Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 $$f(x) = 3 + e^x \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 9x - 5 \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f and of g. [2]

$e^x > 0$

$3 + e^x > 3$

$f > 3$

$g \in \mathbb{R}$

(b) Find the exact solution of $f^{-1}(x) = g'(x)$. [3]

Let $y = 3 + e^x \rightarrow e^x = y - 3$ $\quad g'(x) = 9$

$x = \ln(y-3)$

$f^{-1}(x) = \ln(x-3)$

$= \ln(x-3) = 9$

$x - 3 = e^9$

$\underline{\underline{x = 3 + e^9}}$

(c) Find the solution of $g^2(x) = 112$. [2]

$g^2(x) = 9g(x)$

$= 9g(x) - 5 = 9(9x-5) - 5$

$= 81x - 45 - 5$

$= 81x - 50$

$81x - 50 = 112$

$81x = 112 + 50$

$\cancel{81}x = \frac{162}{81}$

$\cancel{81}$

$x = \underline{\underline{2}}$

2 **(a)** Given that $\log_2 x + 2\log_4 y = 8$, find the value of xy. [3]

$2\log_4 y - \log_4 y^2$

$\frac{\log_2 y^2}{\log_2 4} = \frac{\cancel{2}\log_2 y}{\cancel{2}}$

$= 2\log_2 2$

$\log_2 x + \log_2 y = 8$

$\log_2 xy = 8$

$xy = 2^8$

$= \underline{\underline{256}}$

(b) Using the substitution $y = 2^x$, or otherwise, solve $2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$. [4]

$2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$

$2y^2 - 2y - y + 1 = 0$

$2y^2 - 3y + 1 = 0$

$2y^2 - 3y + 1 = 0$

$(2y-1)(y-1) = 0$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{1}{2}$

$y = \underline{\underline{\frac{1}{2}}}$

$y - 1 = 0$

$y = \underline{\underline{1}}$

$y = 2^x$

$2^x = \frac{1}{2}$

$\ln 2^x = \ln \frac{1}{2}$

$x = \ln \frac{1}{2}$

$2^x = 1$

$\ln 2^x = \ln 1$

$x = \frac{\ln 1}{\ln 2} = \underline{\underline{0}}$

$\ln \frac{1}{2} = \ln 2^{-1}$

$= -\ln 2$

$x = \frac{-\cancel{\ln 2}}{\cancel{\ln 2}}$

$x = \underline{\underline{-1}}$

3 At time t s, a particle travelling in a straight line has acceleration $(2t+1)^{-\frac{1}{2}}\,\text{m s}^{-2}$. When $t = 0$, the particle is 4 m from a fixed point O and is travelling with velocity $8\,\text{m s}^{-1}$ away from O.

(a) Find the velocity of the particle at time t s. [3]

$t = 0,\ s = 4$

$v = 8$

$a = (2t+1)^{-1/2}$

$\int v = \int a\,dt = \int (2t+1)^{-1/2}\,dt$

$= \dfrac{(2t+1)^{1/2}}{\frac{1}{2}\cdot 2} + c$

$v = \sqrt{2t+1} + c$

$8 = \sqrt{1} + c$

$c = \underline{7} \qquad v = \underline{\sqrt{2t+1} + 7}$

(b) Find the displacement of the particle from O at time t s. [4]

$s = \int v\,dt = \int (2x+1)^{1/2} + 7\,dt$

$s = \dfrac{(2t+1)^{3/2}}{\frac{3}{2}\cdot 2} + 7t + c$

$s = \frac{1}{3}(2t+1)^{3/2} + 7t + c$

$4 = \frac{1}{3}(0+1)^{3/2} + 0 + c$

$c = 4 - \frac{1}{3}$

$= 3\frac{2}{3} = \underline{\frac{11}{3}}$

$s = \underline{\frac{1}{3}(2t+1)^{3/2} + 7t + \frac{11}{3}}$

4 **(a)** Write $2x^2 + 3x - 4$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

$2x^2+3x-4$

$2x^2+3x \quad -4$

$2\left(x^2+1.5x+\left(\frac{1.5}{2}\right)^2\right)-4-2\left(\frac{1.5}{2}\right)^2$

$2(x+0.75)^2 - 5.125$

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + 3x - 4$. [2]

$x+0.75=0$

$x=-0.75$

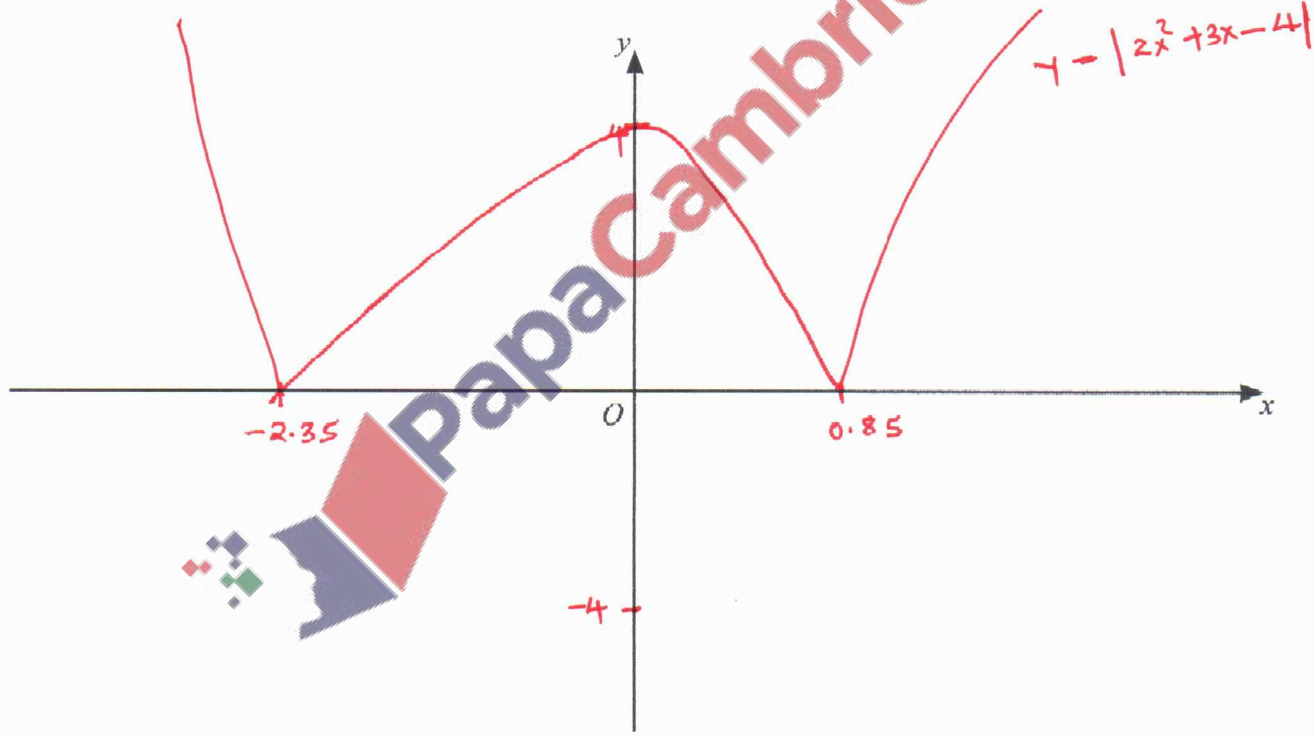
Coordinates $(-0.75, -5.125)$

y-axis $=0$

$y=0+0-4$

$y=-4$

(c) On the axes below, sketch the graph of $y = |2x^2 + 3x - 4|$, showing the exact values of the intercepts of the curve with the coordinate axes. [3]



At x-axis $y=0$

$2(x+0.75^2)-5.125=0$

$(x+0.75)^2 = 2.5625$

$x+0.75=\pm 1.6$

$x=0.85$

$x=-2.35$

(d) Find the value of k for which $|2x^2 + 3x - 4| = k$ has exactly 3 values of x. [1]

$y=k$

$y=5.125$ $\quad k=5.125$

5 $\quad p(x) = 6x^3 + ax^2 + 12x + b$, where a and b are integers.

$p(x)$ has a remainder of 11 when divided by $x-3$ and a remainder of -21 when divided by $x+1$.

(a) Given that $p(x) = (x-2)Q(x)$, find $Q(x)$, a quadratic factor with numerical coefficients. [6]

$P(3) = 6(3^3) + a(3^2) + 12(3) + b$

$= 6(27) + 9a + 36 + b$

$162 + 9a + 36 + b = 11$

$162 + 9a + b + 36 = 11$

$9a + b = -187$ --(i)

$P(-1) = 6(-1) + a(1) + 12(-1) + b = -21$

$-6 + a - 12 + b = -21$

$a + b = -3$ --(ii)

$9a + b = -187$

$a + b = -3$

$b = -187 - 9a$

$a + (-187 - 9a) = -3$

$-8a = -184$

$a = -23$

$b = -187 - 9(-23)$

$b = -187 + 207$

$b = 20$

$P(x) = 6x^3 - 23x^2 + 12x + 20$

$$\begin{array}{r|l} & 6x^2 - 11x - 10 \\ x-2 & 6x^3 - 23x^2 + 12x + 20 \\ & 6x^3 - 12x^2 \\ \hline & -11x^2 + 12x + 20 \\ & -11x^2 + 22x \\ \hline & -10x + 20 \\ & -10x + 20 \\ \hline \end{array}$$

$P(x) = (x-2)(6x^2 - 11x - 10)$

(b) Hence solve $p(x) = 0$. [2]

$(x-2)(6x^2 - 11x - 10) = 0$

$x = 2 \qquad (3x+2)(2x-5)$

$\frac{3x}{3} = \frac{-2}{3}$

$x = -2/3$

$2x - 5 = 0$

$\frac{2x}{2} = \frac{5}{2}$

$x = 2.5$

6 **(a)** Find the unit vector in the direction of $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$. [1]

$$\frac{\begin{pmatrix} +5 \\ -12 \end{pmatrix}}{\left|\begin{matrix} 5 \\ -12 \end{matrix}\right|} = \frac{\begin{pmatrix} 5 \\ -12 \end{pmatrix}}{\sqrt{25+144}} = \frac{\begin{pmatrix} 5 \\ -12 \end{pmatrix}}{\sqrt{169}} = \frac{\begin{pmatrix} 5 \\ -12 \end{pmatrix}}{13} = \begin{pmatrix} 5/13 \\ -12/13 \end{pmatrix}$$

(b) Given that $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} -2 \\ 3 \end{pmatrix} = r\begin{pmatrix} -10 \\ 5 \end{pmatrix}$, find the value of each of the constants k and r. [3]

$4 - 2k = -10r \quad$ -- (i)

$1 + 3k = 5r \quad$ -- (ii)

$\frac{1+3k}{5} = \frac{\cancel{5}r}{\cancel{5}}$

$r = \frac{1+3k}{5}$

$4 - 2k = \cancel{-10}^{-2}\left(\frac{1+3k}{\cancel{5}}\right)$

$4 - 2k = -2(1+3k)$

$4 - 2k = -2 + (-6k)$

$4 - 2k = -2 - 6k$

$6 = -6k + 2k$

$\frac{6}{-4} = \frac{-4k}{-4}$

$k = -3/2$

$r = \frac{1+3k}{5}$

$r = \frac{1+3(-3/2)}{5}$

$r = \frac{1-(-9/2)}{5} \qquad r = 1/10$

$k = -3/2$

$r = 1/10$

(c) Relative to an origin O, the points A, B and C have position vectors $\mathbf{p}$, $3\mathbf{q}-\mathbf{p}$ and $9\mathbf{q}-5\mathbf{p}$ respectively.

$OA = p,\ OB = 3q - p,\ OC = 9q-5p$

(i) Find $\overrightarrow{AB}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [1]

$AB = OB - OA$

$3q - p - p = 3q - 2p$

(ii) Find $\overrightarrow{AC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [1]

$\overrightarrow{AC} = OC - OA$

$= 9q - 5p - p = 9q - 6p$

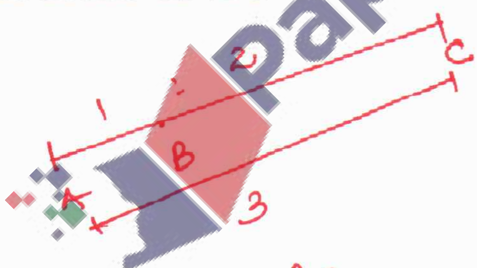
(iii) Explain why A, B and C all lie in a straight line. [1]

From (i) and (ii) so $\overrightarrow{AB} = 3q - 2p$

$\overrightarrow{AC} = 3(3q-2p)$

A B C all lie in a straight line.

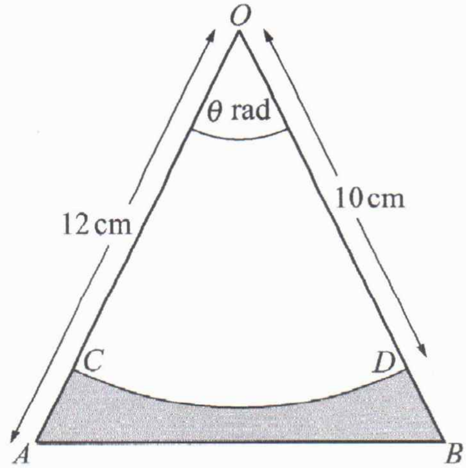
(iv) Find the ratio $AB : BC$. [1]



$AB : BC$

$= 1 : 2$

7



The diagram shows an isosceles triangle OAB such that $OA = OB = 12$ cm and angle $AOB = \theta$ radians. Points C and D lie on OA and OB respectively such that CD is an arc of the circle, centre O, radius 10 cm. The area of the sector $OCD = 35\,\text{cm}^2$.

(a) Show that $\theta = 0.7$. [1]

$\frac{1}{2}(10)(10)\theta = 35$

$\frac{\cancel{50}\theta}{\cancel{50}} = \frac{35}{50}$ $\quad \theta = 0.7$ rads

(b) Find the perimeter of the shaded region. [4]

$AB = \frac{\cos 0.7}{1} = \frac{12^2+12^2-AB^2}{2(12)(12)}$

$288\cos 0.7 = 288 - AB^2$

$AB^2 = 288 - 288\cos 0.7$

$\cancel{AB^2} = \sqrt{67.725}$

$AB = 8.22955$

Perimeter $= 2 + 8.22955 + 2 + 7$

$= 19.2$ cm

(c) Find the area of the shaded region. [3]

Area $= \frac{1}{2}\sin 0.7\,(12\times 12) - 35$

$= 46.384 - 35$

$= 11.38$

$= 11.4\,\text{cm}^2$

8 **(a)** An arithmetic progression has a first term of 7 and a common difference of 0.4. Find the least number of terms so that the sum of the progression is greater than 300. [4]

$a=7, \quad d=0.4$

$U_n = a+(n-1)d$

$S_n = \frac{1}{2}n(2a+(n-1)d)$

$S_n > 300$

$\frac{1}{2}n(14+(n-1)0.4) > 300$

$n(14+(n-1)0.4) > 600$

$14n + 0.4n(n-1) > 600$

$14n + 0.4n^2 - 0.4n > 600$

$0.4n^2 + 13.6n - 600 > 0$

$\frac{4n^2}{4} + \frac{136n}{4} - \frac{6000}{4} > 0$

$n^2 + 34n - 1500 > 0$

$n = \frac{-34 \pm \sqrt{34^2 - 4\times1\times-1500}}{2}$

$n = \frac{-34 \pm \sqrt{7156}}{2}$

$n = 25.296$

$n \simeq 26$

$n = 26$

(b) The sum of the first two terms of a geometric progression is 9 and its sum to infinity is 36. Given that the terms of the progression are positive, find the common ratio. [4]

$U_n = ar^{n-1}$

$U_1 = ar^0 = a$

$S_\infty = \frac{a}{1-r}$

$U_2 = ar^1 = ar$

$a + ar = 9 \quad -(i)$

$\frac{a}{1-r} = 36$

$a = 36(1-r)$

$a = 36 - 36r \quad -(ii)$

$36 - 36r + (36 - 36r)r = 9$

$36 - \cancel{36r} + \cancel{36r} - 36r^2 = 9$

$36 - 9 = 36r^2$

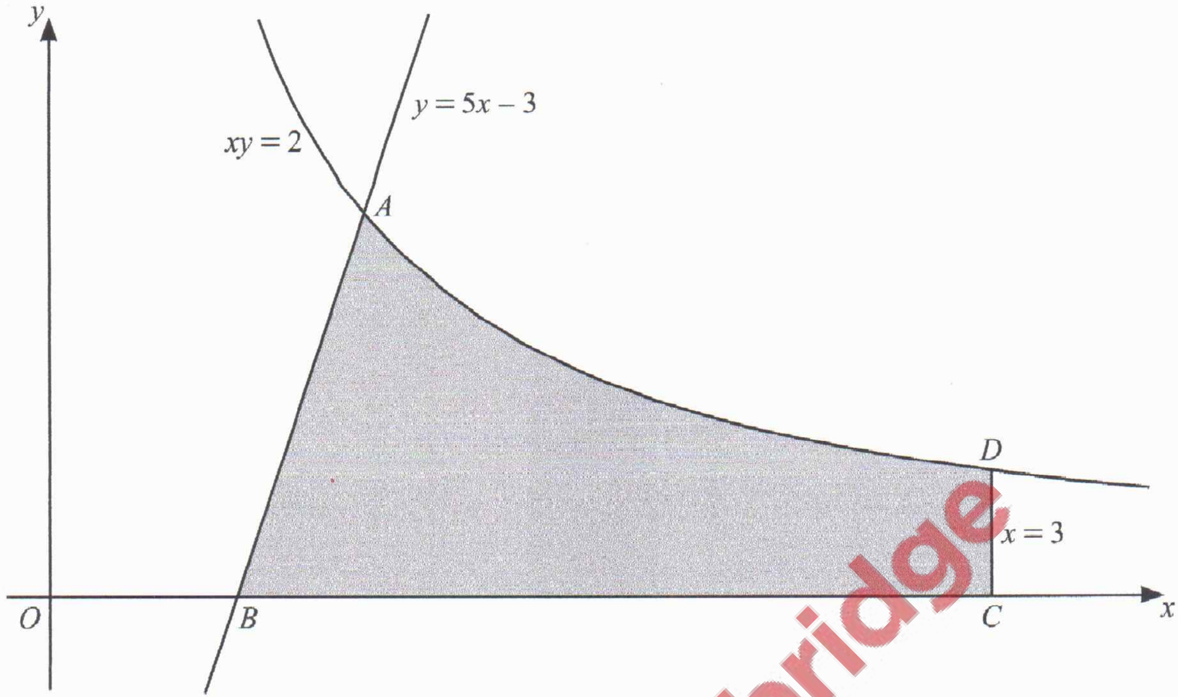
$\frac{27}{36} = \frac{\cancel{36}r^2}{\cancel{36}}$

$r^2 = \sqrt{\frac{27}{36}}$

$r^2 = \sqrt{0.75}$

$r = 0.866 \quad r = \sqrt{0.75}$

9



The diagram shows part of the curve $xy = 2$ intersecting the straight line $y = 5x - 3$ at the point A. The straight line meets the x-axis at the point B. The point C lies on the x-axis and the point D lies on the curve such that the line CD has equation $x = 3$. Find the exact area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are constants. [8]

$xy = 2$, $y = 2$

$y = 5x - 3$

$x(5x-3) = 2$

$5x^2 - 3x = 2$

$5x^2 - 3x - 2 = 0$

$(5x+2)(x-1) = 0$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{-2}{5}$ | $x - 1 = 0$

$x = -2/5$ | $x = 1$

B: at x-axis, $y = 0$

$0 = 5x - 3$

$\frac{3}{5} = \frac{\cancel{5}x}{\cancel{5}}$

$x = 3/5$ or 0.6

$xy = 2$ $y = \frac{2}{x}$

$= \int_1^3 \frac{2}{x}\,dx$

$= [2\ln x]_1^3 = 2\ln 3 - 2\ln 1$

$= 2\ln 3$

$= \ln 3^2$

$= \ln 9$

Shaded region $= 0.4 + \ln 9$

Additional working space for question 9.

10 **(a)** Given that $y = x\sqrt{x+2}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{Ax+B}{2\sqrt{x+2}}$, where A and B are constants. [5]

$$y = x\sqrt{x+2}$$

$$y = x(x+2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (x+2)^{\frac{1}{2}} \cdot 1 + x \cdot \tfrac{1}{2}(x+2)^{-\frac{1}{2}}$$

$$= \frac{\sqrt{x+2}}{1} + \frac{x}{2\sqrt{x+2}}$$

$$\frac{dy}{dx} = \frac{2(x+2) + x}{2\sqrt{x+2}}$$

$$\frac{dy}{dx} = \frac{2x+4+x}{2\sqrt{x+2}}$$

$$\frac{dy}{dx} = \frac{3x+4}{2\sqrt{x+2}}$$

(b) Find the exact coordinates of the stationary point of the curve $y = x\sqrt{x+2}$. [3]

$$\frac{dy}{dx} = \frac{3x+4}{2\sqrt{x+2}} = \frac{0}{1}$$

$$3x+4=0$$

$$\frac{\cancel{3}x}{\cancel{3}} = -\frac{4}{3}$$

$$x = -\frac{4}{3}$$

$$Y = -\frac{4}{3}\sqrt{-\frac{4}{3}+2}$$

$$= -\frac{4}{3}\sqrt{\frac{2}{3}}$$

$$\left(-\frac{4}{3}, -\frac{4}{3}\sqrt{\frac{2}{3}}\right)$$

(c) Determine the nature of this stationary point. [2]

$$\frac{dy}{dx} = \frac{3x+4}{2(x+2)^{1/2}}$$

$$\frac{d^2y}{dx^2} = \frac{\left[2(x+2)^{\frac{1}{2}} \cdot 3 - (3x+4) \cdot 2 \cdot \frac{1}{2}(x+2)^{-\frac{1}{2}}\right]}{\left[2(x+2)^{\frac{1}{2}}\right]^2}$$

$$= \frac{6\sqrt{x+2} - \frac{3x+4}{\sqrt{x+2}}}{4(x+2)}$$

$x = -\frac{4}{3}$

$$= 4.89897 - 0$$

$$= 4.89897 > 0, \quad x = -\frac{4}{3}$$

Since $\frac{d^2y}{dx^2} > 0$ stationary point $\left(-\frac{4}{3}, -\frac{4}{3}\sqrt{\frac{2}{3}}\right)$ is a Minimum Point.