving *b* in the expansion of $\left(1-\frac{b}{x}\right)^2$. Many candidates were able to obtain method marks for realising that they needed to consider two terms from their expansion to find the value of *b* and three terms from their expansion to find the value of *c*.

For the candidates who attempted to identify separate terms rather than write out an expansion, similar sorts of errors were seen, with these candidates being more likely to omit terms when attempting to find the values of *b* and *c*.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should be reminded to work calculations to more than 3 significant figures as premature rounding rarely obtains full marks. They should also read each question carefully and pay attention to the required form of an answer. Candidates could have helped themselves with taking more care, especially when manipulating expressions and equations where errors in signs often occurred. They also need to ensure that they do not omit essential brackets.

General comments

There seemed to be no issues with timing in this examination. Those who omitted parts seemed to be doing so due to a shortage of knowledge. When integration is necessary, it was not always clear at which point an integral was undertaken as the notation was often confused, which did not assist in the assessment process.

Care needs to be taken when using a 'dummy variable'. In **Question 3(b)** it was not helpful if $\log_3 x$ or $\log_x 3$ was replaced with x whilst a suitable equation was being produced. A similar misunderstanding was also seen in **Question 10** when replacing $3x+1.2$ with x led to a loss of subsequent marks.

Candidates were generally well prepared on topics such as completing the square, factor and remainder theorems, area and perimeter of circular measures. At times, the amount of work presented was out of proportion to the marks available. Candidates should take note of the number of marks available – a question worth one mark will not usually require much working. The adoption of an appropriate strategy in solving a mathematical problem was not always followed, especially with questions on trigonometry where there were many occasions when misreads occurred.

Comments on specific questions

Question 1

Nearly all candidates knew the shape of a sine curve and by substitution correctly plotted the start and end points. Most knew that the maximum and minimum were 2 and -6 and that the curve passed through the y-axis at $(0,-2)$ even when their curves were otherwise incorrect. Many gave two or more cycles and some did not translate the curve down 2. Those who used rulers to join points did not score.

Question 2

(a) Only a minority of candidates earned both marks. However, many candidates made arithmetical errors, such as working with $2\left(x+\frac{1}{2}\right)^2$. Others wrote down a correctly but did not have a correct answer for b. It was quite common to see $-\frac{119}{8}$ for b, but there were a few others who gave answers such as $-\frac{241}{16}$. A few candidates initially worked with the correct value for a, but spoiled their solution by writing the answer as $2(4x+1)^2$. A number of candidates did not know what to do and gave answers such as $2(x+1)^2-15$ or $2\left(x+\frac{x}{4}\right)^2-\frac{x^2}{16}-15$.

(b) Some candidates used their correct answer to **part (a)** to write down a correct answer to this part, but a few made a sign error, usually with *a*. Some earned both marks from the follow through. Some candidates differentiated, but often went wrong, usually in calculating *b*. A number of candidates apparently misinterpreted the question as a request to factorise and find the *x*-intercepts.

(c) This was often quite well done, even by candidates who earned no marks in any of the other parts. The vertex was often misplaced in the first quadrant resulting in many candidates only being awarded two of the three marks. Sometimes the curvature of the two branches was incorrect and a few of the weaker responses drew straight lines but not always in the correct place. Most showed the *x*-intercepts correctly but the *y*-intercept was often given as -15 or omitted.

(d) A minority of candidates connected the request with earlier work and earned the mark here, some from the follow through. A few lost the mark by leaving the negative sign in. Some candidates clearly did not understand what was being asked and wrote down their *y*-intercept or gave the *x*-intercepts.

Question 3

(a) This was a well answered question. The majority of candidates were able to solve the simultaneous equations correctly by rearranging one of the given equations and substituting into the other. Common errors were algebraic manipulation where, for example, the second equation was rearranged to $y = \frac{x}{2}$, so they did not achieve the necessary quadratic equation. Having achieved a three-term quadratic, most then went on to correctly find two solutions for one of the variables, but some forgot to substitute back in to find the second set.

(b) This question required candidates to be able to change the base of one of the logarithm terms. Many were able to do this but their working was often confused such that they did not realise that they could multiply through by the appropriate logarithm term to create a quadratic equation. Many made basic errors in manipulating the logarithmic terms, for example $\log x \times \log x = \log x^2$. Where the correct quadratic was found, some candidates found it difficult to interpret their solution correctly, that is, they could not proceed from $\log_3 x = 2$ to $x = 3^2$. Some candidates lost one of the final answer marks for rejecting $\log_3 x = -5$.

Question 4

(a) To show that $b = -9$, candidates were expected to accurately differentiate p(x) to leave just b, once the value of $x = 0$ was substituted. Many did not differentiate, perhaps not understanding the notation $\mathrm{p}'(x)$ and thus gained no credit.

(b) This was generally well attempted. Most correctly found $\mathrm{p}\left(-\frac{2}{3}\right)$ though some sign errors did appear and not all showed p(x)= 0. Those that tried to find $\mathrm{p}\left(\frac{2}{3}\right)$ were not successful. The equation for p(–1) = 6 was usually found accurately and an attempt to eliminate either *a* or *c* from the two equations normally produced the final solutions of 6 and –10. Many arithmetic errors occurred when multiplying through the equations. Only a few tried a long division method and, when it was attempted, it was rarely successful.

(c) Candidates who achieved full marks in **part (b)** and used algebraic long division correctly in this part were successful in this part as well. The use of synthetic division meant that some candidates had an extra coefficient of 3 in their expression which made their result incorrect.

(d) It was clear that some did not understand the difference between roots and factors, with the former often being offered in **part (d)** though this was overlooked if the correct factors were also available. Those that did not find the correct values for *a* and *c* could not complete this part.

Question 5

(a) There were many fully correct responses to this question. Some candidates used the wrong formulae. Candidates should be reminded that the formulae for arithmetic and geometric progressions are given on page two of the examination paper. Many misread the question, for example, $8ar^{14} = ar^{11}$, leading to $r = 2$ and $a = –5$, without realising that $–1< r <1$.

(b) Only a minority of candidates produced a completely correct solution, usually, but not always, from working with = rather than >. A good number of scripts lost the accuracy marks for incorrect manipulation of the negative sign when working with inequalities. Candidates who did less well just earned the first method mark then had arithmetic errors, usually from writing, for example, $-\frac{5}{2}\times\left(\frac{1}{2}\right)^n = -\left(\frac{5}{4}\right)^n$. Some candidates equated their expression to 5 and were obviously unable to complete the solution. There were many candidates who left this question blank.

Question 6

(a) This was usually correct. Common errors were $f \geqslant -4$, $f > -3$, $f \in \mathbb{R}$.

(b) There were very many completely correct answers. Some candidates lost the accuracy mark by omitting the brackets or making a sign error. A few candidates could not manage the manipulation of taking logs and were unable to gain any marks.

(c) A number of candidates lost one or two marks by decimalising $\frac{\ln 4}{3}$ and, in some cases, the inverse function was either curving down steeply in the first quadrant or curving away from the asymptote. Occasionally there was a sign error with labelling (–3, 0) or (0, –3), and sometimes the intercepts were on (–4, 0) and (0, –4). Some candidates drew a parabola, a hyperbola or some other curve which indicated no knowledge of the shape of either curve.

Question 7

Many candidates did not appear to know how to integrate trigonometry, and some confused it with differentiation. The multipliers were often wrong, especially the sign. More often the +1 disappeared, stayed the same or was incorporated into the cosine term. Any reasonable attempt gained at least the mark for substituting the limits in the x term e.g. $\frac{\pi}{2}$. However, substitution of limits was not always clear and decimal answers were common as was leaving the sine and cosine unevaluated.

Question 8

(a) The expected calculation was that arc AB, which was 12.25, would be divided by 7. Some candidates appeared to be unclear with regard to radians with 1.75π being sometimes offered as the solution implying that some candidates thought it was a symbol for radians. Some candidates worked with the reflex angle which was shown with the additional working out of $2\pi –$ angle AOB. Despite the request for radians some calculated the answer in degrees.

(b) This was usually well answered. Most candidates realised the need to add the lengths of the two arcs to twice the radius, although a few added in an extra 7 for OB as well. The main error was in finding angle BOC which should have been easily available using the inverse $\tan\left(\frac{24}{7}\right)$. A few candidates made an incorrect assumption and tried angle $BOC = \pi$ – angle AOB. The sine rule was often seen (in degrees) to find angle BOC. Where the intended calculations were clearly shown follow through marks were available.

(c) There were two areas of sectors to add together to obtain the solution. One was often found correctly being 0.5×7^2 x angle AOB, with follow through from the angle AOB that was found in **part (a)**, although $0.5 \times 7 \times 12.25$ negated the need for that to be used. Candidates should be advised to give information provided in the question if possible, rather than values they have calculated.

Question 9

(a) (i) This was usually calculated correctly using logic or $^{12}P_6$. The main incorrect solution was where $^{12}C_6$ was offered instead.

(ii) This was mostly well done but a few did not include the symbols except when considering the end character and so calculated 8 × 7 × 6 × 5 × 4 × 4.

(b) The expected calculation was 8 × 7 × 24 with the 8 and 7 accounting for not having a symbol at the start or finish. The four symbols would then have to be in the other places and giving the calculation 4 × 3 × 2 × 1 = 24. Seeing either the 8 × 7 or the 24 as part of a product was sufficient to gain a method mark. Many candidates had the correct idea but had a choice of 8 non-symbols at each end thus giving 1536. Some found 1344 but felt they had not accounted for the symbol being at either end and so doubled their answer.

Question 10

Candidates found this to be a challenging question. A small number of candidates gave concise, neat and accurate solutions. Those who made some progress often earned 3 or 4 marks. Some candidates missed the last accuracy mark because one of the correct answers was rounded prematurely or incorrectly: 0.853 being the most common such error. Similarly, a significant number of candidates missed one of the roots, usually -1.24. Some candidates made a slip early on and worked with an incorrect base angle. A small number of candidates worked in degrees and successfully converted back to radians: often, such candidates earned no more than the first method mark because they mixed degrees and radians throughout.

Question 11

About a third of candidates scored 5 or 6 marks on this question. However, many candidates were unable to gain much credit. This was often due to not recognising the log integral.

Many candidates made a good start but then were not able to apply the logarithm laws correctly. Some candidates only earned the second B mark when they showed a valid attempt in substituting the limits in, recovering from the omission of bracket at the initial stage. A few candidates made substitution slips with the limits, or lost a B mark for giving an integral as, for example, $\ln(3x + 1)$ or, more commonly, $3\ln(3x + 2)$ but still went on to earn the mark for applying the logarithm laws. Many candidates obtained an incorrect quadratic equation and showed insufficient working to earn the method mark allocated for solving their quadratic.

Question 12

(a) Most candidates knew how to differentiate a quotient. Some found the differentiation of the numerator to be difficult, generally not including the $6x$ from the derivative of $3x^2 – 2$. Where this was found, in some cases, it was lost in subsequent algebraic manipulation. Some candidates needed to take more care with the use of brackets.

The subsequent manipulation of the derivative was a challenge for most candidates. They found it difficult to deal with the two $(3x^2 – 2)$ brackets where they were raised to the powers of $-\frac{1}{3}$ and $\frac{2}{3}$. Where candidates did manage this, they then did not use brackets writing expressions such as $4x^2 - x - 3x^2 - 2$ rather than $4x^2 - x - \left(3x^2 - 2\right)$ which led to a sign error in the required quadratic expression.

(b) Many candidates did not know how to attempt this part of the question. Some thought that $2 + p$ or just p had to be substituted into their derivative. Those candidates who had a correct derivative from **part (a)** and who realised that this had to be evaluated when $x = 2$ generally found the correct answer.

Question 13

(a) This part of the question proved accessible to the majority of candidates, with over half gaining full marks. Most of the others earned a mark for working out either the mid-point or the gradient. A few earned both of these marks or one of these and then the method mark for the straight line using their mid-point and perpendicular gradient.

(b) This part of the question proved to be challenging for most candidates. Many candidates were either unable to be awarded any marks or omitted the question completely. Some earned partial credit from a vector approach but made a slip with the arithmetic to earn only the method mark. Candidates who opted to work with distances were seldom successful.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to do well in this examination, candidates need to take careful note of key words and phrases. When exact answers are required, candidates should understand that decimals that fill their calculator screen are rarely exact. An exact answer will generally involve, for example, a fraction, surd, logarithm or exponential. When the instruction in the question is Show that …, candidates should make sure that they write down all method steps needed to find the answer that has been given. When a question uses the key word Hence…, candidates should be aware that there is a link to a previously found result. Sometimes this is to provide candidates with the most straightforward method of solution and at other times it is to assess a particular method. In all cases, candidates are advised to use the link indicated when the word Hence is used. It is also important that candidates are aware of the Mathematical formulae given on page 2 of the examination paper. Some candidates are trying to recall formulae that have been given, and are doing so incorrectly. It is important that candidates understand that, to be credited for follow through values, they must show their method of calculation, as correct method will not be assumed from incorrect values alone.

General comments

The presentation of responses was often good. Candidates whose working was neatly laid out generally offered solutions that were more logical. Candidates whose working was more haphazard tended to make errors such as miscopying their own figures. Those who ran out of room in the main examination script, through deleting attempts at answers, should have used the blank page at the end of the examination paper or additional writing paper for their solutions. Some candidates did do this and, helpfully, indicated that their response was written elsewhere. Other candidates tried to compress work into a very small space. This usually did not assist them in providing accurate solutions.

The laws of logarithms were often misapplied in this examination. This was evident in **Question 2** and **Question 4**. For example, candidates were rewriting $\log_a(x-y)$ as $\dfrac{\log_a x}{\log_b y}$, $\dfrac{b}{\log_a x}$ as $\dfrac{\log_x a}{b}$ or $-b\log_x a$, $\ln(xy)$ as $\ln x \ln y$, and $\ln\left(ab^x\right)$ as $x\ln(ab)$.

When the instruction in a question is Do not use a calculator, candidates must show clearly how they found their values. In **Question 5** in this examination, candidates were not permitted to use their calculator to solve the cubic equation formed. It was essential that the method of solution of the quadratic factor equal to zero also needed to be shown and be correct. A few candidates used unusual algorithms to factorise the quadratic factor. These needed to take care to produce factors that had integer coefficients and constants, or to show sufficient detail of what they were attempting, to be credited.

Candidates may need to be reminded to locate errors sensibly in questions where parts are clearly linked. For example, in **Question 8(b)** in this examination, candidates needed to derive an expression for the volume of an object. This solution required the use of the previous part of the question. Some candidates made several attempts to find the expression for the volume. Many of these candidates were trying to adjust their answer to ‘fit’ the given answer, instead of going back and correcting the previous part of the question where they had made an error.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Most candidates found this to be an accessible start to the examination. The majority of candidates provided fully correct solutions. The substitution $x = -5y - 4$ was the most popular and most successful method. A few candidates needed to take more care as some made sign errors when using this substitution. Candidates who used substitutions which involved an algebraic fraction, or fractions, were less successful. Sometimes these candidates omitted to multiply all terms when clearing the fraction or, again, they made sign errors. A few candidates incorrectly discarded a solution, either because it was negative or because it was not an integer. Other candidates had a quadratic equation in terms of y and then solved this as if it were x, going on to incorrectly work out y from these 'x' values. Checking or reading over their work once they had completed the question or examination might have helped these candidates.

Question 2

This question was not well answered. The simplest method of solution for this question was to combine the exponentials into a single term and then take logarithms. Candidates who used this approach were the most successful. Common errors for this method were rewriting as: $\frac{e^{2x-3}}{e^{5-x}} = \frac{7}{4} \rightarrow \frac{2x-3}{5-x} = \ln\frac{7}{4}$, $\frac{4e^{2x}}{4e^3} = \frac{7e^5}{7e^x}$, and $e^{4(2x-3)} = e^{7(5-x)}$. The alternative method of taking logarithms in the first step was much more likely to result in a method error. No recovery from such errors was permitted. Common errors for this method were rewriting as: $(2x-3)\ln 4e = (5-x)\ln 7e$, $\ln 4 \times \ln e^{2x-3} = \ln 7 \times \ln e^{5-x}$, and $4\ln\left(e^{2x-3}\right) = 7\ln\left(e^{5-x}\right)$.

Question 3

Candidates found this question challenging. Fully correct methods were not common. A good number of candidates understood the need to differentiate, although this was not always carried out correctly as sign errors and misinterpretation of the constant value a were seen. Candidates who did differentiate correctly did not always state the normal gradient accurately. The reciprocal of the gradient of the tangent, or similar such as $\frac{1}{a+3}$ was commonly seen. Those who showed that they were clearly attempting to find $\frac{-1}{\textit{their } \left.\frac{dy}{dx}\right|_{x=1}}$ were able to gain further credit. Those who did not show what they were trying to do, but simply wrote down an incorrect algebraic fraction for the gradient of the normal, were unable to make further progress.

An alternative, simpler approach, was to equate gradients of tangents. However, not many candidates observed that the gradient of the tangent was 4. Many candidates who had differentiated seemed not to know how to progress correctly from that stage. Commonly, candidates equated the gradient function for the tangent to 0 or to $-\frac{1}{4}$. These candidates were then, also, unable to make further progress.

On occasion candidates equated gradients for normals or tangents using functions that still included x. This work was not credited until the value $x = 1$ had been substituted. Other candidates stated that the gradient of the normal was $-\frac{1}{4}x$ and evaluated this when $x = 1$ as $-\frac{1}{4}$. This was incorrect and was penalised.

The simplest method for finding b was to use the value of a to evaluate y when $x = 1$ and use that in the given normal equation. A few candidates, having found a value for a using a sufficiently accurate method, made an error when finding b. This commonly happened when candidates generated a relationship between a and b with an error. This was unnecessary and so not condoned. On other occasions, candidates confused $x = 1$ and $a = -1$ when using the given normal equation and solved $0 = \frac{1}{4} + b$ which was not condoned.

Weaker responses used $y = 0$ without justification in both $y = a + 1$ and $y = -\frac{1}{4}x + b$ to find a and b. These were not credited. Other weak responses either incorrectly calculated the value of a using $\frac{a}{1} + 3(1) - 2 = 0$ or indicated that the gradient of the curve was $\frac{a}{1} + 3(1)$ and found a by equating this to 0.

Question 4

Some fully correct, neat and logical solutions were presented for this question. Candidates needed to take great care not to make an error in the early stages of this question as no recovery was permitted should an uncorrected error in method be made. Several candidates found what seemed to be a correct quadratic equation using an incorrect method. These responses, which were from wrong working, were not credited. As an initial step, candidates needed to change the base so that all logarithms were in the same base. A good number of candidates carried out this step correctly. A common error at this stage, however, was to deal with the 2 incorrectly when changing the base of the term $\frac{2}{\log_x 3}$. For example, candidates sometimes rewrote this as $-2\log_3 x$, as $\frac{\log_3 x}{2}$ or as $\log_3 3^2 \times \log_3 x$ which then became $\log_3 9x$. The weakest responses often began by rewriting the left-hand side of the given equation as $\frac{\log_3 11x}{\log_3 8}$ or $\log_3 11x - \log_3 8$. Some candidates treated log not as a function but as a multiplier, so they divided in order to cancel the logs.

Once the change of base was completed correctly, candidates needed to manipulate the log equation so that the quadratic equation, free of logarithms, could easily be obtained from it. This was reasonably well done, although a step of $11x – 8 = x^2 + 3$ at this stage was seen quite often. The final steps in the solution were fairly well carried out by those who had successfully worked their way through to this stage. The only common error in the solving of the quadratic tended to be to forget to discard the value $x = 1$.

Question 5

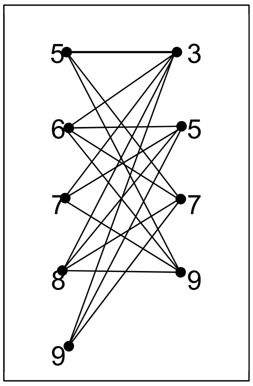
Candidates were not permitted to use a calculator in this question. For this reason, the use of the linear factor $x – 1$ to find a quadratic factor was expected. Candidates were allowed to use the factor $x – 1$ if they had found $x = 1$ to be a root by inspection as this was fairly obvious. Candidates who used one of the other linear factors needed to show that the root had not been found using a calculator. A reasonable number of candidates produced good, clear and fully-justified solutions. The first step in the method was to equate the equations of the curves and form a cubic equation which could then be solved. The quadratic factor $6x^2 + x - 12$ needed to be found and then factorised or equated to 0 and solved. The method of finding the roots had to be seen. A few candidates omitted to state that $x = 1$ was a root somewhere in the solution. Candidates who simply wrote down a product of 3 linear factors and then 3 roots, without derivation, were not credited. Candidates who gave correct roots following incorrect working were also not credited. The weakest responses often used the derivatives of the two given curves. Another common weak response was to find the correct cubic and then to ‘adjust’ it in some way to produce a quadratic. For example, $x\left(6x^2 - 5x - 13\right) + 12 = (x + 12)\left(6x^2 - 5x - 13\right)$.

Question 6

This question used assumed knowledge of properties of numbers. Some candidates were clearly unsure of the meaning of ‘odd’ and ‘prime’.

In both parts of this question, only a few fully correct responses were seen. A small number of candidates offered a fully correct method but made an arithmetic slip when finding the final answer. Some candidates earned a mark for showing method which was equivalent to one of the two most efficient method steps. A few candidates made good use of matching diagrams to count the number of possibilities and then multiplied by ${}^6P_2$.

For example, in **part (a)**:



Candidates who listed possibilities sometimes omitted one or more cases or miscounted. Correct responses should have used permutations and/or the product rule for counting. Commonly, weaker responses used combinations or incorrect permutations, such as ${}^{7}P_{2}$, or factorials in the product rule for counting.

Question 7

(a) In this question, candidates needed to prove a trigonometric identity. Candidates needed to work to show that the left-hand side of the identity was equal to the right-hand side. Most candidates did this, although a small number still worked as if they were manipulating an equation, which was not permitted. There were a good number of fully correct responses seen. A variety of approaches could be used, but by far the most common was to write the fractions or combine the fractions with a common denominator, take the necessary steps to show that the numerator was $2 - 2\cos x$, factorise and then complete the proof. A few candidates did not earn the final mark as they had omitted brackets several times or had used notation such as $\sin x^2$ rather than $\sin^2 x$. A few candidates were unsure of how to transform $\dfrac{2-2\cos x}{\sin x - \cos x \sin x}$ to the given answer and made no real progress after this point. A small number of candidates needed to take more care with signs as sign errors were seen. Some candidates incorrectly replaced $1 - \cos x$ with $\sin x$ at some stage. Others gave a correct first step and then spuriously cancelled terms, such as $\sin x$ or $1 - \cos x$, making no further progress. Weaker responses often started with $(1 - \cos^2 x)$ in the numerator rather than $(1 - \cos x)^2$ and usually did not recover from this. Other weak responses omitted brackets at every stage and so could not be credited.

(b) The key word Hence in this part of the question directed candidates to use the previous part of the question to help them answer this part. This was the most straightforward method of solution. A good number of candidates took notice of this instruction. The few candidates that did not, tended to make no real progress here. A good proportion of candidates formed the correct quadratic equation in $\sin x$. A small number made sign slips or were unable to manipulate $\dfrac{2}{\sin x} = 3\sin x - 1$ correctly, with $2 = 3\sin^2 x - 1$ fairly common in these cases. Those who did have a 3-term quadratic equation in $\sin x$ often managed to solve this correctly. Many candidates who used a substitution before factorising followed good practice, first stating their substitution, such as $y = \sin x$. Candidates who omitted to state their substitution or used the substitution $x = \sin x$ need to be aware that this is not good practice and can result in errors. A few candidates who used the factorising by grouping approach commonly made errors by writing things like $\left(3\sin^2 x - 3\sin x\right)\left(2\sin x - 2\right)$ rather than $\left(3\sin^2 x - 3\sin x\right) + \left(2\sin x - 2\right)$. However, most of these candidates recovered by then writing the product of correct factors. A small number of candidates unfortunately rejected the solution $\sin x = -\dfrac{2}{3}$ without fully considering what solutions could arise from it. Others rejected $x = -41.8°$, again without fully considering what solutions could arise from it. A good number of candidates found all 3 correct angles and stated them to at least 1 decimal place. A small number of candidates included incorrect angles in the range such as 41.8°, 138.2°, 180° or 270°.

Question 8

(a) This question should have been straightforward. However, correct expressions for the surface area were not often seen. Expressions which had no multiple or an incorrect multiple of πr^2, such as $2\pi rh$ or $2\pi r^2 + 2\pi rh$ or $4\pi r^2 + 2\pi rh$ or $5\pi r^2 + 2\pi rh$ were far more common. The weakest responses involved expressions for the total surface area with components of different dimensions, such as $2\pi r^2 + \pi r^2 h$ or$2\pi r + 2\pi r^2 + 2\pi rh$. Some responses made no use at all of the information given that the total surface area was 300 cm^2.

(b) Candidates were more successful in this part of the question. Those who had been successful in **part (a)** were often successful in earning at least two marks in this part. A few of these candidates made slips in completing the argument they were making. Usually these were sign slips or carelessness with powers or with π. Many candidates were able to use their expression for h in terms of r in a correct expression for the volume of the object. A few candidates omitted to use the correct expression for the volume of the hemisphere with $\frac{4}{3}\pi r^3$ and $\frac{8}{3}\pi r^3$ being fairly common in these cases. These were still able to earn some credit if they formed the sum of this expression with the correct expression for the volume of the cylinder using their h. Weaker responses were typically very untidy and often these candidates were attempting to work back from the given answer, without success.

(c) Candidates found the most success in this part of the question as it used information they had been given in **part (b)**. A good number of candidates understood the need to differentiate and were able to do this successfully. Many of these candidates were able to find a sufficiently accurate value of r and use this correctly to find the required volume. Some candidates made an error in finding the derivative but were still able to earn the subsequent method marks if they continued to try to work with it in the correct way. Some candidates made rearrangement errors when finding r and so lost accuracy. The value $r = \sqrt{60\pi}$ was fairly common in these cases. Some candidates did not discard the negative value for r, even though this did not make sense in the context of the question. Other candidates thought that they needed to discard the positive value of r, perhaps showing some confusion with the second derivative test. A few candidates substituted their value of r into the expression for the derivative instead of the expression for the volume. Some candidates did not attempt to find the volume and these may have benefitted from rereading the question once they had finished their attempt to answer. Weaker responses found the derivative and made no more progress or went on to find the second derivative, equate it to 0 and solve. The weakest responses made no use of calculus whatsoever and often attempted to solve $V = 0$ to find a value of r.

Question 9

Throughout this question, candidates were required to give exact values. Calculators were permitted but candidates needed to use them sensibly to ensure that all values remained exact. Those candidates who chose to use decimals at an early stage were heavily penalised. Weaker responses typically showed that candidates considered the triangle to be right-angled and this resulted in them making no progress in any part of the question.

(a) Candidates performed fairly well in this part of the question. Most candidates used the side-angle-side area formula $\frac{1}{2}bc\sin A$ with the correct lengths and equated to the given area. This formula is given in the list of Mathematical formulae on page 2 of the examination paper. A few candidates omitted brackets or omitted $\frac{1}{2}$ when forming this initial equation and these were not always recovered in subsequent working. A small number of candidates misread or miscopied the given area as $\frac{2\sqrt{3}}{5}$. A few candidates were able to derive, for example, $2\sin A = \frac{2\sqrt{5}}{3}$ but then were unable to complete correctly and commonly stated $\sin A = \frac{4\sqrt{5}}{3}$. Many candidates went on to find angle A. This was ignored as it was not necessary. A few candidates found the length of the perpendicular from B to AC using $\frac{1}{2}\times\text{base}\times\text{height}$. Once this had been found, they used right-angled triangle trigonometry to find sin A. This approach varied in its success as there was a greater chance of making an error.

(b) Candidates found this part to be very challenging. Fully correct responses were seen but were not common. A small number of candidates found the exact value of cos A using Pythagoras' theorem, for example, but then did not find the value of x. These candidates might have benefitted from rereading the question once they had finished their answer. A few candidates only had the value $\frac{2}{3}$ for cos A embedded in an equation for finding x and did not state it separately as cos A. As the question required candidates to find the value of cos A, as well as x, this was not condoned for full credit. The majority of candidates were unable to state the exact value for cos A and so were unable to access most of the marks for finding x. Commonly, cos48.2 or 0.667 or similar were stated. A method mark was available to these candidates for a correct statement of the cosine rule using correct lengths. However, some candidates made slips in this, even though the formula was given in the list of Mathematical formulae on page 2 of the examination paper. Other candidates misapplied the rule and used the lengths and structure appropriate for finding cos C. Weaker responses often used decimal values or stated that $\frac{12-x^2}{8}$ and $\sqrt{12-8\cos A}$ were their 'values' of cos A and x.

(c) At this stage, only the best candidates were able to gain full marks. Those who had decimal values were able to earn a method mark for correctly using these in the sine rule or in a correct equation using the side-angle-side formula for the area once again. A few slips were made in using the sine rule, most notably omitting brackets which were not recovered, using $\sin\frac{\sqrt{5}}{3}$ rather than $\frac{\sqrt{5}}{3}$, using $\sqrt{5}-1$ rather than their value of x or rather than using $\sqrt{5}+1$. When using the area of the triangle approach the most common errors were, again, omitting brackets that were not recovered, using $\sqrt{5}+1$ rather than $\sqrt{5}-1$ or omitting the $\frac{1}{2}$. Candidates who had accurate values earned an accuracy mark at the initial stage. These candidates sometimes went on to simplify sufficiently to a correct form, but a fair number made slips in attempting to do this.

Question 10

(a) This part of the question was very well answered, with many candidates offering fully correct solutions. Most candidates formed a correct pair of equations in a and r, although some did work through to the correct values without ever stating these. A good number of these went on to solve these equations correctly. The most common approach was to eliminate a and find $r^3 = 3.375$. Many did this. Most were able to find the correct value of r from this, although some slips were made, most notably taking the square root rather than the cube root. Candidates who attempted to eliminate r and find a first were much more likely to find incorrect values. Some candidates rounded 15.1875 to 15.2, for example. This was not condoned unless the correct values for a and r were derived. Some candidates did not realise that they had found the first term when finding a and went on to divide by 1.5 again. This was not condoned. Weaker responses typically indicated the two equations as $ar^3 = 4.5$ and $ar^6 = 15.1875$ or, when trying to solve, found the difference of the terms rather than the ratio of the terms. Some candidates used expressions appropriate for an arithmetic progression.

(b) Candidates found this part of the question to be challenging, with many not able to form a correct plan to solve the problem. Candidates who were successful often found the 16th term of the GP and then used this as the first term in S_{10} with $r = 1.5$. Some of these candidates made premature approximation errors and so lost the final accuracy mark. Another successful plan, although not as commonly attempted, was to find $S_{25} - S_{15}$ or similar. A few candidates were successful through listing all 10 terms and summing them. This was more time consuming and more prone to rounding and calculation errors, however. Although the question required the answer to be given to the nearest integer as a helpful guide to candidates, more accurate correct answers were accepted on this occasion. A few candidates gained marks for correct plans using their values from **part (a)**. Some candidates earned a mark for a partially correct method, using S_{25} or S_{15}, for example. The most common incorrect plan was to try to find $S_{26} - S_{16}$, which was of no merit. Other fairly common incorrect plans were to find $S_{16} - S_{10}$ or $S_{16} - S_6$, again, these were of no merit. Weaker responses listed sums and totalled those or found the 16th term of the GP and, without connection, S_{10} using $a = 2$, 16 or 15.1875 and $r = 1.5$. These were not credited.

Question 11

(a) It was expected that candidates would use translations or displacement vectors in order to solve this problem. Some candidates did this and were successful. A few candidates used the ratio theorem. Even though this is beyond this syllabus, these were often successful and so credited. A few candidates were given the benefit of the doubt that they had made mental connections between gradients and vectors as, with minimal written work shown, they gave the correct coordinates for *C*. A small number of candidates incorrectly used the relationship $AC = 2CB$ when structuring their vectors, resulting in an answer of (1, –2). Candidates who attempted to find *C* using algebraic equations were almost always unsuccessful. The most common incorrect strategies were to find the mid-point of *AB* or to then also find the mid-point of *A* and $\left(-\frac{1}{2}, 0\right)$ or to write $\overrightarrow{AC}$ as $\overrightarrow{OC}$ and state $C(3, -4)$ or $\frac{1}{3}\overrightarrow{AB} = \frac{1}{3}\begin{pmatrix}-5+4\\6-6\end{pmatrix}$ as $\overrightarrow{OC}$ and state $\left(-\frac{1}{3}, 0\right)$ or similar.

(b) Candidates were more successful in this part, with many able to earn at least three of the four marks available. Those who were correct in **part (a)** often earned all four marks in this part also. A few candidates may have done better if they had reread the question once they had completed their answer, as some omitted to state the equation in the required form and this was penalised. The most successful way to find the gradient of *CD* was to first find the gradient of *AB*. This used given coordinates, not coordinates generated by the candidate. Most candidates did this and the majority were successful in finding both the gradient of *AB* and the gradient of *CD* then forming the equation for their point *C*. Candidates were only allowed to use the mid-point to form the equation in this part of the question if $\left(-\frac{1}{2}, 0\right)$ had been their answer to **part (a)**. Some candidates misinterpreted the question as requiring the equation of the perpendicular bisector and used the mid-point in this part even when it was not their *C* from **part (a)**. This was not condoned.

(c) Most candidates found the problem-solving skills needed to answer this part of the question to be beyond them. A small number of candidates did produce neat, clear and accurate solutions using algebra and solved equations. A few other candidates were successful using a geometric approach with Pythagoras' theorem and vectors, or a mixture of geometry and algebra. Some candidates earned a mark for finding a correct length or setting up a correct length equation. Most candidates attempted to form and solve equations and soon confused themselves with the algebra that was needed. Whilst some understood the need to substitute the equation of the line *CD* into the length equation they had derived, many had made errors at this point or were using an incorrect equation for *CD* and so only partial credit could be awarded. Weaker responses offered incorrect length formulae and/or invalid separation of the equation $(x-4)^2 + (y+6)^2 = 125$ into $(x-4)^2 = 125$ and $(y+6)^2 = 125$, for example. Many candidates made no real attempt to answer.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

It is important that candidates

- are familiar with the requirements of the rubric on the front page prior to the examination, particularly with reference to the degree of accuracy required
- are familiar with the mathematical formulae provided on the second page of the examination paper
- understand the requirements of the question by taking note of the key words and phrases in a question
- show sufficiently clear and logical steps in their method, ensuring correct use of brackets
- check that the requirements of the question have been met before moving on to the next question.

General comments

Good responses were well-structured with clear and logical steps. Those with minimal or missing steps, and disorganised responses were more likely to lose a greater number of marks in the event of minor errors. Showing all method steps was especially important in 'Show that…' questions. The need for this was seen in **Questions 4(b) and 5(a)** in this examination. Candidates also needed to follow the rubric and make sure that they showed all necessary method steps and did not rely on their calculators. This was especially the case in **Question 7**, for example, in this examination where trials on values of *n* were heavily penalised.

Many candidates gave their final answers to the required degree of accuracy as stated in the rubric on the front page. Candidates who gave correct answers to a greater degree of accuracy did not lose marks when rounding errors were subsequently made. A lack of familiarity with the level of accuracy required for angles in degrees was evident in **Question 5(b)** in this examination. Errors in the final answer resulting from prematurely rounding answers in intermediate work was also seen in **Question 5(b)**.

Some candidates would improve by taking greater care with bracketing. Errors due to omission of brackets were most evident in **Questions 5(a), 10(a), 11(a)** and **(b)**.

When an answer space provided insufficient space for a candidate's response, it was helpful to see additional sheets annotated with the appropriate question number, whilst also annotating the main response space to indicate that their response continued elsewhere. Where candidates wish to delete work, it is advised that this is done with a single line, so that work beneath is still legible.

Most candidates appeared to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Many candidates demonstrated a good understanding of the steps involved in arriving at the correct inequality. Some candidates could improve on expanding brackets and collecting like terms correctly, by ensuring they use a method to help them minimise sign errors. The better solutions exhibited a clear method of factorisation for the solution of the quadratic, and a diagram to help identify the correct inequality by ordering the critical values on a number line correctly.

Question 2

A clear plan was the key to making good progress in this question. Although rarely seen, a sketch of the curve and the line might have helped candidates develop a plan for responding to this question.

The most efficient method was to differentiate the curve and substitute $x = 2$ to obtain the gradient of the tangent in terms of a, and then equate this to 7, the gradient from the given equation of the tangent $y = 7x + b$. This was then solved to find $a = 3$. The next step was to obtain y using $x = 2$ and their a in the equation of the curve, then find b using $y = 7x + b$. Candidates who chose this approach made the fewest mistakes and earned their marks efficiently. Some candidates did not spot the connection $\frac{\mathrm{d}y}{\mathrm{d}x} = 4a - 5 = 7$ and made no progress beyond an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

A less popular method was using the fact that the tangent and curve touch at just one point. Candidates using this approach equated the line to the curve and then rearranged their expression to a three-term quadratic. Realising there was a unique point of contact and solving $b^2 - 4ac = 0$ gave the equation $144 - 8a + 4ab = 0$. A second equation was then found most popularly by substituting $x = 2$ into $y = ax^2 - 5x + 2$ and $y = 7x + b$ and equating them to get the equation $4a - b = 22$. Solution of their two equations simultaneously gave values for a and b. Many candidates found the equation $4a - b = 22$ initially, and then made no further progress down this route. This scored no marks, unless they also attempted the discriminant of their three-term quadratic in x.

Candidates sometimes used circular methods, making a or b the subject and then substituting back into the same equation, often with slips so that a value was erroneously found. Other common misconceptions were to let $y = 0$, or putting the gradient = 0 after finding an expression in terms of a for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Question 3

Almost all candidates gained at least the first mark, for reaching $\lg\,(2x-1)(x+2)) = 2 - \lg 4$. Most candidates saw the need to change the integer 2 in the equation into a logarithm so they were successful in the use of the laws of logarithms to simplify both sides of the equation into a single logarithm. Again, the majority of the candidates were able to equate the argument on both sides of the equation free of logarithms to arrive at the required quadratic equation. Many competently solved the resultant quadratic equation correctly by factorising or by the use of the quadratic formula. Some candidates omitted to check if both values obtained satisfied the equation and incorrectly included the invalid negative solution.

Some candidates were not able to deal with the logarithms and gained no credit when they incorrectly split $\lg(2x–1)$ into $\lg 2x – \lg 1$, etc., or incorrectly combined $\lg(2x-1)+\lg(x+2)$ as $\lg\,(2x-1+x+2)$.

Question 4

(a) This was a very accessible part, with most candidates scoring full marks. The most common misconception was to substitute $x = 2$ into $0 = kx + 6$ and/or $0 = x^3 – 4x^2 + 3kx + 2$ and solve for k.

(b) Many candidates arrived at the correct cubic $x^3 – 4x^2 + 6x – 4 = 0$ with their value of k from **part (a)**, by equating the curve to the line and rearranging the resulting expression to equal zero. It was not uncommon to see this work done in **part (a)** and copied into this part. Factorisation of the cubic was well-executed by most, often performed by algebraic long division by $(x – 2)$. The last mark for showing that there are no real roots for $x^2– 2x + 2$, so only one root of the cubic (at $x = 2$) was more elusive. It was not sufficient to say 'no solution' or 'does not factorise' etc. Candidates were expected to evaluate the discriminant and compare it with zero, in this case $(– 2)^2– 4(1)(2) < 0$ implying there were no real roots and hence only one root for the cubic. When the quadratic formula was used candidates needed to clearly identify the discriminant being negative. This is an example of where the instructions in the question, 'Show that…', ought to guide the candidate to offer a complete and reasoned solution. Some candidates used their calculators to find the three roots of the cubic including the two complex roots $x = 2$, $x = 1 + i$ and $x = 1 – i$. This use of the calculator was not acceptable here. Others went on to factorise their quadratic incorrectly and still declare that there was only one root.

Question 5

(a) Most candidates realised the need to work through steps sequentially from the left-hand side (LHS) expression to the right-hand side expression. The most common method employed was to write the LHS terms over the common denominator $(1-\sin x)\cos x$, although some omitted the brackets which occasionally resulted in errors when these were not recovered in subsequent steps. The numerator was usually expanded correctly and then correct use of the Pythagorean identity in the numerator gave $2-2\sin x$. The final mark was for a convincing finish, via the factorised forms of the numerator and the denominator, cancelling of the common factor, and finishing by demonstrating their knowledge of the relationship between cos *x* and sec *x*. Errors seen included $(1-\sin x)^2 = 1-\sin^2 x$, $1-\sin x = \cos x$, and the incorrect notation $\sin x^2$. Much less common was the alternative method, involving knowledge of the difference of two squares, although it was often executed well when it was seen.

(b) Most candidates realised the instruction 'Hence …' meant that they needed to use **part (a)**. They converted $2\sec\frac{\theta}{2} = 8\cos^2\frac{\theta}{2}$ to $\cos^3\frac{\theta}{2} = \frac{1}{4}$, then took the cube root. Many candidates, however, made errors in the algebra, reaching incorrect equations such as $\cos\frac{\theta}{2} = \frac{1}{4}$. Some candidates disregarded the 'Hence…' instruction and embarked on lengthy and fruitless manipulations of the equation as given in the question. Those who got as far as $\cos\frac{\theta}{2} = \sqrt[3]{\frac{1}{4}}$ often obtained at least the correct solution $\theta = 101.9^\circ$, although extra incorrect solutions were frequently included. Some prematurely rounded to $\pm 102^\circ$. Others reached $\frac{\theta}{2} = 50.9^\circ$ and then halved to give $\theta = 25.5^\circ$.

Question 6

This was an exceptionally well-answered question with a high proportion of candidates gaining full marks with correct values of *a*, *b* and *c*, which indicated a correct expansion method had been used. Of those candidates not scoring full marks, the most common error was in not applying the powers to the '*a*' as well as the '*x*' in the third and fourth terms. Those who used brackets correctly were much less likely to make this error.

Question 7

The majority of candidates were successful in the use of the combination formula to write *n*! in an expanded form. Most candidates were also able to simplify their equation by recognising and cancelling out common factors on both sides to arrive at the correct quadratic equation. In addition, most then correctly factorised or used the quadratic formula to solve their quadratic equation correctly to arrive at the correct values for *n*. Finally, most knew the value of *n* cannot be a negative number so were successful in the choice of the final value for *n*, and so were able to find the value of the given expression, ${}^nC_8$. Some candidates had difficulty in moving from the factorial form to the correct factorial-free form. The small number of candidates who chose to use trial and improvement with their calculators were only able to access the final mark, as method was required to be shown, as instructed in the rubric on the front of the examination paper.

Question 8

Only a minority of candidates scored full marks in this question. It was evident that many candidates did not know how to deal with vectors in this context.

(a) There appeared to be confusion about what a position and a direction vector are in both this part and the next. A correct response was to find the unit direction vector, multiply it by 26*t* and add it to the position vector. However, many candidates were finding (12**i** + 5**j**) + (3**i** – 2**j**) = (15**i** + 3**j**) or $(12\mathbf{i}+5\mathbf{j})-(3\mathbf{i}-2\mathbf{j}) = (9\mathbf{i}+7\mathbf{j})$ or finding the modulus of those resultant vectors or the modulus of the position vector (3**i** – 2**j**). Some knew that they needed to find the modulus of the direction vector which gained them the first mark.

(b) This proved to be a more challenging question for many candidates. It was common to see the angle α = 36.86… found and no further progress. Others went on to find the vertical and horizontal components, e.g. adjacent = 20cos(36.86…) and opposite = 20sin(36.86…) but either did not put them together as a vector or confused x and y values effectively using 3**i** + 4**j** as their direction vector. However, those who just made this mistake in forming the direction vector but understood the correct method for the overall question went on to get the position vector of B to be $(67\mathbf{i}-18\mathbf{j})+t(12\mathbf{i}+16\mathbf{j})$ and were able to gain two marks of the four available. Similar confusions as in **part (a)** were seen with candidates using the position vector (67**i** – 18**j**) as a direction vector in their final answer, which gained no credit. Answers of (67**i** – 18**j**) + t(20cos36.86**i** + 20sin36.86**j**) did not score the final mark until this was written as (67**i** – 18**j**) + t(16**i** + 12**j**).

(c) The first step in the method for this part was to equate x or y components of their two vectors from the previous parts of the question, providing those components were of the form of $a + bt = c + dt$. Some candidates seemed unfamiliar with the idea of equating scalars. It was also acceptable to rearrange the vectors to the form e.g. t(8**i** – 2**j**) = (64**i** – 16**j**) and find t by dividing the modulus of the two vectors. Those who performed one of these methods correctly with correct equations usually went on to get $t = 8$ and the correct position vector 195**i** + 78**j** scoring full marks. Some candidates with both **parts (a)** and **(b)** fully correct got as far as finding $t = 8$ and did not go on to find the position vector, a slip which might have been avoided if the question had been re-read to check that it had been answered fully.

Question 9

The first two parts of this question were much more accessible than the second two parts, which proved to be challenging.

(a) This question was answered well, with many candidates gaining both marks. It was fairly common to award one mark for the correct differentiation of e^{-2x}, by implication in an incorrect expression. Another common situation was to award one mark for the product ruled applied correctly with an incorrect derivative of e^{-2x}. When this occurred, detailed setting out of their method allowed correct application of the product rule to be more easily identified. Some candidates re-wrote the expression as a quotient and used the quotient rule here, although this route was more likely to result in errors being made.

(b) Nearly all candidates realised the needed to put their derivative from **part (a)** equal to zero. However, if their **part (a)** was incorrect and not of similar form, candidates could not make progress here. A significant number of candidates were unable to deal with the e term correctly and ended up with two x values. Others tried to use logarithms to solve the equation. Although a few rare responses managed to successfully use a logarithm method, most attempts incorrectly used log laws. Some realised that e to a power cannot equal 0, so successfully divided through by e^{-2x}, rather than factorising their expression.

(c) This proved to be a challenge for many. Few candidates made the link successfully to **part (a)**, and despite the instruction to use their answer to **part (a)**, many attempted to integrate directly. This was done incorrectly in most cases, although a very small minority of candidates were successful in applying integration by parts. Of those candidates that were successful in relating this question to **part (a)**, a significant number of these gained two out of three marks because they omitted the constant of integration. Partial marks were often scored by those working in k, or working with an incorrect, but consistent value for k.

(d) Candidates needed to use a correct answer from **part (c)** to score marks here, so those with 2 or 3 marks in **part (c)** generally scored at least one mark here. The most common error for those able to access this part was to go directly from substitution to a decimal final answer, missing or misunderstanding the instruction to find the 'exact value of …' the expression.

Question 10

(a) This arithmetic sequence question, requiring candidates to find the first term and common difference, was answered extremely well with most candidates scoring full marks. The majority used the standard sum and n^{th} term formulae and then solved their equations in a and d simultaneously. A small number of candidates found the answer by inspection, manually summing individual terms to eight terms, although this method was prone to slips and is not advised. The most common error made was to misinterpret the information in the question and use an incorrect formula, e.g. using the sum formula twice; or to mix up the values given in the question and assign them to the incorrect aspects of the formulae.

(b) In contrast to the previous part, it was rare to see a correct answer in this part. The information in the question was often misunderstood, with few candidates appreciating that they needed to subtract S_{11} from S_{30}, or take the sum of 19 terms beginning with $T_{12} = 37$. Common wrong approaches were to find $S_{19} - S_{11}$, $S_{19} - S_{12}$ or $S_{31} - S_{12}$, or to add the eight terms from T_{12} to T_{19}. A minority of candidates took the route of calculating all 19 terms, starting with the twelfth term, but this method was prone to slips and is not advised.

Question 11

(a) The majority of candidates gained full marks in this part. The most efficient solution used the route $\overrightarrow{OX} = \overrightarrow{OP} + \lambda\overrightarrow{PS}$, although $\overrightarrow{OX} = \overrightarrow{OS} - (1-\lambda)\overrightarrow{PS}$ was seen occasionally. The most common error in this part was to reverse the direction of the vectors. Partial marks for $\overrightarrow{PS}$ and/or the correct route were frequently earned, although detailed method was required to be seen in order to identify the route the candidate had used when their final answer was incorrect.

(b) Similarly, this was carried out competently by the majority of candidates. When no marks were scored here it was usually because there were sign errors made in forming $\overrightarrow{OQ}$ or the candidate had little idea of forming a route for $\overrightarrow{OQ}$. A correct unsimplified expression for $\overrightarrow{OQ}$ was partially credited.

(c) Many candidates with correct expressions for $\overrightarrow{OX}$ in **parts (a)** and **(b)** were able to gain full marks in this part. However, it was clear that some did not recognise that equating scalars was necessary here. Some responses never cleared the *a* and *b*, so did not find values for λ and μ. Those with incorrect expressions for $\overrightarrow{OX}$ in **parts (a)** and **(b)** were still able to gain partial credit if they equated coefficients correctly from their expressions, provided they contained *a*, *b*, λ and *a*, *b*, μ. A very small number of candidates restarted this question and did not use the two expressions for $\overrightarrow{OX}$ they had already found. This often proved to be laborious and prone to errors.

(d) Many candidates did not answer this question – either because they had no values for λ and μ in **part (c)** or, possibly, because they were unable to form a correct plan. Of those that attempted the question many did not spot the connection $\frac{OX}{OQ} = \mu$ so started to divide vectors for *OX* and *OQ* which usually did not conclude successfully. Only those obtaining a correct value for μ in **part (c)** were likely to score here.

(e) Many candidates did not answer this part, for similar reasons to those mentioned in **part (d)**. Of those who attempted the question this proved even more challenging than **part (d)** as the connection was more complicated, with $\frac{PX}{XS} = \frac{\lambda PS}{(1-\lambda)PS}$. Again candidates got lost in dividing vectors unsuccessfully and giving vectors as answers.