Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

Cambridge IGCSE

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

6715046212

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **February/March 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

DC (JM/CB) 165270/2

Cambridge Assessment International Education
[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Given that $\mathscr{E} = \{x : 1 < x < 20\}$,

$A = \{\text{multiples of } 3\}$, $\{3, 6, 9, 12, 15, 18\}$

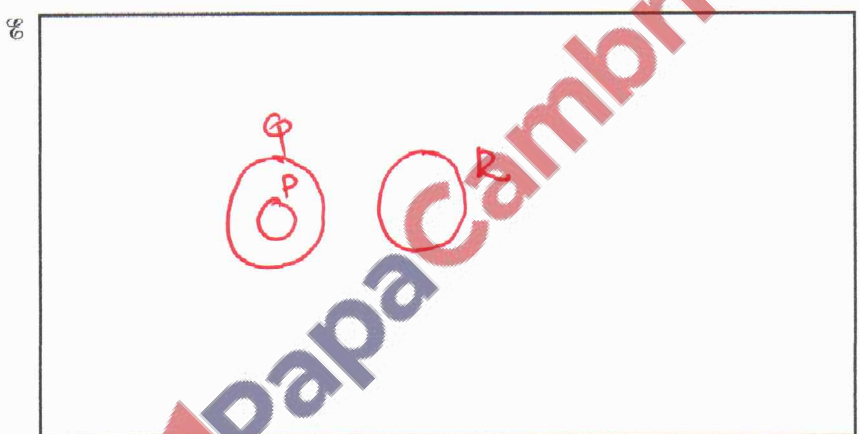
$B = \{\text{multiples of } 4\}$, $\{4, 8, 12, 16, 20\}$

find

(i) $n(A)$, $n(A) = 6$ [1]

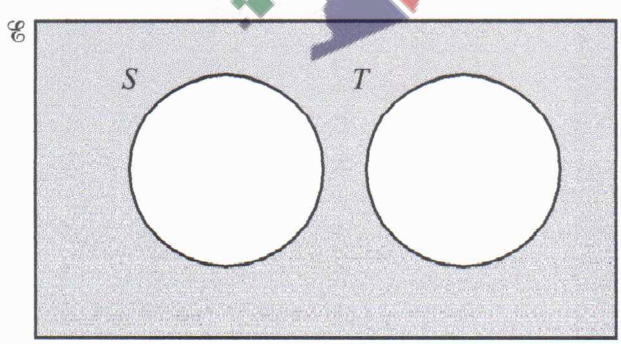
(ii) $n(A \cap B)$. (12) $= 1$ [1]

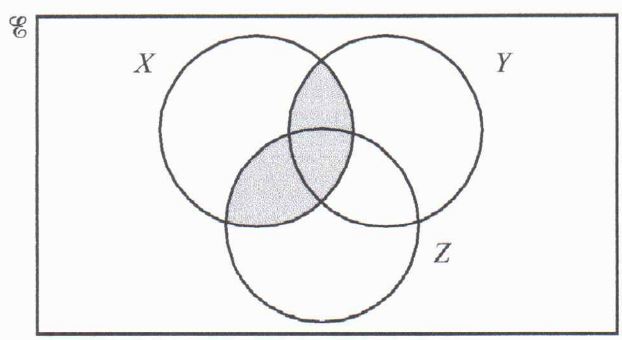
(b) On the Venn diagram below, draw the sets P, Q and R such that $P \subset Q$ and $Q \cap R = \varnothing$.



[2]

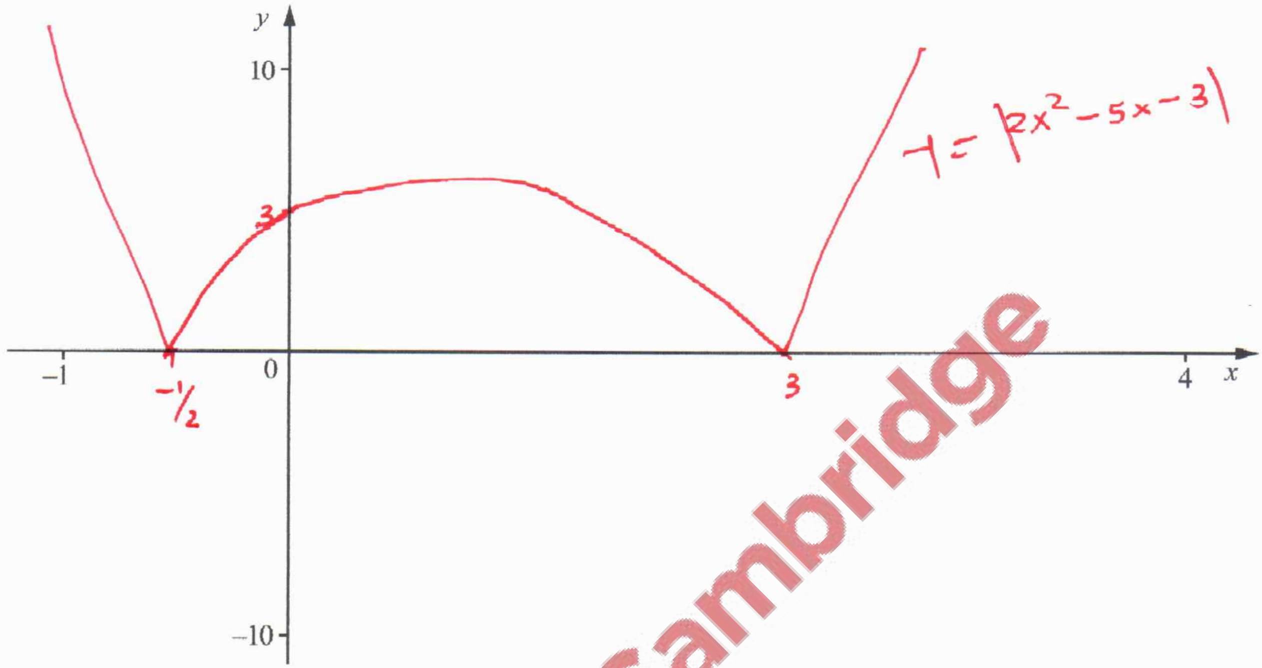
(c) Using set notation, describe the shaded areas shown in the Venn diagrams below.





$(S \cup T)'$ or $S' \cap T'$ .... $(X \cap Y) \cup (X \cap Z)$ .... [2]

2 On the axes below, sketch the graph of the curve $y = |2x^2 - 5x - 3|$, stating the coordinates of any points where the curve meets the coordinate axes.



[4]

When $x = 0$, $2(0^2) - 5(0) - 3$

$= -3$

$= 3$

x-intercept $y = 6$

y-intercept, $x = 0$.

$2x^2 - 5x - 3 = 0$

$P = -6$ $(-6, 1)$

$S = -5$

$2x^2 - 6x + x - 3 = 0$

$2x(x-3) + 1(x-3) = 0$

$2x = -\frac{1}{2}$ $x = 3$

$x = -\frac{1}{2}$

3 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $\left(3-\frac{x}{9}\right)^6$. Give the terms in their simplest form. [3]

$$\left(3-\frac{x}{9}\right)^6 = 3^6 + {}^6C_1(3)^5\left(-\frac{x}{9}\right)^1 + {}^6C_2(3^4)\left(-\frac{x}{9}\right)^2$$

$$= 729 - 162x + 15x^2$$

(ii) Hence find the term independent of x in the expansion of $\left(3-\frac{x}{9}\right)^6\left(x-\frac{2}{x}\right)^2$. [3]

$$\left(729 - 162x + 15x^2\right)\left(x^2 - 4 + \frac{4}{x^2}\right)$$

$$= 729(-4) + 15(4)$$

$$= -2856$$

4 The polynomial $p(x) = 2x^3 + ax^2 + bx - 49$, where a and b are constants. When $p'(x)$ is divided by $x+3$ there is a remainder of -24.

(i) Show that $6a - b = 78$. [2]

$x+3=0$

$x=-3$

$p'(x) = 6x^2 + 2ax + b$

$p'(-3) = -24$

$p(-3)^2 + 2a(-3) + b = -24$

$-6a+b=-78$

$6a-b=78$

It is given that $2x-1$ is a factor of $p(x)$.

(ii) Find the value of a and of b. [4]

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$p(\frac{1}{2}) = 0$

$2(\frac{1}{2})^3 + a(\frac{1}{2})^2 + b(\frac{1}{2}) - 49$

$1 + a + 2b - 196 = 0$

$a = 195 - 2b$

$6(195-2b) - b = 78$

$1170 - 12b - b = 78$

$\frac{1092}{13} = \frac{13b}{13}$

$b = 84$

$6a - b = 78$

$6a - 84 = 78$

$\frac{6a}{6} = \frac{162}{6}$

$a = 27$

(iii) Write $p(x)$ in the form $(2x-1)Q(x)$, where $Q(x)$ is a quadratic factor. [2]

$p(x) = 2x^3 + 27x^2 + 84x - 49$

$$\begin{array}{r|l} & x^2 + 14x + 49 \\ 2x-1 & 2x^3 + 27x^2 + 84x - 49 \\ & 2x^3 - x^2 \\ \hline & 28x^2 + 84x \\ & 28x^2 - 14x \\ \hline & 98x - 49 \\ & 98x - 49 \\ \hline & 0 \end{array}$$

$p(x) = (2x-1)(x^2+14x+49)$

(iv) Hence factorise $p(x)$ completely. [1]

$p(x) = (2x-1)(x+7)^2$

$= (2x-7)(x+7)(x+7)$

$x^2 + 14x + 49$

$p = +49$

$s = 14$

$(x+7)(x+7)$

5 It is given that $\log_4 x = p$. Giving your answer in its simplest form, find, in terms of p,

(i) $\log_4(16x)$, [2]

$\log_4 16 + \log_4 x$

$= 2 + p$

(ii) $\log_4\left(\dfrac{x^7}{256}\right)$. [2]

$\underset{7p}{\underline{\log_4 x^7}} - \log_4 256$

$= 7p - 4$

Using your answers to **parts (i) and (ii)**,

(iii) solve $\log_4(16x) - \log_4\left(\dfrac{x^7}{256}\right) = 5$, giving your answer correct to 2 decimal places. [3]

$(2+p) - (7p-4) = 5$

$-6p + 6 = 5$

$-6p = -1$

$\dfrac{6p}{6} = \dfrac{1}{6}$

$p = 1/6$

$\log_4 x = 1/6$

$x = 4^{1/6}$

$x = 1.26$

6 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & -4 \\ 2 & 5 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & -2 & 0 \end{pmatrix}$, write down the matrix products which are possible. You do not need to evaluate your products. [2]

BA, CB.

(b) It is given that $\mathbf{X} = \begin{pmatrix} 2 & -2 \\ 5 & 3 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$.

(i) Find $\mathbf{X}^{-1}$. [2]

$\det x = (2\times3)-(5\times-2)$

$= 6-(-10)$

$= 16$

$x^{-1} = \frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}$

(ii) Hence find the matrix **Z** such that $\mathbf{XZ}=\mathbf{Y}$. [3]

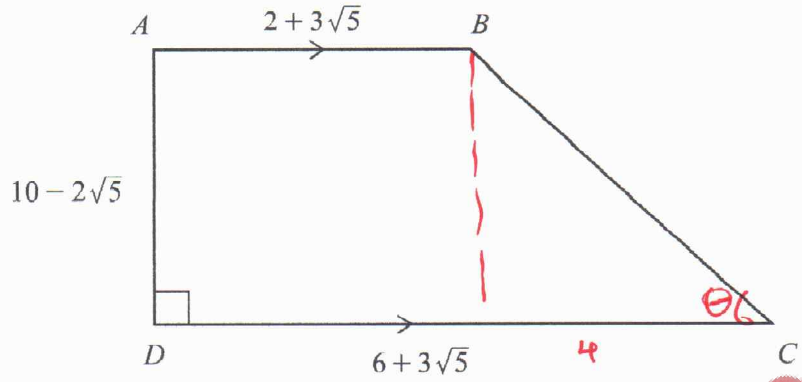
$Z = x^{-1}Y = \frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$

$= \frac{1}{16}\begin{pmatrix} 12+4, & 3+0 \\ -20+4, & -5+0 \end{pmatrix}$

$Z = \frac{1}{16}\begin{pmatrix} 16, & 3 \\ -16, & -5 \end{pmatrix}$

7 **Do not use a calculator in this question.**

All lengths in this question are in centimetres.



The diagram shows the trapezium $ABCD$, where $AB = 2+3\sqrt{5}$, $DC = 6+3\sqrt{5}$, $AD = 10-2\sqrt{5}$ and angle $ADC = 90°$.

(i) Find the area of $ABCD$, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [3]

$$A = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(2+3\sqrt{5}+6+3\sqrt{5})(10-2\sqrt{5})$$
$$= \frac{1}{2}(8+6\sqrt{5})(10-2\sqrt{5})$$
$$= \frac{1}{2}(80-16\sqrt{5}+60\sqrt{5}-60)$$
$$= \frac{1}{2}(20-44\sqrt{5}) = \underline{10-22\sqrt{5}}$$

(ii) Find $\cot BCD$, giving your answer in the form $c+d\sqrt{5}$, where c and d are fractions in their simplest form. [3]

$$\tan\theta = \frac{10-2\sqrt{5}}{4}$$
$$\text{Cot } BAD = \frac{4}{10-2\sqrt{5}} \times \frac{(10+2\sqrt{5})}{(10+2\sqrt{5})}$$
$$= \frac{40}{80} + \frac{8}{80\sqrt{5}}$$
$$= \underline{\frac{1}{2} + \frac{1}{10\sqrt{5}}}$$

8 **(a)**

The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity $v\,\mathrm{ms}^{-1}$ at time t seconds after leaving a fixed point.

(i) Write down the value of the acceleration of P when $t = 5$. [1]

The acceleratio = 0, it is at rest.

$a = 0$

(ii) Find the distance travelled by the particle P between $t = 0$ and $t = 10$. [2]

Area of Trapezium A $= \frac{1}{2}(6+4)10$

$= \underline{\underline{50}}$

Area of Trapezium B $= \frac{1}{2}(20+10)4$

$= \frac{1}{2} \times 30 \times 4$

$= \underline{\underline{60}}$

Distance $= 60 + 50$

$= \underline{\underline{110}}$

(b) A particle Q moves such that its velocity, $v\,\text{ms}^{-1}$, t seconds after leaving a fixed point, is given by $v = 3\sin 2t - 1$.

(i) Find the speed of Q when $t = \dfrac{7\pi}{12}$. [2]

$V = 3\sin 2t - 1$

$V = 3\sin 2\left(\frac{7\pi}{12}\right) - 1$

$V = -5/2$

$v = \underline{5/2\ \text{m/s}}$ or $\underline{2.5\ \text{m/s}}$

(ii) Find the least value of t for which the acceleration of Q is zero. [3]

$\frac{dv}{dt} = 0$

$6\cos 2t = 0$

$\cos 2t = 0$

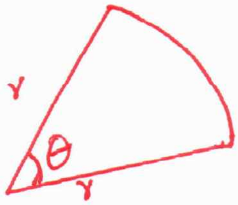
$2t = \pi/2$

$t = \underline{\pi/4}$

$t = \underline{\frac{\pi}{4}}$ or 0.785

9 The area of a sector of a circle of radius r cm is $36\,\text{cm}^2$.

(i) Show that the perimeter, P cm, of the sector is such that $P = 2r + \dfrac{72}{r}$. [3]



$A = \frac{1}{2} r^2 \theta$

$2 \times 36 = \frac{1}{\cancel{2}} r^2 \cancel{\times 2}$

$\theta = \frac{72}{r^2}$

$P = 2r + r\theta$

$P = 2r + r\left(\frac{72}{r^2}\right)$

$P = 2r + \frac{72}{r}$

(ii) Hence, given that r can vary, find the stationary value of P and determine its nature. [4]

$\frac{dP}{dr} = 2 - \frac{72}{r^2}$

$0 = 2 - \frac{72}{r^2}$

$r^2 \times 2 = \frac{72}{\cancel{r^2}} \cancel{\times r^2}$

$\frac{\cancel{2} r^2}{\cancel{2}} = \frac{72}{2}$

$r^2 = 36$

$r = \sqrt{36}$

$r = 6$

$P = 2(6) + \frac{72}{6}$

$P = \frac{12 + (72)}{6}$

$P = 24$

$\frac{d^2P}{dr^2} = \frac{144}{r^3}$

$= \frac{144}{6^3}$

$= 0.66666$

$= 0.667$

$\frac{d^2P}{dr^2} > 0$ It is a Minimum Value.

10 A curve is such that when $x = 0$, both $y = -5$ and $\frac{dy}{dx} = 10$. Given that $\frac{d^2y}{dx^2} = 4e^{2x} + 3$, find

(i) the equation of the curve, [7]

$y = \frac{dy}{dx}$

$\frac{dy}{dx} = \int \frac{d^2y}{dx^2}$

$= \int 4e^{2x} + 3$

$\frac{dy}{dx} = \frac{4e^{2x}}{2} + 3x + c$

$10 = 2e^{2(0)} + 3(0) + C$

$10 = 2 + C$

$C = 8$

$y = \int 2e^{2x} + 3x + 8$

$y = \frac{2e^{2x}}{2} + \frac{3x^2}{2} + 8x + d$

$-5 = 1 + d \quad d = -6$

$y = e^{2x} + 3/_2 x^2 + 8x - 6$

(ii) the equation of the normal to the curve at the point where $x = \frac{1}{4}$. [3]

Equation of Normal

$\frac{dy}{dx} = 2e^{\frac{1}{2}} + 3(1/4) + 8$

$= 12.05$

$M \times m_2 = -1$

$m_2 = -0.083$

$y = e^{\frac{1}{2}} + 3/_2 (1/_{16}) + 2 - 6$

$y + 2.26 = -0.083(x - 1/4)$

$y = -0.083x + 0.0075 - 2.26$

$y = -0.083x - 2.25$

$y + 2.26 = -1/_{12}(x - 1/4)$

11 **(a)** Solve $\sin x \cos x = \frac{1}{2}\tan x$ for $0° \leqslant x \leqslant 180°$. [3]

$$\sin x \cos x = \frac{1}{2}\left(\frac{\sin x}{\cos x}\right)$$

$$\sin x \cos x - \frac{\sin x}{2\cos x} = 0$$

$$\sin x \left[\cos x - \frac{1}{2\cos x}\right] = 0$$

$\sin x = 0$

$x = 0,\ 180°$

$\cos x = \frac{1}{2\cos x}$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$x = 45°,\ 135°$

(b) **(i)** Show that $\sec\theta - \dfrac{\sin\theta}{\cot\theta} = \cos\theta$. [3]

$$\frac{1}{\cos\theta} - \frac{\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}}}{}$$

$$= \frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta}$$

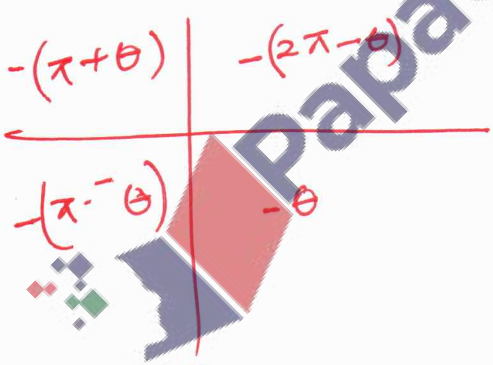
$$= \underline{\underline{\cos\theta}}$$

(ii) Hence solve $\sec 3\theta - \dfrac{\sin 3\theta}{\cot 3\theta} = \dfrac{1}{2}$ for $-\dfrac{2\pi}{3} \leqslant \theta \leqslant \dfrac{2\pi}{3}$, where θ is in radians. [4]

$\cos 3\theta = \frac{1}{2}$

$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{3}$

$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, -\frac{\pi}{9}, -\frac{5\pi}{9}$



-30°

-150°

$-\pi/3$,

$\theta = \pm\frac{\pi}{9}, \pm\frac{5\pi}{9}$

$\frac{\pi}{9}, \frac{5\pi}{9}, -\frac{\pi}{9}\ -\frac{5\pi}{9}$