ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

The paper provided a good range of responses showing that many candidates understood the syllabus objectives and were able to apply them appropriately. Candidates appeared to have no timing issues. The vast majority of candidates made good use of the space provided on the paper and responses were generally straightforward to mark with a good standard of presentation.

General comments

Candidates should read each question carefully and pay attention to the required form of the answer. Sometimes additional care could have prevented slips, especially when manipulating expressions and equations, where errors in signs often occurred. There was also a frequent omission of essential brackets.

If it is stated in a question that a calculator must not be used, candidates must show all steps of their working.

Questions that start with the word 'Hence' require that the method employed should use the result from the previous part.

When integration was necessary, it was not always clear at which point an integral was undertaken as the notation was often confused and the arbitrary constant was omitted.

Topics which elicited strong responses included completing the square, the factor and remainder theorems and the equation of a perpendicular bisector. Topics which elicited the weakest responses were velocity-time graphs, combinations and permutations and arithmetic and geometric progression.

Comments on specific questions

Question 1

(a) This question was well answered, with many responses gaining full credit. The most common error was having a denominator of 25 on the *c* value or having an incorrect expression in the brackets, for example $5(x^2 - \frac{7}{5})$ or $5(x^2 - \frac{7}{5}x)$.

(b) As this question part started with the word 'Hence', candidates were required to use *their* result from the previous part, and not re-start by using differentiation.

(c) The vast majority of candidates knew the shape of a quadratic curve and correctly plotted the *x*- and *y*-intercepts. Most responses correctly identified the minimum point of the quadratic curve as (1.4, –1.8). However, some candidates did not know how to deal with the modulus function and were not able to gain credit because they did not produce a correct shape for the curve.

(d) This part of the question was a follow through from candidates' responses in **part a**. However, the question proved challenging and many responses did not gain credit. Some candidates mistakenly thought they needed to factorise the quadratics. There were a lot of errors in the inequality signs ($\leqslant$ instead of $<$).

Question 2

(a) This question was very well answered, with many responses gaining full credit. Most candidates understood that the notation required them to differentiate twice. The most common error, which did not occur frequently, was not to differentiate at all, but to substitute the value of $\frac{1}{2}$ into the original polynomial.

(b) The value of *a* was given in **part (a)** and so candidates needed to find the integers *b* and *c* in the polynomial. Most candidates correctly attempted to apply the remainder and factor theorems and progressed to find values for *b* and *c*. A small proportion substituted $x = -1$, but equated the expression found to 0 rather than 7. Some candidates made errors with fractions or with negative numbers.

(c) If the values of *b* and *c* found in **part (b)** were incorrect, this part became far more challenging. With correct values of *b* and *c*, several methods were available. The most common approach was algebraic division, and this was also the most successful. Synthetic division was also popular but was often not completed correctly. A few candidates chose to compare coefficients. This generally proved successful but required a significant amount of correct algebra. Some responses did not show the factor and quadratic as a product as required, so did not gain full credit.

(d) If the quadratic found in **part (c)** was correct, most candidates correctly factorised the quadratic and listed all three factors as products. A significant proportion worked backwards from roots found on their calculators, but incorrectly included fractions in the factors.

Question 3

(a) This question was well answered. Many candidates found the correct value of *x*. Candidates were very successful at finding the gradient of *AB* and its midpoint. However, some were then unable to find the perpendicular gradient or make further progress. When attempting to find the gradient of the perpendicular line, some candidates only changed the sign of *AB*'s gradient instead of using the negative reciprocal.

(b) This question proved very challenging. The most common incorrect answer was (4, 9). Some candidates attempted to answer this question using vectors, but they were rarely successful.

Question 4

(a) This question proved challenging, with few candidates recognising that they needed to find the area under the graph. Some candidates were unable to separate the area to find the value of *V*, with the most common error being to try to find the area of a big triangle from 20 to 40.

(b) A significant minority produced a completely correct solution. The most common error was to divide or multiply by 35 instead of dividing by –15.

Question 5

(a) Candidates must be aware of the implications of the instruction not to use a calculator in this question. While most candidates knew how to answer both parts of **Question 5**, the level of detailed working required in answers was not always present. Many candidates clearly knew how to apply the cosine rule correctly to solve this part. However, often candidates did not provide enough evidence to show that they had completed the work without a calculator. Some candidates treated the triangle as a right-angled triangle and were unable to gain credit.

(b) Many candidates knew how to use the sine rule, but many responses did not contain sufficient evidence that they had been completed without the use of calculator. As in **part (a)**, some candidates treated the triangle as a right-angled triangle and did not gain credit.

Question 6

(a) Candidates used many different approaches to develop their proofs. The most common and most successful was to rewrite each of the terms in terms of sin and cos. Almost all knew the conversion for tan and cot but some thought sec to be the reciprocal of sin rather than cos. Having converted the terms, most candidates were successful in simplifying the expression, using appropriate trigonometric rules to reach the required outcome. Those who chose to work with tan in the denominator were slightly less successful but often made some good progress. A common mistake was assuming that tan + 1 = sec.

(b) This question proved challenging. The question started with 'Hence' so candidates who did not use the identity given in **part (a)** made no progress. Common errors included using only the positive square root of $\frac{1}{2}$, not square rooting $\frac{1}{2}$ but using $\sin\phi = \frac{1}{2}$, and not multiplying by 3 after finding $\sin^{-1}\left(\frac{\phi}{3}\right)$.

Question 7

(a)(i) This question was very well answered. The most common error throughout the question was using permutations rather than combinations.

(ii) This proved a challenging question. Candidates tended to be more successful when they considered 'with family' and 'without family' separately. Few candidates gained full credit, but many gained partial credit for having considered 'with family' or 'without family' or both.

(b) This question proved challenging, with many responses demonstrating difficulty manipulating factorials. The most common error was to use combinations rather than permutations, even though the notation for permutations was in the question. Simplifying factorials proved challenging and few responses reached the final quadratic. Written working for this question part was often unstructured and challenging to follow.

Question 8

(a) Most responses correctly stated the derivative of the function, although some omitted the 3 multiplier when differentiating $(3x-4)^{\frac{1}{3}}$. Many responses gained partial credit for correctly applying the quotient rule, but few successfully completed the algebraic manipulation to put the derivative in the required form $\frac{dy}{dx} = \frac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$.

(b) This question part proved very challenging. Although some concise, neat and accurate solutions were seen, few candidates realised that when $\frac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}} = 0$, they needed to consider $Ax + B = 0$.

Question 9

(a) Some complete responses to this question were seen. However, some were unable to work with the natural logs. For those who correctly found the common difference of 3ln$q$, not all managed to eliminate natural logs and leave a straightforward quadratic to solve, in order to find n.

(b) Most responses correctly identified the common ratio as P^{-2x}, although this was sometimes kept unsimplified. After finding the initial form of the expression for the nth term, some candidates made sign errors when expanding $P^{-2x(n-1)}$, while others included the (n – 1) but as a product rather than as a power. In most cases, those who correctly expanded the powers were able to give a final correct answer in the required form.

(c) The majority of candidates correctly found the common ratio of $\frac{4}{3}\cos^2 3\theta$. A common error was to then omit the index 2 in the next stage of their working.

Very few candidates found both required values for 3θ, and even fewer gave the final inequality in the correct form, with < being used inside $\leqslant$.

Question 10

(a) This question was very well answered, with many responses gaining full credit. Candidates demonstrated confidence using the product rule and differentiating logarithms.

(b) This question proved challenging and was omitted by many candidates. Some candidates gained partial credit for achieving a multiple of $k(3x+1)^2\ln(3x+1)$ or $\frac{1}{3}\times\frac{1}{2}\times(3x+1)^2$. A few candidates reached a complete answer but omitted to include +c.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

It is essential that candidates follow the instructions concerning non-use of calculators when given and show sufficient detail in their solutions as evidence that a calculator has not been used.

Candidates should work to at least 4 significant figures when dealing with intermediate calculations so that their final 3 significant figure answers are accurate.

Candidates should read the demands of the question carefully and ensure that they have answered the question fully and used all the information that they have been given.

Candidates are reminded of the rubric on the front of the examination paper which states that all necessary working should be shown clearly and that no marks will be given for unsupported answers from a calculator.

General comments

There appeared to be no timing issues and most candidates had sufficient space to write their responses. Candidates generally made appropriate use of the blank page at the end of the paper or additional pages to ensure that solutions were clearly presented after errors had been made previously. Most candidates alerted Examiners to the fact that solutions were elsewhere.

The questions provided differentiation so that all candidates were able to show what they had learned.

Comments on specific questions

Question 1

Most candidates were able to identify the amplitude of the given curve as the value needed for the unknown constant a, and to give a correct answer of 4. The value of the unknown constant c was more challenging, with responses of 2 rather than –2 being quite common. Many candidates were unable to use the information given in the graph to determine the period of the graph and hence the value of the unknown constant b. Responses of $\frac{8}{3}$, rather than the correct $\frac{3}{8}$, were quite common.

Question 2

It was important that candidates took note of the fact that the use of a calculator was not permitted in this question. Sufficient evidence of working needed to be provided in the form of intermediate calculations.

It was expected that the given equation initially be written as a 3-term quadratic equated to zero.

The most common method of solution of this quadratic equation was the quadratic formula. Candidates needed to show their calculation of the value of the discriminant by showing the expansion of brackets and the subsequent simplification. Use of this value could be either calculated separately and then substituted into the quadratic formula or calculated as part of the quadratic formula. Having gained two solutions with a surd term in the denominator, sufficient working was required to show evidence of rationalisation. Expansion and simplification of relevant terms in the denominator of either of the two solutions was sufficient to gain a method mark for this process.

Some candidates factorised the quadratic expression gained after re-arrangement. This was usually very successful and resulted in one solution in the required form being obtained without further simplification. The second solution needed to be rationalised and so the necessary steps needed to be seen.

Some candidates used their calculators to simplify terms and for rationalisation. In some cases, candidates did not gain any marks because of clear calculator use, while others did not gain the last 2 marks as rationalisation was not shown.

Question 3

(a) Most candidates recognised that the product of the factors $(x+2)(x-1)(x-4)$ was part of the solution. Many candidates went on to deduce that a factor of 3 was also involved, but fewer realised that there were two possible equations for *y*.

Candidates were instructed to find the possible expressions in factorised form. However, some candidates used an identity and found the expression or expressions for y in expanded out form.

(b) Most candidates found the critical values of 3 and $\frac{1}{9}$. The methods used were equally popular.

Some candidates found the two possible linear equations resulting from dealing with the modulus correctly. Some candidates had extra critical values as a result of not applying the modulus correctly. Other candidates chose to form a quadratic equation using the squaring method.

The final inequality proved to be challenging for some candidates. Candidates who had used the quadratic equation method tended to be more successful in finding the correct inequality. Many candidates did not combine their results or check them to ensure that a correct final inequality was given.

Question 4

(a) It was essential that radian measure was considered throughout this question as the angle AOB was given in radians. This part of the question required candidates to recall the relevant formulae, substitute in the given values and solve 2 simultaneous equations. This question was very well answered by candidates who worked in radians, but those who attempted to use degrees rarely made progress.

(b) There were two ways of finding the area of the shaded region. The first way was to find the length of *AC* and use the area of right-angled triangle *AOC*, from which the sector area was subtracted. The second way was to find the length of *OC* and use the sine rule for the area of triangle *AOC*, from which the sector area was subtracted. Most candidates realised that they already knew the area of the sector.

In some cases, accuracy marks were not gained due to premature approximation of the length of either *AC* or *OC*. Candidates should work to at least 4 significant figures when dealing with intermediate calculations.

Question 5

(a) Most candidates recognised this question as involving a disguised quadratic and were able to deal with it appropriately to obtain $6p^{\frac{2}{3}}-13p^{\frac{1}{3}}-5=0$ or an equivalent quadratic equation using a relevant substitution. Most candidates were then able to solve this quadratic equation and determine the possible values that *p* could take. Some candidates erroneously discarded the solution $p^{\frac{1}{3}}=-\frac{1}{3}$. Exact solutions were required for this question and most candidates did give their answers in exact form.

(b) Many correct solutions were seen, although some candidates were unable to deal with $2\lg(2x+5)$ correctly. Most candidates realised that base 10 logarithms were involved and were able to obtain a quadratic equation by using the laws of logarithms correctly. Subsequent solution of this quadratic equation was usually done correctly, and exact solutions were given.

Question 6

(a) Many candidates answered this part very well, realising that the trigonometric terms needed to be eliminated using trigonometric identities. The easiest way to do this was to make use of the identities $\cot^2\theta = \frac{1}{\tan^2\theta}$ and $\tan^2\theta + 1 = \sec^2\theta$. There were other acceptable methods, but errors often occurred in the simplification process.

(b) Some excellent solutions were seen. Most candidates were able to re-write the equation correctly as $\sin\left(2\phi + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2}$ and use a correct order of operations to find a solution of $-\frac{5\pi}{24}$. The number of subsequent possible solutions varied, with many omitting any positive solutions or omitting the solution $\frac{23\pi}{24}$.

Most candidates worked in radians in terms of π and did not resort to the use of degrees.

Question 7

(a) Many correct solutions were seen, with most candidates realising that they needed to reduce the number of available people as the groups were selected. However, many candidates, having obtained the correct combinations of ${}^{14}C_2$, ${}^{12}C_3$, ${}^{9}C_4$ added the combinations rather than multiplying them. Additional combinations which, when evaluated, were equal to 1, were ignored.

(b) (i) Many correct solutions were seen, showing a good understanding of the need to consider permutations.

(ii) Many correct solutions were seen, showing a good understanding of the need to consider permutations.

(iii) Most candidates realised that ${}^{8}P_4$ was an essential part of dealing with this problem. Candidates then needed to consider the number of ways the numbers could either start or end. It was hoped that candidates would obtain either $8 \times {}^{8}P_4$ (13440) or $15 \times {}^{8}P_4$ (25200). Many candidates were able to obtain at least one of these values. Others determined that there were 23 different ways that the numbers could start and finish. This was equally acceptable.

Question 8

(a) Responses were varied, with many candidates not realising that for the function to exist the logarithmic term needed to be greater than zero. It was important that candidates gave their responses in the correct form. For example, responses of $x > \frac{4}{3}$ or $a = \frac{4}{3}$ were acceptable, but $x = \frac{4}{3}$ or $a > \frac{4}{3}$ were not. Candidates must read the question carefully and ensure that their response makes sense in the context of the question.

(b) Again, responses were varied. Many candidates did not show familiarity with the range of values for logarithmic functions. It was also important that responses were given using the correct notation. For example, the expected answer was $f \in \mathbb{R}$ or a correct equivalent. A response of $x \in \mathbb{R}$ was not acceptable.

(c) Many candidates were not familiar with the basic shape of the logarithmic curve. Most candidates did not realise that the information concerning the number of solutions that the equation $f(x) = f^{-1}(x)$ has indicated that the graphs of each function intersected with the other at two distinct points. Many candidates sketched reasonable shapes but with either one or no points of intersection. Many sketches were in the first quadrant only and many others did not have the coordinates of the intersections with coordinate axes stated or marked on the graph. Candidates should read the demands of the question carefully and ensure that they have answered the question fully and used all the information that they have been given.

(d) (i) Most candidates were able to provide the correct simplified expression for the composite function.

(ii) Candidates were meant to use their response to **part (i)** so that they were solving the equation $f(4x-9)=0$. Most candidates did just this and proceeded to obtain a correct exact answer. However, some candidates miscopied their work as they proceeded through their solution, writing 13 as 3, or 12 as 2.

Question 9

This was a completely unstructured question designed to assess candidates' problem-solving skills. Many completely correct solutions were seen, as were partially correct solutions where arithmetic slips were the only error.

Candidates are reminded of the rubric on the front of the examination paper which states that all necessary working should be shown clearly and that no marks will be given for unsupported answers from a calculator. There were instances where candidates found the points of intersection of the straight line and the curve using the SOLVE function on their calculator. There were also instances where the integrand was written down, but no integration was seen, with a decimal answer only being given. Such solutions were unable to get the allocated method and accuracy marks for these parts.

There were several acceptable methods of solution depending on how the candidate decided to view the situation. Most candidates considered the area under the curve and the area under the straight line separately. Others used a subtraction method to obtain one integral, but sometimes sign errors caused inaccuracies. A few candidates chose to consider the area as the sum of a triangle and the area enclosed by the curve and the *y*-axis.

Question 10

This question proved the most challenging on the paper for most candidates, and completely correct solutions to all of its parts were relatively rare.

(a) (i) This part required candidates to express the sum of three terms of an arithmetic progression in a given form. Candidates found it challenging to determine the correct expression for the common difference, with $(2x+1)$ often being seen rather than the correct $3(2x+1)$. Candidates made it more complicated than necessary by using the common difference as $6x+3$ rather than spotting the factor of $(2x+1)$ and extracting that first. The ensuing algebra in multiplying out expressions such as $(n-1)(6x+3)$ proved challenging for many. This part was answered successfully in about half of the responses seen.

(ii) Candidates were required to use the answer to **part (a)** and equate it to the given expression. Manipulation of the resulting equation into a three-term quadratic was then required. Some candidates saw no problem in achieving a solution, from an incorrect **part (a)**, that was not a positive integer. Candidates should consider the validity of their answer in the context of the question. The number of terms in a sequence of this type will always be a positive integer.

Candidates should demonstrate their method of solution e.g. by factorisation or by using the quadratic formula, rather than just writing down solutions from a calculator. They can then gain credit for knowing how to solve the equation.

(iii) No progress could be made unless the answer to **(a)(ii)** was a positive integer and only a minority of candidates were able to proceed correctly to obtain the solution of -0.25.

(b) Many candidates were unable to write down a correct initial equation to work from. A common error was to write r^{n-1} as $3(2y+1)^{n-1}$ rather than $(3(2y+1))^{n-1}$ or $(6y+3)^{n-1}$. Following on from that, some correct work was seen to achieve an acceptable equation although many candidates expanded brackets at an early stage and found the subsequent algebra too difficult to proceed with.

(c) Fully correct solutions were rare, and this question part was very challenging for many candidates. Some candidates began writing down expressions for sums to infinity but did not make progress. Of those that realised the limits on the common ratio were relevant, many could not proceed any further than a statement like $-1 < 2\sin^2\theta < 1$. There were many protracted attempts to solve various trigonometric equations which proved fruitless. Answers were often seen that were just lists of values from these solutions rather than a range of values.

Of those who did progress to seeing the $\theta < \frac{\pi}{4}$ part of the solution, only a minority could then give the fully correct solution involving zero as well.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates are reminded that when a sketch has been asked for, they should check to ensure the relevant details have been included.

Candidates should also realise the importance of the word 'Hence' when used in the context of an examination paper.

Candidates should check that they have answered the question demands in full.

General comments

There appeared to be no timing issues and most candidates had sufficient space to write their responses.

The questions provided differentiation so that all candidates would be able to show what they had learned.

Comments on specific questions

Question 1

(a) Many candidates were unable to calculate the period of $3\tan\frac{\theta}{2}-3$. Answers of 4π rather than the expected 2π were common, showing that many candidates did not know that $\tan\theta$ has a period of π.

(b) Although many candidates were familiar with the basic shape of a tangent curve, most were unable to produce a correct sketch. Candidates who had identified a correct period in **part (a)** were usually more successful than those who had not. Candidates are reminded that when a sketch has been asked for, they should check that all relevant details have been included. In this question, it was expected that the coordinates of the points where the graph meets the axes were included either on the sketch itself or elsewhere in the answer space.

Question 2

(a) Many candidates were able to write the given expression in the form required.

(b) It is essential that candidates realise the importance of the word 'Hence' when used in the context of an examination paper. It means that the previous question part must be used in the solution of the current part. In this question, candidates were expected to use their completed square form answer from **part (a)** and write down the coordinates of the stationary point. Candidates should also appreciate the difference between 'write down', which implies no actual working is required, and 'calculate', which implies that some working is necessary. Candidates were not expected to use calculus to determine the required coordinates.

(c) Responses were varied. It was intended that candidates again make use of their completed square form from **part (a)** to help with the solution of the inequality. Forming a 3-term quadratic equation to find the critical values was acceptable, but simplification errors tended to be more common using this method.

Question 3

(a) Most candidates were able to obtain partial credit by applying the laws of logarithms correctly. Most problems occurred when attempting to deal with 3, with some candidates not recognising that 3 can be written as lg 1000.

(b) Many correct responses were seen, with candidates realising that a change of base and simplification to a 3-term quadratic equation in either base c or base 3 were needed.

Question 4

This unstructured question proved challenging for many candidates, although some completely correct solutions were seen. The question assessed candidates' problem-solving skills. A logical approach, with key steps identified, was needed. The coordinates of the points of intersection of the line and curve needed to be found first and then the equation of the perpendicular bisector of the line joining these two points. The value of *a* could then be calculated.

Question 5

(a) Many correct responses were seen. Any errors were usually sign errors rather than calculation errors.

(b) Many candidates were able to produce the correct solution by considering the product of the result from **part (a)** and the expansion of $\left(x^2 + \frac{2}{x^2}\right)^2$. Most errors tended to be sign errors, but some candidates did not consider all the terms that involved x^{16}.

Question 6

(a) This question assessed candidates' ability to recall the two basic formulae involved in the circular measure part of the syllabus. Although there were many correct solutions, some candidates found this question very challenging. The question gave the angle θ in radians; attempts to change to degrees and progress from there were not successful.

(b) This part proved very challenging, and few correct responses were seen. Candidates without a perimeter in the form $ar + \frac{b}{r}$ were unable to progress. Those candidates who did have a perimeter in the necessary form usually gained a method mark for differentiation. Candidates should check that they have answered the question demands in full. In this case, some candidates found the value of the radius required to give a stationary value and showed that this value for the radius gave a minimum value, but then omitted to find this minimum value.

Question 7

(a) Many candidates found the values of lnx and lny and proceeded to draw a correct straight-line graph.

(b) This question part was challenging for many candidates. It was necessary to use $\ln y = \ln A + b\ln x$, obtained from $y = Ax^b$ given in the stem of the question. Some candidates realised that they needed to find the gradient, but often did not know what this represented. A similar problem happened with the vertical intercept of the graph.

(c) Again, this question part was challenging for many candidates. Most did not realise that they could make use of their often correct straight-line graph, but instead attempted to use the values of *A* and *b* from **part (b)** which were usually incorrect.

Question 8

(a) This question part proved challenging, and few responses gave a correct vector.

(b) Few responses gained full credit in this part. Errors usually occurred from incorrect use of the given ratio.

(c) Many candidates misinterpreted or were unable to use the fact that $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$, so few correct responses were seen.

(e) Many candidates gained full credit in this part by making correct use of the fact that $\overrightarrow{BZ} = \mu b$.

(e) Many candidates omitted this part, but many of those who did attempt it were able to gain credit for equating their results from **parts (c)** and **(d)**, equating like vectors and then solving the resulting simultaneous equations.

Question 9

(a) Many candidates realised that they needed to differentiate the given expression for the displacement to find the velocity and then find the times when the velocity is zero. The most efficient way was to differentiate a product and factorise, making use of the fact that $(t-4)$ is a factor common of both terms. It was equally acceptable to expand out the expression for the displacement and then differentiate, although sign errors and arithmetic slips tended to be more common.

(b) Many candidates did not recognise that a sketch of part of a cubic curve was needed. With the results from **part (b)** being used as well, candidates should have been able to see that there was a minimum point at the point where $(t-4)$. It was essential that the intercepts with the axes were marked either on the sketch itself or stated in the answer space.

(c) Many candidates omitted this part and those that did attempt it were often unsuccessful. It was necessary to make use of the expression for the velocity obtained in **part (a)** and sketch an appropriate quadratic curve, again realising that the results of $t = 0$ and $t = 4$ also obtained in **part (a)** were important points to mark.

(d) (i) Again, quite a few candidates did not attempt this part and those that did often did not realise that they needed to differentiate their expression for the velocity obtained in **part (a)** to obtain an expression for the acceleration.

(ii) Although many candidates did not attempt this part, of those that did, many realised that the acceleration could be represented by a straight line, often correctly positioned. However, the intercepts with the axes were often incorrect or missing.

Question 10

(a) This question proved very challenging. It was essential that each step was shown clearly. The first required step was to write $\cos^4\theta - \sin^4\theta$ as $(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$ and then make use of $\cos^2\theta + \sin^2\theta = 1$.

(b) Candidates needed to realise that **part (a)** was to be made use of in **part (b)** so that the given equation could be written as $2\cos^2\frac{\phi}{3} = \frac{1}{2}$ and so the equations $\cos\frac{\phi}{3} = \pm\frac{1}{2}$ needed to be solved. Many candidates did not attempt this part and of those that did and made some progress, the solution of the equation $\cos\frac{\phi}{3} = -\frac{1}{2}$ was often omitted.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

Candidates need to show full and clear methods. Where the final answer to a question is required in a given form, full credit cannot be gained unless this is done. Candidates should be aware of the instructions on the front page of the examination paper and ensure that all answers are given to the accuracy indicated. In questions where the answer is given, fully explained solutions with all method steps shown are required. Repeating information given in the question alone cannot be credited. In such questions, candidates are encouraged to use consistent notation, such as using the same variable throughout their working, and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being awarded. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit, and answers with no working are unlikely to gain credit. Candidates should write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solution. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator. Where diagrams are required, candidates should include as much annotation as possible, and must include all information requested in the question.

General comments

Some responses were well presented, with clearly organised work that demonstrated wide-ranging mathematical skills. These responses were generally clear to follow. However, some responses contained a lot of unlinked working, often resulting in little or no credit being awarded.

Several questions were unstructured, and candidates needed to plan their method carefully. There were many strong solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required knowledge of standard methods were well answered. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Candidates need to read the questions carefully and ensure that, when a question requests the answer in a particular form, they give the answer in that form. This was particularly the case when the question stated that an exact answer was required. Some candidates could improve their solutions by keeping their working relevant.

Candidates must ensure that each part of a question is answered and that the answer is clearly identified. Candidates who use the blank page or an additional booklet should have made it clear which question is being answered. When this was not done, it was not always possible to connect work to a specific question.

When a question demands that a specific method is used, little or no credit can be awarded for the use of a different method. This is also the case when part of a question is linked to the previous part by the key word 'Hence…'

When using calculators, candidates should check that they are using the appropriate unit of angle measure. When a question indicates that a calculator should not be used, candidates must show clear and complete method steps. Where there is insufficient evidence of working, full credit will not be awarded.

When candidates are given, or have derived, an expression and need to substitute a value into that expression, the substitution should be shown. This is also the case when applying limits to an integral. Candidates should take care with the accuracy of values in their working. They must work to a greater degree of accuracy than that required by the final answer.

Any work that candidates wish to delete should be crossed through with a single line so that it can still be read. Sometimes such work may be marked, and it can only be marked when it is readable. Where a candidate feels they have made an error and they are unable to offer any alternative work, they are advised not to cross out their work. Rubbing work out and then writing over work can sometimes result in responses being difficult to read.

Comments on specific questions

Question 1

This question was well answered, and many fully correct solutions were seen. Some candidates found the gradient and intercept correctly but did not put them into the appropriate linear form. A common error was to add rather than multiply when removing the log from a correct linear equation. The question required the final answer to be given in a specific form, so it was not sufficient to only state the values of A and b.

Question 2

(a) This question required a sketch, and the majority of candidates recognised the need to draw a cosine graph over two cycles with the correct amplitude. A certain degree of accuracy was required, and some responses showed incorrect curvature, particularly at both ends. This spoiled an otherwise good response. In some responses, the symmetrical centre section was very skewed.

Some candidates drew more than two cycles across the domain. Most of these also had an incorrect amplitude. Some responses contained sine curves.

(b) This question was very well answered, with most candidates giving correct responses. The most common error was confusing the amplitude and period. Answers of 2.6 and 8 were also seen.

(c) As the function was defined in degrees, answers in radians were not accepted. Some responses confused the period and amplitude, and some candidates indicated that they needed to calculate $360 \div 6$ but gave 90 as the answer.

Question 3

(a) This proved to be a more challenging question, with many candidates unable to make the link between the given equation and the graph. Some recognised the x-intercepts as the required points but made sign errors in identifying b and c. Many candidates tried to expand the brackets of the equation for $\mathrm{h}(x)$ and made no progress. A very small number of candidates tried to give the answer in the factorised form but were often not successful.

(b) Most candidates correctly identified the critical values, and many fully correct solutions were seen. Some candidates did not correctly place the inequality signs, or only stated the critical values.

Question 4

(a) Many responses contained errors in application of the log rules. Many multiplied the 6 and 3 to give 18. The omission of brackets around $2y - 1$, when multiplying it by $\log 5$, caused additional difficulty for candidates who had otherwise successfully used the log rules. Many responses contained decimal approximations for log values early in the solutions, sometimes resulting in loss of accuracy in the final answer. Candidates should note the rubric on the front page of the examination paper which indicates that all necessary working should be shown. A correct answer without evidence of method was not credited as the equation could be solved using a suitable calculator.

(b) The majority of candidates used a substitution and worked with a quadratic in that substituted variable. Candidates were reasonably split between two methods, using either $u = \mathrm{e}^{2x}$ and

$u^2 - 4u + 3 = 0$ or $u = e^x$ and $u^4 - 4u^2 + 3 = 0$. These proved successful in nearly all cases. Most candidates successfully reversed their substitution back to values for e^{2x} or e^x. However, some did not recognise that $\ln 1$ was 0 and a few did not give their answers in exact form. Many candidates saw an equation involving indices and as a result assumed they needed to use logs. This invariably led to an error straight away and no further progress was made.

Question 5

This question was well answered. Most candidates recognised that this was a rate of change question requiring the use of the chain rule and applied it clearly and accurately. Poor notation using inappropriate variables occasionally led to errors. The accuracy of the final answer was sometimes insufficient with answers not given to 3 significant figures, as stated in the general rubric. Candidates should note that 0.053 is accurate to 3 decimal places. Some candidates made slips in their final step, simplifying $\frac{24}{144\pi}$ to $\frac{\pi}{6}$ or $\frac{1}{6}$. Candidates who did not recognise the intention of the question often substituted into the formula for volume and made no progress.

Question 6

(a) Candidates who found the vectors $\overrightarrow{QR}$ and $\overrightarrow{PR}$ correctly and substituted these into the given equation invariably solved the equations they derived correctly. Algebraic errors were very rare. Some candidates summed the position vectors instead of subtracting them. There were other longer methods applied, some of which led to a correct solution, but often these resulted in the use of inappropriate vectors.

(b) (i) This part proved challenging for many candidates. Of the candidates who made a good attempt, some made errors by giving their answer as column vectors and others by not evaluating the trigonometric ratios. There also needed to be more care taken with signs, as these were often reversed when the components for **i** and **j** were of the correct magnitude. Sometimes the components themselves were reversed.

(ii) Where incorrect values for **a** and **c** were found in **part (i)**, **part (ii)** was rarely correct. When **a** and **c** were correct, then this part was usually fully correct too. Many candidates knew how to find the magnitude of a vector and some of these candidates found an argument, although this did not always lead to a correct bearing.

Question 7

(a) This part was very well answered. The most common successful method was to find the difference between the area under the curve and the area of the triangle, having found the areas separately. Some candidates did not show the substitution of 5, the upper limit, and candidates are reminded of the need to show all working. A very small number of candidates did not realise the need to use integration and a few omitted to subtract the area of the triangle. Very rarely, the limits were reversed, and the negative sign was then ignored.

(b) (i) Some efficiently and fully correct solutions were seen. Occasionally solutions omitted to divide the first integral by 2. Candidates who found this question challenging were generally unable to deal with the first term. The common error was to assume a natural logarithm was involved. A significant number of responses had an incorrect sign in front of the sine term.

(ii) This part proved to be more challenging, as many candidates did not expand the numerator and were unable to make progress. These candidates often tried to apply the quotient rule in some way or integrated the numerator and denominator separately. When candidates expanded the numerator, they mostly went on to do well. The main errors tended to involve algebraic slips or not integrating $\frac{1}{2x}$ using natural logarithms.

Question 8

(a) (i) Some efficient and fully correct solutions were seen. Some candidates realised the need to state inequalities but were confused between domain and range. The domain should have been in terms of x and the range in terms of $f^{-1}(x)$ or y. These were often reversed. Some candidates found the question challenging and seemed to misunderstand what was required. They often did not state any inequalities, but instead attempted to manipulate the given function.

(ii) Many candidates solved the equation and found the correct value of 1.6. It was much rarer for candidates to realise that 0 was also a solution. This was usually because candidates divided by x or cancelled it early in their solution. Occasionally candidates had only the zero correct.

(iii) Many candidates were able to sketch a graph with an appropriate shape. Some solutions would have been improved by more careful positioning. The previous part of the question indicated that the curves should intersect at $x = 1.6$, which was approximately halfway along the given x-axis scale. Frequently, however, this point of intersection was too close to the right-hand side of the diagram and in some cases at $x = 3$ or beyond. There were also some curves which went from the y-axis to the x-axis. Some candidates omitted this part.

(b) (i) Most candidates demonstrated knowledge of the process for finding the inverse function. Errors usually involved algebraic slips, for example not taking the cube root of the whole function at the end, or square-rooting rather than cube-rooting. Some candidates had an incomplete process as they did not convert their final answer into a function of x. Some candidates made an error immediately, by removing the cube root on the right-hand side and replacing it with a cube root, rather than a cube, on the left.

(ii) There were fewer correct answers seen for this part, with a whole range of incorrect values seen. A single value was required, and several responses gave the answer as $k \geqslant 0$. Some candidates found $gh(x)$ in this part and omitted to state the value of k as required.

(iii) This part was generally well answered, with many correct functions seen. Sometimes correct work was spoilt by incorrect simplification, usually involving the sum of the cube root of each term.

Question 9

(a) This part was very well answered. Candidates usually applied the formulae for the area of a sector and the length of an arc for a correct, efficient solution. Some candidates worked with fractions of a circle, which occasionally resulted in the inclusion of an erroneous π in their value for θ.

(b) (i) Candidates who considered the most efficient approach were the most successful. The most efficient method was to apply the cosine ratio in the triangle OAD to find OD and to double it to find $2y\cos\alpha$. There were many other possible correct expressions. These were usually from longer methods which resulted in much more complicated expressions. These expressions were much more difficult to apply in the next part of the question.

(ii) Candidates who recognised the correct plan, to calculate the area of the triangle and subtract the area of the sector, made progress in this part of the question. Candidates who had more complex expressions from **part (i)** often found the algebraic manipulation required to be too challenging. Candidates who realised their expression did not give the required result often tried to restart, usually attempting to use the given answer in some way. It is not acceptable to assume the validity of the given answer and attempt to work backwards. These attempts were usually circular arguments involving expanding brackets and then removing common factors.

Question 10

This question was well answered by many candidates, with a high number of fully correct solutions seen. There were also some candidates who made little progress, either giving a full unsimplified expansion without identifying the key terms or showing no working of merit. Where candidates did identify the key terms, there were occasional miscalculations or slips in the indices for a and b. Some candidates did not show that $a^2b = -12$ at the expected point in the solution. Instead, they used the values of a and b to show this at the end of the solution. This was condoned on this occasion. However, because of that approach, some candidates omitted it altogether. Most candidates who did progress used the given statement in their solution. Other candidates solved the problem without using $a^2b = -12$. This led to working such as $2^{18} = 262144$ and was more likely to lead to errors in the algebra.

Question 11

Many different methods were attempted to solve this problem. Some candidates produced complete and efficient solutions. Others omitted the question or made no real progress in solving it. Eliminating x to form an equation in y and then using the discriminant to find k was the most popular and successful approach. Eliminating y was almost as popular but was more likely to lead to algebraic slips due to the fractions involved. Having found k, some candidates did not substitute it back in to find the coordinates. Due to the nature of the terms in the quadratic in y or x, some candidates were confused about which values to substitute in, particularly in realising that k^2 was a constant. Candidates who attempted to use calculus and to equate $\frac{dy}{dx}$ to the gradient rarely made any progress. No fully complete solution by this method, which could be approached in several ways, was seen. The key issue was that candidates were unable to deal with an equation containing both x and y.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a question indicates that a calculator must not be used, candidates need to show sufficient and convincing method to be credited. Also, when values are incorrect and the method used to find them is not seen, marks cannot be awarded. Full method should always be shown. For example, candidates should not rely on calculators to solve quadratic equations or to work out the values of derivatives or integrals for particular values. Candidates should use their calculators as efficient checking tools and should understand the need to find and correct errors when they arise. When a specific method of solution is required in the question, it is expected that candidates will answer using that method and the use of other methods is unlikely to result in credit being awarded. The ability to carry out algebraic manipulation is expected, and candidates must make sure that their skills in this area are secure.

General comments

Candidates were often able to demonstrate secure knowledge and understanding of mathematical techniques and were able to apply these to solve problems. Some questions required the use of many skills. This was seen in **Questions 3(b), 8** and **10** in this examination.

When a question uses the key phrase 'Show that', showing a clear and complete method for every step is essential. The marks are awarded for complete justification of the result being shown. This was required in **Questions 3(b)** and **5(a)** for example.

When a part of a question includes the key word 'Hence...', it is expected that candidates use a previously found result or results to answer the current part of the question. This indicates that a specific skill or method was being assessed. This was seen in **Questions 1(b)(ii), 7(c)(ii)** and **10**.

Some candidates gave neat and clear responses, making their work easy to follow. These candidates were much less likely to make a miscopying error. Other candidates could have presented their work in a more organised way. Presentation was often poor in **Questions 2(a)**, **3(b), 4** and **7(c)(ii)**, for example. Work in these questions was sometimes hard to follow. This was especially the case when candidates first wrote in pencil and then wrote over the top of this using pen.

It was very helpful when candidates indicated that a solution continued or had been written elsewhere in their script, when the need arose, and also indicated where that continuation or solution could be found.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

(a) Most candidates found this to be an accessible start to the examination paper. A good number of candidates gained full credit. Almost all candidates simplified to the correct quadratic expression and most provided a correct method for finding the critical values or the correct critical values themselves. A few candidates gave an incorrect final inequality. Common incorrect inequalities were $1 < x < 4$ or $1 > x > 4$ or $x > 1$ and $x > 4$ or $x > 4$ only. A few other candidates stated the critical values only.

(b) (i) Again, a good number of fully correct answers were seen. Candidates needed to give their answer in the form required, with integer values of *a*, *b* and *c*. Candidates finding the values of *a*, *b* and *c* by solving equations did not gain full credit if they did not state the correct final form. The most common errors were incorrect values of *c*, with -4 or $\frac{4}{3}$ or 12 or 16 seen. A few candidates omitted to include the value of *a* and stated an answer of $(x-2)^2+4$. The weakest responses often offered $3(x-4)^2+16$ or $3(x-6)^2-20$ or similar or attempted to solve $3x^2-12x+16=0$.

(ii) Candidates needed to interpret the completed square form to answer this part of the question. This was a strict follow through of their form from **part (i)**. As this was the case, candidates needed to state 'y = *their* value of c' as the solution in this part. Therefore, candidates who checked the answer using calculus and stated an equation from that work were credited only if their equation also corresponded to **part (i)**. Some candidates stated only the coordinates of the minimum point. This did not gain credit.

Question 2

(a) A good number of fully correct solutions were seen to this part of the question. Candidates needed to manipulate the expression they were given. The simplest and most successful method of solution was to rewrite the second term as $\frac{x^{-2}}{8}$ before differentiating. A good number of candidates did this and went on to differentiate and solve correctly. A small number of candidates rewrote the second term as $8^{-1}x^{-2}$ then differentiated. This was also relatively successful. Some other candidates rewrote the second term as $(8x^2)^{-1}$. This required the use of the chain rule and results were more varied from this form. A few candidates combined the two terms into a single algebraic fraction. The success of this approach, which required the use of the quotient rule, was also variable. Some candidates omitted to state the negative value of *x*. Other candidates stated both *x*-coordinates but omitted to find the *y*-coordinates. A good proportion of candidates did attempt to find at least one *y*-coordinate. The question required more than one point to be found and this was emphasised in **part (b)**, so candidates who stated one point only might have improved if they had checked their working or read the question a little more carefully. The most common error prior to differentiating was to rewrite the second term as $8x^{-2}$ and state the derivative as $64x-16x^{-3}$. If this was the only error, candidates could earn special case marks for finding the exact stationary values from this form.

(b) It was necessary for candidates to use the second derivative test in this part of the question and any other test was not credited. A good number of candidates were able to state a sufficiently accurate second derivative. Candidates who had used the chain or quotient rules in **part (a)** often made this part of the question much more complicated than necessary as they had not simplified their first derivative. The accuracy marks for the test were dependent on the correct second derivative and also the correct value or values of *x* having been found in **part (a)**. Some candidates simply stated, or showed by correct evaluation, that the second derivative was positive. This was sufficient for one of the accuracy marks. To earn both accuracy marks, it was necessary for candidates to indicate at what value of *x* they were carrying out the test and to state that the points were minimum points. This needed to be done once, or twice without contradiction. A few candidates tested both points and stated that one point was a minimum and the other a maximum. These candidates might have improved if they had re-read the question as it should have been clear that both points had the same nature. Alternatively, for both accuracy marks, candidates could make a comment such as 'the second derivative is positive for all values of *x*' and state the points were minimum points. This was seen on occasion.

Question 3

(a) Most candidates chose the simplest method of solution, which was to substitute $x=-3$ into the cubic expression and show that the result was 0. Some candidates used long or synthetic division and this was acceptable if done correctly and a remainder of 0 was indicated in some way. A small number of candidates used factorisation by inspection. In a question such as this, where candidates were required to 'Show that…' $x+3$ was a factor, this was not acceptable unless this was multiplied out to show that it resulted in the original cubic expression.

(b) This was a non-calculator problem-solving question. Candidates needed to solve the equations of the line and the curve simultaneously and understand that $x+3$ was a factor of the resulting cubic expression. A good proportion of candidates did this and went on to find the correct quadratic factor. A significant number of candidates, however, used $x - 4$ or $2x - 1$ without sufficient justification of their use. It was not enough to make a statement such as 'using hit and trial' or 'using trial and error method' and then show that $x - 4$ or $2x - 1$ was a factor using the factor theorem, or similar, as this did not justify the initial selection of these factors. Very few candidates showed sufficient, evaluated trials using the factor theorem or products of values that were factors of 2 and 12 as justification for their use of these factors. There was, in fact, evidence to suggest that these candidates were using their calculator and working back. This was not permitted. Those candidates who did have a correct quadratic factor sometimes omitted to show the method of solution and simply stated the values of x. Again, this was penalised as this was a non-calculator question and it was expected that candidates show evidence of how they had found their values. Some candidates used the factor $x - \frac{1}{2}$. This form is generally stated by candidates who are working back from the roots found using their calculator and was not accepted. Algebraic factors should be written in the form $ax + b$ where a and b are integers.

Once candidates had found the three values of x, it was necessary to show that B was the mid-point of AC. Many candidates did this successfully by finding (x, y) pairs of coordinates for all three points and using the mid-point formula. This was the simplest approach, although other valid methods were acceptable. A common error when finding the coordinates was to state all the y-coordinates as 0. Those candidates who only considered the x-coordinates of the three points usually omitted the justification that B had to be the mid-point as the points were on the same straight line. A few candidates misinterpreted what needed to be done and, without first finding the coordinates of B, simply found the mid-point of AC. These candidates might have improved if they had re-read the question once they had found the three values of x. Other candidates who confused the points A, B and C might also have improved if they had re-read the question.

Question 4

A good proportion of fully correct responses were seen to this question assessing connected rates of change. Candidates needed to find an expression for $\frac{dy}{dx}$ in terms of x, the value of x when $y = 3$ and then apply the chain rule. A good number of candidates found the correct derivative in its simplest form, $\frac{dy}{dx} = -\sec^2(1-x)$. A common error was to state $\frac{dy}{dx} = \sec^2(1-x)$. A few candidates either omitted to differentiate 2 or did so incorrectly. A few other candidates omitted brackets around $1 - x$ or included a spurious constant '+ c'. Some candidates made the differentiation more complex than necessary as they rewrote the initial equation as $y = 2 + \frac{\sin(1-x)}{\cos(1-x)}$ or $y = 2 + \sin(1-x)\left(\cos(1-x)\right)^{-1}$ and applied the quotient or product rule. These approaches usually resulted in more than one sign error. The weakest responses:

- altered the argument of the trigonometric function in some way when differentiating or
- stated $\frac{dy}{dx} = \sec(1-x)$ or $\frac{dy}{dx} = \sec^2 x(1-x)$ or $\frac{dy}{dx} = \sec(1)$ or
- rewrote $\tan(1 - x)$ as $\tan 1 - \tan x$ before differentiating.

A good number of candidates found $1 - x = \frac{\pi}{4}$. Some candidates benefitted from subsequent incorrect work being ignored once this statement had been seen, as it was fairly common to see $1 - \frac{\pi}{4} = \frac{3\pi}{4}$. Sometimes, when evaluating $\tan^{-1}(1)$ candidates had their calculator in degree mode. Other candidates wrote the value as a decimal and rounded it to 2 significant figures. This loss of accuracy was not always recovered in later, more accurate, work. When attempting to solve $3 = 2 + \tan(1 - x)$, a few candidates offered $\tan(1 - x) = \frac{3}{2}$.

For the final stage in the solution, candidates needed to apply the correct chain rule and correctly write their derivative in terms of cosine or tangent for entry into the calculator. Candidates who had fully correct work to this point were able to show correct method using -2×0.04. Candidates whose only error was to write their derivative as an incorrect multiple of $\sec^2(1-x)$ were able to show correct method using '*their* multiple of $2' \times 0.04$. Candidates who had made other errors needed to show substitution of an angle into their derivative correctly written in terms of cosine or tangent and the multiplication by 0.04. Candidates who did not show the conversion to cosine or tangent and whose derivatives and/or angles were incorrect were not credited at this stage. Correct method is not determined from an incorrect value only. Candidates who wrote the answer –0.08 without any evidence of a derivative in terms of *x* were only able to gain credit for finding the value of 1 – *x* or *x*.

Question 5

(a) The first step was to write the given relationship in the form $\lg P = \lg A + T\lg b$ or an equivalent form with variables $\lg P$ and T. Many candidates were able to complete this step successfully. A few candidates wrote $\frac{P}{A} = b^T$ and then took logarithms to base *b*. This resulted in a change of base being needed and made the solution more complex than necessary. Once a correct equation in terms of $\lg P$ and T had been stated, candidates needed to compare it with $y = mx + c$, or a similar linear form. It was sufficient to write both equations in matching forms, and some candidates did this. It was also sufficient to write, for example, $y = \lg P$, $x = T$, $m = \lg b$ and $c = \lg A$, which was seen on occasion. Comparisons were often incomplete or simply omitted. Many candidates simply stated '$y = mx + c$' without indicating which elements corresponded, or made statements such as '$\lg A$ and $\lg b$ are constants' which was not sufficient without further comparison. The weakest responses often started with $\lg P = T\lg Ab$, used natural logarithms or attempted to find the equation of the line shown in **part (b)**.

(b) Candidates were more successful in this part of the question and a good number gained full credit. Some candidates used $\lg b = m$ and $\lg A = c$ successfully to find the values required. Other candidates formed the equation $\lg P = \frac{3}{7}T + 6$ and transformed this equation successfully to $P = 10^6 \times 10^{\frac{3}{7}T}$, which was acceptable. A few candidates used base e when anti-logging. A small number of candidates used points other than (0, 6) and (14, 12). This was totally unnecessary as accurate values had been given. It introduced errors and was only accepted if it resulted in sufficiently accurate values, which was rare.

(c) The simplest method of solution was to write $\lg P = 8$ and $\lg P = 9$ and use the graph to read off the values of T. A good number of candidates were able to do this. Some candidates used values other than 8 and 9, commonly 2 and 3 or 9 and 10. Use of such values was not accepted. Some candidates used the equations they had generated. When these were correct, full credit could be given. When the equations were incorrect, candidates were penalised. Candidates then needed to subtract the times they had found and state an acceptably accurate answer. Again, a good number of candidates were able to do this. The most common error, which was fairly frequently seen, was to misinterpret the question and to write the answer as an inequality, such as $4.6 \leqslant T \leqslant 7$, instead of a number of units of time. This was not condoned. An alternative approach was to use the gradient as the $\frac{\text{Change in } \lg P}{\text{Change in } T} = \frac{3}{7}$ with the change in $\lg P$ found from 9 – 8. This was used successfully when seen but was not common.

Question 6

(a) (i) This part of the question was well answered. As the question required the first three terms in the expansion of the expression given, it was expected that the terms be written as a sum. Candidates who listed the 3 terms separately were not given full credit. A few candidates stated a correct answer and then spoiled it by multiplying all the terms by 49, for example. This was also not given full credit. Candidates who included powers of 1 in their coefficients were not credited as it was necessary for the coefficient of each term to be simplified. A few candidates stated, for example, that $1^5 = 5$. A few other candidates stated $x^0 + \frac{5}{7}x^1 + \frac{10}{49}x^2$. This was accepted. However, candidates should be aware that the requirement in the question to write the terms in ascending powers of *x* does not mean that the constant 1 should be rewritten in this way. It simply indicates that they should be finding the sum of the constant, the term in x and the term in x^2 and not start with the term in x^5.

(ii) Solutions to this part of the question were more varied. Some fully correct solutions were seen. A few candidates were able to find a correct equation but made slips when solving. A few other candidates were able to write an equation using ${}^nC_1$ but not able to rewrite this as n. There was a great deal of incorrect algebraic manipulation in this question. It was not uncommon for candidates to confuse themselves when expanding to find the terms needed. This was especially the case when they wrote more than the first two terms from each expansion and then multiplied. The equation $7n = 89$ was fairly common and $89 = 7n + 5n + 5$ was also seen. There was a lot of incorrect manipulation of the 7, with $(7 + 7x)^n$ also commonly stated or used.

(b) A reasonable number of candidates were able to earn at least partial credit in this question. Care needed to be taken with signs and mathematical form. As a first step, candidates needed to state the correct required terms or the coefficients of these terms. As part of this step, they needed to show that they understood that $(-2)^4$ and $(-2)^2$ were part of those coefficients. Using at least one correct term, candidates who formed an equation with the coefficients they had found and the given proportion $\frac{5}{8}$ were able to gain further credit. It was important that candidates did not include powers of x at this stage, as it was the division of the coefficients that was equal to $\frac{5}{8}$ and not the division of the terms themselves. Some candidates persisted with powers of x in several subsequent steps of their solution or included a spurious x^2, so that $\frac{5}{8}$ became $\frac{5}{8}x^2$ on one side of the equation. A small number of candidates needed to take a little more care when forming the equation, as method errors were not condoned. A few other candidates made slips when simplifying the correct equation. Commonly these slips were in the manipulation of the term k^{-2}.

It was very common for the answer $k = 4$ to be stated, following an otherwise correct solution. Candidates who gave this answer needed to give more consideration to the information regarding the coefficient of x. Those candidates who wrote some working for this term were almost always successful in their choice of value.

Question 7

(a) This part of the question involved rewriting the square root as an index and applying the chain rule twice. A small number of candidates produced fully correct answers in the form required. A few candidates offered correct answers which were not in the fractional form required or not fully simplified, so could not gain full credit. Some candidates were able to differentiate $(4x - 2)^5$ correctly and were partially credited. Other candidates made a reasonable attempt to differentiate, such as $\frac{1}{2}\left(3+(4x-2)^5\right)^{-\frac{1}{2}} \times 5(4x-2)^4$. This also earned partial credit. The weakest responses typically offered a first step of $3^{\frac{1}{2}} + (4x-2)^{\frac{5}{2}}$ or rewrote the square root incorrectly, for example as the index $\frac{1}{3}$, or misinterpreted the notation used for the derivative and found the inverse function. A few candidates expanded $(4x - 2)^5$. This was unnecessary and there was a much greater risk of an error being made using this approach.

(b) In this part of the question, it was expected that candidates find a correct expression for $\frac{dy}{dx}$, find the value of x when $y = 10$ and use a correct small changes relationship to find the change in x. The majority of candidates used this approach. An alternative method, transforming the equation to make x the subject, finding an expression for $\frac{dx}{dy}$ and then use a correct small changes relationship, was also used by a few candidates. Both approaches were reasonably successful.

Some candidates solved $\frac{dy}{dx} = 10$ instead of $\frac{5x}{3x+2} = 10$ and made no further progress. Sign errors were fairly common in this question, both in the finding of the value of x and in the substitution of this value into the derivative. Those candidates who did find the correct value of x sometimes did not apply a correct small changes relationship. The incorrect calculations 0.01×62.5 and $62.5 \div 0.01$ were regularly seen. Similar errors were made using the alternative method. The weakest responses found the value of x for each of $y = 10$ and 10.01 and then subtracted these values of x. This was not accepted as the question clearly indicated that differentiation was to be used. The notation used by candidates was not always correct as sometimes it was notation appropriate for rates of change. On this occasion poor use of notation was condoned. However, it should be noted that confusion over notation can sometimes indicate that candidates are confused about methods.

(c) (i) This was a straightforward application of the product rule for differentiation and it was generally well answered.

(ii) In this part of the question, candidates needed to link the information found in **part (i)** to the integral under consideration. A reasonable number of candidates gained full credit. Some candidates earned partial credit for a correct answer with the constant of integration omitted, for example. A few candidates earned a mark for demonstrating that they understood the connection to **part (i)** in some way. This could be achieved either by suitable rearrangement of the relationship from **part (i)** so that the integration of the $3x^2\ln x$ could be carried out or by the rearrangement of the given integral so that the integration of $x^2 + 3x^2\ln x$ could be carried out. Some candidates were unable to manipulate the 6 correctly. Other candidates added terms such as x^2 to both sides when the integral of x^2 was appropriate. Others added terms to one side of an integral equation but subtracted them from the other. The weakest responses made no attempt to use **part (i)** in this part and integrated the elements of the product 'term by term'.

Question 8

To earn full credit in this question, candidates needed to work with exact values throughout. The question required them to find the exact coordinates of point P and the key word 'exact' was important. Some candidates produced neat and accurate solutions. Other candidates began well, finding the derivative, the value of y and values of the gradient of the tangent and the gradient of the normal, all in exact forms. Many also formed the equation of the normal using exact values. However, when they rearranged and found the value of x, they converted their exact values to decimals. Some candidates did not gain the final mark as they did not state both coordinates or they reversed the coordinates. This was not condoned. Some candidates stated an incorrect y-coordinate for P after using $y = 0$ correctly to find the x-coordinate.

The derivative needed to be seen in terms of x. Candidates who used their calculators to find the gradient of the tangent when $x = \frac{4\pi}{3}$ did not gain full credit.

The trigonometric functions needed to be evaluated for $x = \frac{4\pi}{3}$. This was necessary as candidates needed to demonstrate that they were working in radians at all stages. Candidates who did not evaluate the gradient of the tangent, but attempted to take the negative reciprocal of the trigonometric function with $x = \frac{4\pi}{3}$, often made errors. Some attempts to find the normal gradient either used the reciprocal **or** the negative of the gradient of the tangent but not both. A few candidates did not attempt to use the gradient of the normal but formed the equation of the tangent instead. These candidates may have improved if they had re-read the question. Weak responses offered a normal gradient of 4 following the gradient of the tangent stated as $-\frac{1}{4}$ from the derivative $-\frac{1}{4}\sin\frac{x}{4}$. Other weak responses did not find $y = \frac{1}{2}$ but instead misinterpreted the information in the question and used the point $\left(\frac{4\pi}{3}, 0\right)$ or stated this as their answer.

Question 9

At some point in their solution, candidates needed to integrate each of the expressions given for v in terms of t. A good number of candidates were able to integrate at least one of the expressions correctly. Many candidates were able to indicate that they were attempting to carry out a correct plan. Of the two possible approaches to solving this problem, using the sum of two appropriate definite integrals and equating that sum to 13.4 was by far the more successful. Candidates who tried to find expressions for the displacement functions very often confused themselves by mixing the two methods. This generally resulted in an incorrect plan. Other candidates who found a correct displacement function for $4 \leqslant t \leqslant k$ sometimes destroyed their plan by writing this function with $t = k$, but equating it to something other than 13.4. Candidates using definite integrals sometimes reversed limits, used the limits 0 and k only or subtracted the integrals rather than summing them. A few other candidates made errors when solving a correct equation to find the value of k. Also, on occasion, premature approximation errors were made. A decimal answer was expected as the value of k represented time. The weakest responses offered derivatives rather than integrals. Some candidates made no attempt to answer. Candidates who found the correct value of k from incorrect working were not credited. Candidates should be aware of the instruction on the front of the examination paper which indicates that all necessary working must be shown.

Question 10

Candidates were required to find two expression for $\overrightarrow{OE}$. One expression should have been in terms of **b**, **c** and λ and the other expression in terms of **b**, **c** and μ. A good number of candidates were able to find these expressions and some of these went on to produce fully correct solutions. Some candidates were able to find one correct expression only and this was usually in terms of **b**, **c** and λ. A small number of candidates earned partial credit for a sufficiently accurate vector, using correct directions, but which had some incorrect numerical scalars. Some candidates attempted to find two expressions using the same scalar. Other candidates found one expression for $\overrightarrow{OE}$ but then stated two expressions for $\overrightarrow{AE}$, for example, and then continued to use these to answer the question.

Providing candidates had vectors of a suitable form, further credit could be earned by equating components of the two expressions for $\overrightarrow{OE}$ they had found. The equations formed could not be credited until they were free of vectors and in terms of λ and μ only. A good number of candidates went on and found the correct values of λ and μ. Some candidates would have improved if they had been a little more careful with their mathematical form. On occasion, brackets were incorrectly expanded and so errors were made in equating components.

Not all candidates were able to interpret the value of μ in order to write down the correct ratio and a lot of solutions involved ratios of vectors. The ratio required, AE:EB, needed to be stated in the correct form, for example 4:3. Some candidates left their answers as the proportion $\frac{4}{3}$ which was not accepted. The ratio 3:4 was only condoned if it was identified as EB:AE.

A few candidates needed to take more care when reading the key information in the question. On occasion the information regarding C being the mid-point of OA was misinterpreted as $\overrightarrow{OC} = \frac{1}{2}\mathbf{c}$. Other candidates used $\overrightarrow{OB} = \mathbf{b}$ which was also clearly incorrect.

Candidates using non-vector methods to answer the question were not credited. Candidates who offered $\mu : 1 - \mu$ as the final ratio should be aware that, as this could be written down without finding any vectors at all, it was highly unlikely to be the answer required.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a specific method of solution is required in the question, it is expected that candidates will answer using that method and the use of other methods is unlikely to result in credit being awarded. For example, if an equation is required to be solved using a graphical method, a solution found using algebra will not be credited. Necessary method needs to be shown so that credit can be awarded. When values are incorrect and the method used to find them is not shown, marks cannot be awarded. For example, the substitution of limits into an integral should be shown so that the method can be seen and assessed. A calculator should not be used instead of showing this necessary method step. Candidates should ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus. Candidates should also be aware of how to use their calculator to check their solutions.

General comments

Some candidates were able to demonstrate good problem-solving skills and apply multi-technique approaches when needed. A good proportion of candidates were able to form and manipulate algebraic expressions when required. This was particularly evident in **Questions 2, 3, 9** and **10**. A few candidates found the simplification of algebraic factorials to be somewhat challenging. This was seen in **Question 5(b)(ii)**.

When a part of a question includes the key word 'Hence...', it is expected that candidates use a previously found result to answer the current part of the question. This indicates that a specific skill is being assessed. This was seen in **Questions 3(b)** and **5(b)(ii)** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

(a) The most efficient solution was to rewrite the modulus equation as $4x - 5 = \pm 7$ then solve the pair of linear equations formed. A reasonable proportion of candidates approached the solution in this way and these were more likely to find a correct pair of values. Some candidates attempted to solve $4x - 5 = 7$ and $4x + 5 = 7$ or attempted to solve only $4x - 5 = 7$. A few other candidates attempted to solve $4x - 5 = \pm \frac{1}{7}$. Some candidates omitted to write down an initial pair of unsimplified linear equations and made errors in the first step they wrote down. These candidates may have improved if they had understood the need to show all necessary method steps when solving equations. Some candidates made arithmetic slips when solving the linear equations. A few candidates attempted to square both sides and this was also reasonably successful. The most common error when using this approach was to forget to square 7.

(b) This question required candidates to draw and use a graph to solve a modulus inequality. Candidates needed to interpret the inequality to decide which graph was suitable for them to draw to be able to answer the question. Good solutions were accurate, ruled drawings of the graph of $y = |x - 5|$ for at least the section of that graph that intersected with the given graph of $y = |3x + 9|$. Some candidates offered fully correct solutions, interpreting the points of intersection as the critical values needed and writing the correct inequality. Some candidates were able to identify the correct critical values from the correct graph but then stated either an incorrect inequality, such as $-7 < x < 1$, or offered equations. A few candidates gave the answer $1 \leqslant x \leqslant 7$ and this may indicate some confusion when interpreting the absolute value function. Candidates whose solutions were from algebraic working alone, and not from use of a graph, did not gain credit.

Question 2

Candidates were not permitted to use a calculator in this question. Some fully correct responses were seen. It was important that full method was shown so that it was clear a calculator had not been used The two most common approaches were to first take the square root of $98x^{12}$ or to multiply the numerator and denominator by $3-\sqrt{2}$. Occasional slips were made when finding $\sqrt{98x^{12}}$ and the most common slip was to have an incorrect power of x or to write $98x^6$. Some candidates made arithmetic slips which resulted in a loss of accuracy. Other candidates did not follow the instruction to write the answer in the form $\left(a\sqrt{b}+c\right)x^d$, where a, b, c and d were integers.

Question 3

(a) Many candidates were able to differentiate correctly. Some of these candidates were also able to simplify the algebraic fraction formed by cancelling the common factor of x from the numerator and denominator. Some candidates factored out x but omitted to cancel it and this was not given full credit. Other candidates attempted to cancel but did so incorrectly. The weakest responses usually offered $\ln x^3 + \ln 3x^2$, or similar, before they attempted to differentiate.

(b) A small number of fully correct responses were seen. Some candidates gained partial credit for a correct expression in x with the constant of integration omitted. Weaker responses commonly either incorrectly attempted some algebraic manipulation before they integrated, in an attempt to write the algebraic fraction as powers of x, or applied the quotient rule for differentiation.

Question 4

(a) Almost all candidates were able to answer correctly, substituting $x = -4$ into the polynomial and evaluating to show the result was 0. The handful of incorrect or incomplete responses either

- substituted $x = 4$ instead of -4 or
- showed that $x + 4$ was a factor without explaining at some point that if $x = -4$ was a root then $x + 4$ was a factor, or similar, or
- stated only that $p(-4) = 0$ and did not show this using a calculation.

(b) Many candidates were able to gain partial credit by finding the correct quadratic factor using the linear factor $x + 4$. Some candidates went on to earn full credit by evaluating the discriminant of the quadratic to show that the result was negative. A few candidates applied the quadratic formula to $2x^2 + 3x + 10 = 0$ and indicated that this had no real solutions as the discriminant was negative. This was accepted. A few candidates applied the quadratic formula and simply said it had no solutions or was not possible. This did not gain full credit as the candidates had not indicated that the reason for this was that the discriminant was negative. Candidates who attempted to factorise and then said it could not be done did not gain credit.

Question 5

(a) (i) Candidates found this question challenging. Very few correct responses were seen. Partial credit could only be given if the full method was attempted. Some candidates offered a partially correct method that was incomplete. Many candidates used permutations or omitted the question.

(ii) Candidates were more successful in this part of the question with a good number offering a fully correct solution. Weaker responses either offered no solution at all or offered a sum of factorials or permutations or a product of combinations.

(b) (i) Some candidates were able to show algebraically that $(n-3)\times {}^nC_3$ was the same as $4\times {}^nC_4$. A few candidates were able to state the correct expression for either ${}^nC_3$ or ${}^nC_4$ but made errors in the algebraic manipulation required to prove the result. For example, some candidates treated the identity as an equation, moving terms from side to side. This was not accepted. The weakest responses made no attempt to answer or verified the result for a few values of *n*, which was not credited.

(ii) Some candidates were able to both derive the quadratic equation and solve it to state the correct single value of *n*. Some candidates needed to take more care with their presentation as necessary factorials or brackets were sometimes omitted. A few candidates omitted to derive the equation but did manage to solve it successfully and earned partial credit. Some candidates did not reject the negative value of *n* so could not gain full credit. Other candidates omitted the question.

Question 6

Some neat and fully correct responses to this question were seen. Candidates needed to find the coordinates of *A*, find an expression for the first derivative and evaluate this at the point $x = 0$. It was then expected that candidates form the equation of the tangent and use it to find the coordinates of *B*. This should then have been followed by a simple length calculation. Occasionally candidates made slips when recording their values or other slips in method, such as recording the *y*-coordinate of *B* as something non-zero. A few candidates used the equation of the normal rather than the equation of the tangent. These may have improved if they had identified the key word 'tangent' or if they had re-read the question when they had completed their solution. The weakest responses were unable to differentiate $5e^{2x} - 3$ correctly or used no calculus at all. When no calculus was offered, candidates usually found the coordinates of a second point on the curve, commonly the *x*-intercept of the curve, and found the length of the line joining *A* and this point. Some candidates omitted this question.

Question 7

A good proportion of candidates understood the need to use the chain rule and quotient rule in order to differentiate. Those candidates who took care with the accuracy and mathematical form of their answers were the most successful. This included using brackets where needed and making sure that their calculator was in radian mode. A few candidates omitted to show the substitution of 0.1 into their derivative and simply quoted a resulting value. These candidates may have improved if they were aware that use of a correct method cannot be determined from an incorrect value and that the full method should always be shown. A few candidates differentiated correctly but did not attempt to find the approximate change in *y*. The weakest responses made no use of the quotient rule and sometimes used $0.1 + h$ rather than 0.1 or made no use of *h* at all. Candidates who stated the correct answer following incorrect working did not receive credit. While the calculator is a good checking tool, it does not replace method and when an answer does not agree with the calculator value, candidates should find and correct the error in their working and not just alter their answer.

Question 8

(a) (i) Candidates found this part of the question to be very challenging. It was rare to see any solution that included –1 as a boundary value and, in most cases, 0 or 1 were also included.

(ii) Candidates were more successful in this part of the question. The simplest method of solution was to use $x = f\left(\frac{2\pi}{3}\right)$ and then evaluate. This was seen a few times and was usually successful. An alternative method was to find the inverse function, $\cos^{-1}\left(\frac{1}{x}\right)$ and solve $\cos^{-1}\left(\frac{1}{x}\right) = \frac{2\pi}{3}$. This was a little less successful as some candidates were either unable to find the correct inverse function or were unsure how to proceed correctly once they had formed the equation. Similarly, the handful of candidates who understood that $\sec^{-1}(x) = \frac{2\pi}{3}$ was also a correct form, were often unable to make any attempt to solve this. The weakest responses sometimes misinterpreted the inverse function notation as the reciprocal or manipulated $\sec x$ as if it were $\sec \times x$.

(iii) Candidates who understood the nature of composite functions were able to simply write down the domain. It was not necessary to find an expression for the composite function and those who attempted to do so in this part were commonly not successful. A common incorrect answer was that the domain was all real values of *x*. Some candidates made no attempt to answer.

(iv) The simplest method of solution in this part was to find the expression for the composite function, equate this to 1 and solve. Some candidates were able to do this successfully and those who had a correct answer to **part (iii)** usually offered a correct set of values and no others, as required. Some candidates omitted a value from their solution set or included an extra value outside the domain, which was not condoned for full credit. Some candidates forgot to include the negative square root when solving. Other candidates offered solutions in degrees. These candidates may have improved if they had understood that, when an angle is given in terms of π, this indicates that it has been measured in radians. A few candidates were unable to square sec x correctly, offering $\sec^2 x^2$ and other candidates used an incorrect order of composition or used a product of the functions f and g.

(b) A small number of fully correct and very neat solutions were seen. A few candidates were not sufficiently accurate at some stage, but still offered a reasonably good solution. Most candidates found this question somewhat challenging. It was not uncommon to see graphs drawn with incorrect curvature or as straight lines. Some graphs had no asymptotic behaviour. A good number of candidates did earn a mark for recalling that a function and its inverse are reflections of each other in the line $y = x$. Those who tried to find an expression for $h^{-1}(x)$ and draw it independently were rarely successful. In the weakest responses, candidates reflected their incorrect graphs of h in the x-axis or the y-axis or rotated them about the origin.

Question 9

(a) In this question candidates needed to rewrite the expression they were integrating as the sum of two terms which were powers of x. Those candidates that understood this usually made good progress. As candidates were given the answer, 36.6, it was necessary to show full and complete working to earn full credit. This included showing the complete substitution of the upper and lower limits into the integrated expression and the subtraction of these values. The weakest responses either integrated each part of the expression separately and stated $\dfrac{\left(\frac{x}{2}\right)^2 + 4x}{\dfrac{x^{\frac{4}{3}}}{\frac{4}{3}}}$ or similar, or did not integrate at all.

(b) Few candidates understood that the command word 'verify' allowed them to use the given information to find the x-coordinate by substituting $y = 0.1$ into both equations and solve them both for x. This was the simplest method of verification. Most candidates equated the expressions and formed a quadratic equation in x, which they then solved to show that $x = 2$. To complete the method, candidates then needed to substitute this value into either the equation of the line or the equation of the curve to verify $y = 0.1$. A small number of candidates were successful using this approach. Stating that $y = 0.1$ following the finding of $x = 2$ was not sufficient to be credited as the value of y had been given and full method had to be shown. Candidates who used this approach and did not reject the negative value of x did not receive full credit.

Following the verification of the y-coordinate of A, candidates needed to carry out a correct plan to find the plane area required. A good number of candidates made progress with this, and some offered fully correct solutions. When integrating to find the area under the curve, a good number of candidates used natural logarithms, although a few omitted brackets or the multiplier $\frac{1}{3}$. There were various methods of finding the area under the line, using integration or geometric shapes. Some candidates who used integration combined the partial fractions they were integrating into a compound fraction and attempted to integrate that. This was not a sensible plan. Some candidates omitted the work for the plane area completely. These candidates may have improved if they had re-read the question once they thought they had completed their solution.

Question 10

(a) (i) Most candidates followed the instruction in the question and found two expressions for r in terms of a and d. A good number of candidates were able to carry out the algebraic manipulation required to show the given result. Some candidates found one expression and a second for r^2 in terms of a

and d and this was condoned on this occasion. A few candidates used a instead of $a + d$ in their expressions and this misconception was not condoned. A few other candidates omitted to form an equation, or made algebraic slips after equating expressions.

(ii) A good number of candidates were able to find the correct value of r. This was often found by substituting $a = -17d$ into one of the expressions for r they had found in **part (i)** or from forming the three terms $-16d$, $-4d$, $-d$ and using two of these to find the value. Candidates who used a in place of $a + d$ usually gave the answer $\frac{4}{17}$ in this part. This misconception was not condoned. Some candidates made no attempt to answer.

(b) Candidates found this part of the question somewhat challenging. A small number of fully correct solutions were seen. A few candidates made some progress and found the correct value of q. Some of these candidates were then unsure of a sensible next step. Those candidates who persevered with their values could be credited, as marks were awarded for correct method steps using their values as long as full method was shown. A few candidates found the sum to 20 terms of the geometric progression or found the 20th term of the arithmetic progression. Some candidates omitted this question.