Cambridge Assessment International Education

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (RW/SG) 202074/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 (a) Write the expression $x^2 - 6x + 1$ in the form $(x+a)^2 + b$, where a and b are constants. [2]

$x^2 - 6x + 1$

$\left(\frac{-6}{2}\right)^2$ $\left(\frac{b}{2}\right)^2 = c$

$(x-3)^2 - 9 + 1$

$\underline{\underline{(x-3)^2 - 8}}$

(b) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 6x + 1$. [1]

$y = (x-3)^2 - 8$

$x - 3 = 0$

$x = \underline{3}$

$y = -\underline{8}$

Minimum Value $\exists$ $y = -8$ when $x = 3$.

$(3, -8)$

2 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points (6, 5) and (8, 9) is obtained. Show that $y = e^p x^q$ where p and q are integers. [4]

Gradient $(M) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9-5}{8-6}$

$= \frac{4}{2}$

$= \underline{2}$

General equation $\exists$ a line

$y = Mx + c$

$(8, 9)$ $9 = 2(8) + c$

$9 = 16 + c$

$c = 9 - 16$

$c = \underline{-7}$

$y = Mx + c$

$\log y = M \log x + c$

$\log y = 2 \log x - 7$

$e^{\log y} = e^{2\log x - 7}$

$y = e^{\log x^2 - 7}$

$\underline{\underline{y = e^{-7} x^2}}$

$s = -1$

3 **(a)** Solve the inequality $|4x-1| > 9$. [3]

$(4x-1)^2 > 9^2$

$(4x-1)(4x-1)$

$4x(4x-1) -1(4x-1) > 81$

$16x^2 - 4x - 4x + 1 > 81$

$16x^2 - 8x + 1 > 81$

$\frac{16x^2}{8} - \frac{8x}{8} - \frac{80}{8} > 0$

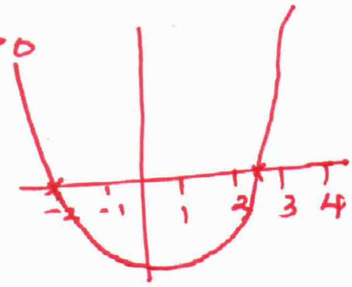
$2x^2 - x - 10 > 0$

$2x^2 - 5x + 4x - 10 > 0$

$x(2x-5) + 2(2x-5) > 0$

$(x+2)(2x-5) = 0$

$x = -2 \quad x = 5/2$



$x > 5/2 \;, \quad x < -2$

(b) Solve the equation $2x - 11\sqrt{x} + 12 = 0$. [3]

$2x - 11\sqrt{x} + 12 = 0$

let $\sqrt{x} = y$

$2(\sqrt{x})^2 - 11\sqrt{x} + 12 = 0$

$2y^2 - 11y + 12 = 0$

$P = 24 \quad (-8, -3)$

$S = -11$

$2y^2 - 8y - 3y + 12 = 0$

$2y(y-4) - 3(y-4) = 0$

$(2y-3)(y-4) = 0$

$2y - 3 = 0$

$\frac{2y}{2} = \frac{3}{2}$

$y = 1.5$

$y - 4 = 0$

$y = 4$

so: $\sqrt{x} = 4$

$x = 4^2$

$x = 16$

$(\sqrt{x})^2 = (3/2)^2$

$x = 9/4$

4 The graph of $y = a + 2\tan bx$, where a and b are constants, passes through the point $(0, -4)$ and has period $480°$.

(a) Find the value of a and of b. [3]

$y = a + 2\tan bx$

$\overset{x}{(0}, \overset{y}{-4)}$

$-4 = a + 2\tan b(0)$

$-4 = a + 0$

$a = -4$

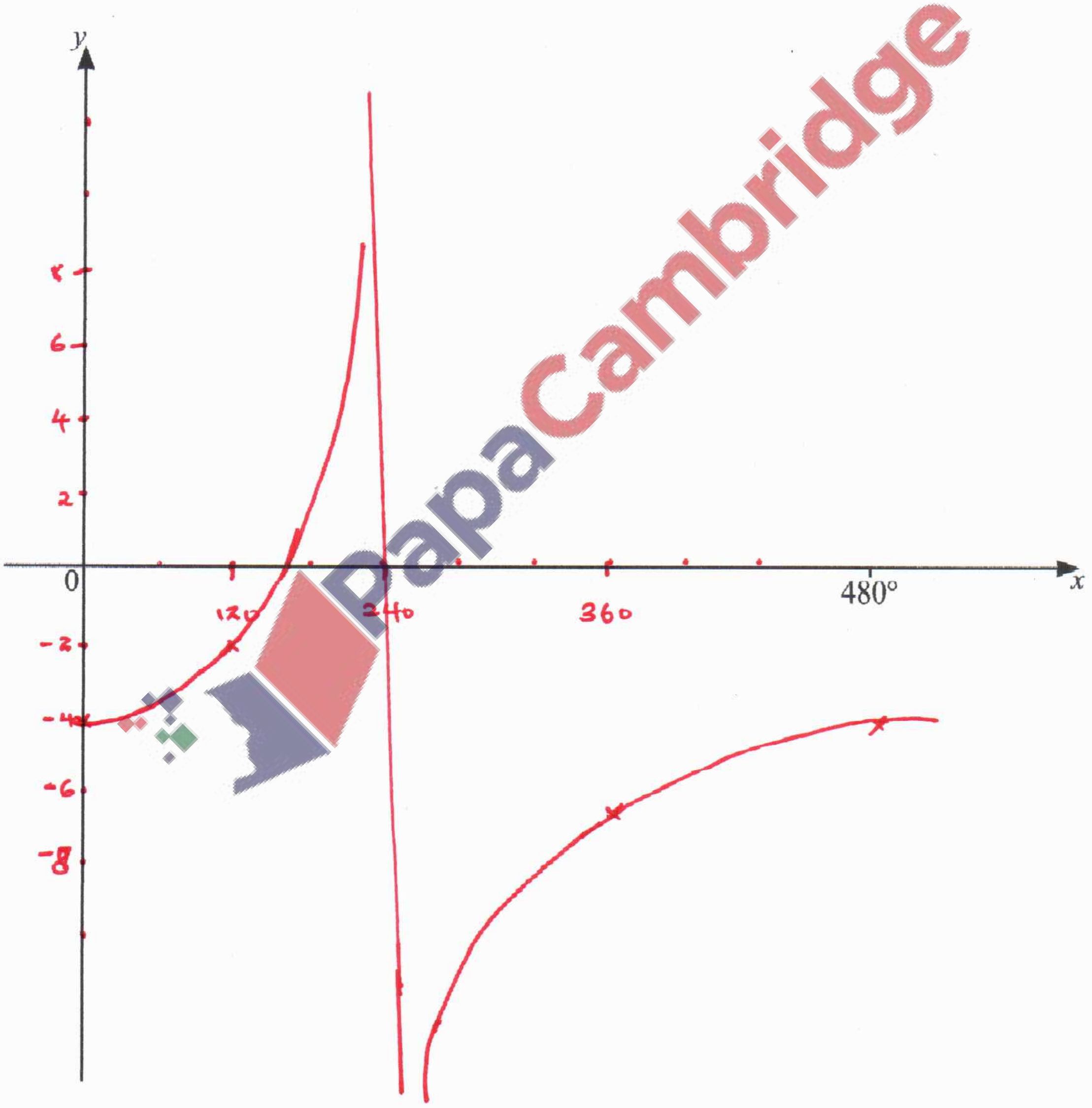
$\text{Period} = \frac{180°}{b}$

$480 = \frac{180}{b}$

$b = \frac{180}{480}$

$b = 3/8$

(b) On the axes, sketch the graph of y for values of x between $0°$ and $480°$. [2]



$Y = -4 + 2\tan \frac{3}{8}x$

$-4 + 2\tan \frac{3}{8} \times 120$

$= -2$

x	0	120	240	360	480
y	−4	−2	×	−6	−4

asymptote

5 The curves $y = x^2$ and $y^2 = 27x$ intersect at $O(0, 0)$ and at the point A. Find the equation of the perpendicular bisector of the line OA. [8]

$y = x^2$, $y^2 = 27x$

$(x^2)^2 = 27x$

$x^4 = 27x$

$x^4 - 27x = 0$

$x(x^3 - 27) = 0$

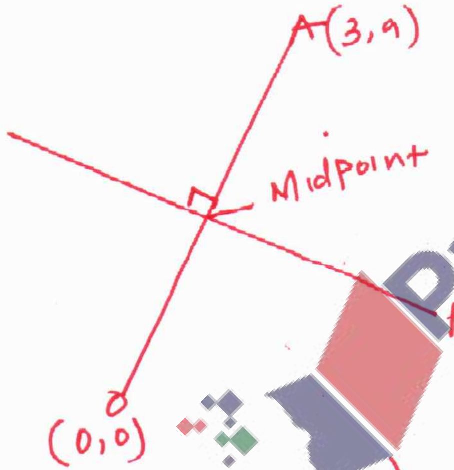
$x = 0$ | $x^3 - 27 = 0$

$x^3 = 27$

$x = \sqrt[3]{27}$

$x = \underline{3}$

A $(3, 9)$ $O(0, 0)$



Midpoint $= \left(\frac{0+3}{2}, \frac{0+9}{2}\right)$

$= (1.5, 4.5)$

Gradient of OA $= \frac{0-9}{0-3}$

$= \frac{-9}{-3}$

$= \underline{3}$

Gradient of P $= M_1 \times M_2 = -1$

$3 \times M_2 = -1$

$M_2 = -\frac{1}{\underline{3}}$

Midpoint $(1.5, 4.5)$ Gradient $= -\frac{1}{3}$

$\frac{y - 4.5}{x - 1.5} = -\frac{1}{3}$

$3(y - 4.5) = -1(x - 1.5)$

$3y - 13.5 = -x + 1.5$

$3y = -x + 1.5 + 13.5$

$3y = -x + 15$

$\frac{3y}{3} = \frac{-x}{3} + \frac{15}{3}$

$y = -\frac{1}{3}x + 5$

6 Variables x and y are such that $y = e^{\frac{x}{2}} + x\cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1+h$, where h is small. [6]

$\Delta y = \left.\frac{dy}{dx}\right|_{x=1} * h$

$y = e^{x/2} + x\cos 2x$

$\frac{dy}{dx} = e^{x/2} \cdot \frac{1}{2} + x \cdot -\sin 2x \cdot 2 + \cos 2x \cdot 1$

$\frac{dy}{dx} = \frac{e^{x/2}}{2} - 2x\sin 2x + \cos 2x$

When $x=1$, $\frac{e^{1/2}}{2} - 2(1)\sin 2 + \cos 2$

$\frac{dy}{dx} = 1$

$= \underline{\underline{-1.41038}}$

$\therefore$ Change y $\Delta y = \underline{\underline{-1.41h}}$

7 Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$. [6]

$b^2 - 4ac = 0$

Since the lines Intersect.

$2x + 1 = 4x^2 + kx + k - 2$

$4x^2 + kx - 2x + k - 2 - 1 = 0$

$4x^2 + (k-2)x + k - 3 = 0$

$\underbrace{4x^2}_{a} + \underbrace{(k-2)}_{b}x + \underbrace{k-3}_{c}$

$(k-2)^2 - 4(4)(k-3) = 0$

$k^2 - 4k + 4 - 16(k-3) = 0$

$k^2 - 4k + 4 - 16k + 48 = 0$

$k^2 - 20k + 52 = 0$

$P = 52$

$S = -20$

Using quadratic equation formula

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{20 \pm \sqrt{-20^2 - 4 \times 1 \times 52}}{2}$

$\dfrac{20 \pm \sqrt{400 - 208}}{2}$

$\dfrac{20 \pm \sqrt{192}}{2}$

$\sqrt{192} = \sqrt{4 \times 48}$

$= 2\sqrt{48}$

$k = \dfrac{20 + 2\sqrt{48}}{2}$ or $k = 20 - \sqrt{192}$

$= 10 + \sqrt{48}$ $k = \dfrac{20 - 2\sqrt{48}}{2}$

or $10 - \sqrt{48}$

8 In this question, a, b, c and d are positive constants.

(a) **(i)** It is given that $y = \log_a(x+3) + \log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$. [1]

Both of log is greater than zero.

$x+3>0$ and $2x-1>0$.

(ii) Find the exact solution of the equation $\dfrac{\log_a 6}{\log_a(y+3)} = 2$. [3]

$\log_a 6 = 2\log_a(y+3) = \log_a(y+3)^2$

$(y+3)^2 = 6$

$y+3 = \sqrt{6}$

$y = -3+\sqrt{6}$

$\log_a 6 = 2\log_a(y+3)$

$\cancel{\log_a} 6 = \cancel{\log_a}(y+3)^2$

$6 = (y+3)^2$

$y^2+6y+3=0$

$\sqrt{24} = \sqrt{4\times 6} = 2\sqrt{6}$

$\dfrac{-6\pm\sqrt{24}}{2}$

$y = \dfrac{-6+\sqrt{24}}{2} = y = -3+\sqrt{6}$

(b) Write the expression $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c + d\log_a 9$, where c and d are integers. [4]

$\log_a 9 + \cancel{\log_a b}\left[\dfrac{\log_a 9a}{\frac{1}{2}\cancel{\log_a b}}\right]$

$\log_a 9 + 2\log_a 9a$

$\log_a 9 + \underbrace{\log_a 81a^2}$

$\log_a 9 + \log_a 81 + \log_a a^2$

$\underbrace{\log_a 9 + 2\log_a 9} + 2\log_a a$

$3\log_a 9 + 2\log_a a$

$= 2 + 3\log_a 9$

$\log_{\sqrt{b}} 9a = \dfrac{\log_a 9a}{\log_a \sqrt{b}}$

$\sqrt{b} = b^{1/2}$

$\log_a cb = \log_a c + \log_a b$

9 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\overset{x}{\frac{\pi}{4}}, \overset{y}{\frac{13\pi}{12}}\right)$ on the curve, find the equation of the curve. [7]

To obtain equation of the curve $= y$.

Integrate:

$\frac{dy}{dx} = \int \sin\left(6x - \frac{\pi}{2}\right) dx$

$= \frac{-\cos\left(6x - \frac{\pi}{2}\right)}{6} + c$

substitute $x = \pi/4$ $\quad \frac{1}{2} = \frac{-\cos\left(6(\pi/4) - \pi/2\right)}{6} + c$

$\frac{1}{2} = +1/6 + c$

$c = 1/2 - 1/6$

$c = 1/3$

$\frac{dy}{dx} = -1/6 \cos(6x - \pi/2) + 1/3$

$y =$ Integration of

$\int -1/6 \cos(6x - \pi/2) + 1/3\, dx$

$= \frac{-1/6 \sin 6x - \pi/2}{6} + 1/3 x + d$

$y = -1/36 \sin(6x - \pi/2) + 1/3 x + d$;

Use $x = \pi/4$, $y = 13\pi/12$

$y = 1/36 \sin\left(3\pi/2 - \pi/2\right) + \frac{1}{3}x + d$

$\frac{13\pi}{12} = \frac{\pi}{12} + d$

$d = \frac{13\pi}{12} - \pi/12 = \pi$

$y = -1/36 \sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \frac{1}{3}x + \pi$

$y = \frac{-1}{36} \sin\left(6x - \frac{\pi}{2}\right) + \frac{1}{3}x$

$y = -1/36 \sin(6x -$

$y = -1/36 \sin\left(6x - \frac{\pi}{2}\right) + \frac{1}{3}x + \pi$

10 Relative to an origin O, the position vectors of the points A, B, C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix},\ \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix},\ \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

$\sqrt{80} = \sqrt{16 \times 5}$
$= 4\sqrt{5}$

(a) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= \begin{pmatrix} 10 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ -5 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

Magnitude AB $|AB|$
$|AB| = 4^2 + 8^2$
$= \sqrt{16+64}$

$\sqrt{80}$
Unit Vector $= \dfrac{\begin{pmatrix} 4 \\ 8 \end{pmatrix}}{\sqrt{80}}$

(simplified form) $\dfrac{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\sqrt{5}}$

(b) The point A is the mid-point of BC. Find the value of x and of y. [2]

OA = Midpoint $(6, -5)$

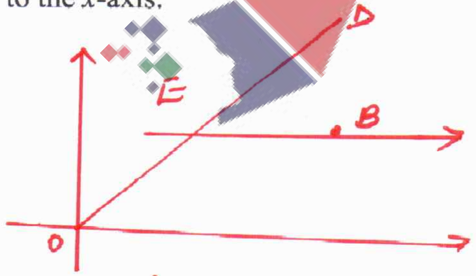
$\dfrac{10+x}{2}, \dfrac{3+y}{2} = 6, -5$

$\dfrac{10+x}{2} = \dfrac{6}{1}$
$10 + x = 12$
$x = 2$

$\dfrac{3+y}{2} = -5$
$3 + y = -10$
$y = -10 - 3$
$y = -13$

$x = 2$
$y = -13$

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x-axis. [3]



$\dfrac{OE}{OD} = \dfrac{1}{1+\lambda}$

$OE = \dfrac{1}{1+\lambda}\begin{pmatrix} 12 \\ 7 \end{pmatrix}$

$\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$

$\begin{pmatrix} \frac{12}{1+\lambda} \\ \frac{7}{1+\lambda} \end{pmatrix} - \begin{pmatrix} 10 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{12}{1+\lambda} - 10 \\ \frac{7}{1+\lambda} - 3 \end{pmatrix}$

$\dfrac{\frac{7}{1+\lambda} - 3}{x} = 0$

$\dfrac{7}{1+\lambda} - 3 = 0$

$\dfrac{7}{1+\lambda} \cancel{\times} = \dfrac{3}{1}$

$7 = 3 + 3\lambda$

$\dfrac{3\lambda}{3} = \dfrac{4}{3}$

$\lambda = 4/3$

11 The 2nd, 8th and 44th terms of an arithmetic progression form the first three terms of a geometric progression. In the arithmetic progression, the first term is 1 and the common difference is positive.

$a = 1$

(a) **(i)** Show that the common difference of the arithmetic progression is 5. [5]

$T_n = a + (n-1)d$

$T_2 = 1 + d$

$T_8 = 1 + 7d$

$T_{44} = 1 + 43d$

$1+d,\ 1+7d,\ 1+43d$

$\frac{(1+d)r}{1+d} = \frac{1+7d}{1+d}$

$r = \frac{1+7d}{1+d}$

$\frac{(1+7d)r}{1+7d} = \frac{1+43d}{1+7d}$

$r = \frac{1+43d}{1+7d}$

$\frac{1+7d}{1+d} = \frac{1+43d}{1+7d}$

$1 + 14d + 49d^2 = (1+d)(1+43d)$

$1 + 14d + 49d^2 = 1 + 43d + d + 43d^2$

$6d^2 - 30d = 0$

$d(6d - 30) = 0$

$d = 0 \quad | \quad 6d - 30 = 0$

$6d - 30 = 0$

$\frac{6d}{6} = \frac{30}{6}$

$d = 5$ ✓

(ii) Find the sum of the first 20 terms of the arithmetic progression. [2]

$Sum = \frac{n}{2}\{2a + (n-1)d\}$

Substitute the values

$a = 1$

$n = 20$

$d = 5$

$S = \frac{20}{2}\{2(1) + (20-1)5\}$

$= 10\{21 + 19(5)\}$

$= 10(97)$

$= \underline{970}$

(b) **(i)** Find the 5th term of the geometric progression. [2]

$a = 1+5 = 6$

$a = \underline{6}$

$T_8 = 1+7(5)$

$= 1+35$

$= \underline{36}$

$r = \frac{36}{6} = \underline{6}$

$T_5 = ar^4$

$= 6(6)^4$

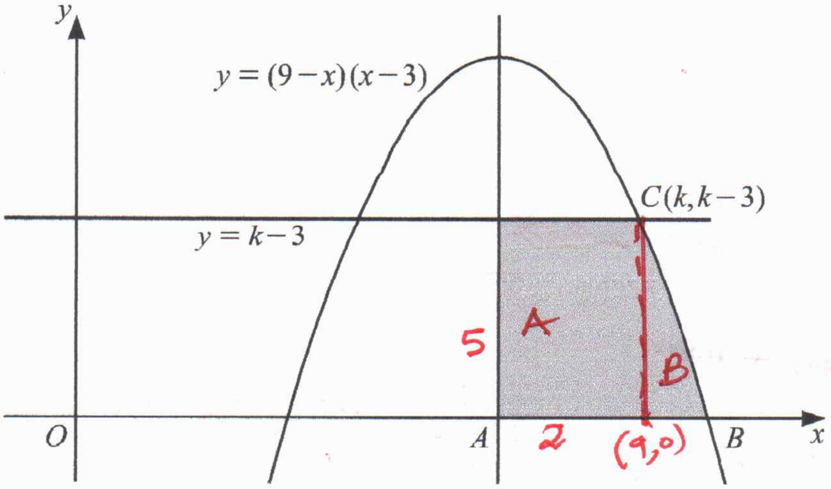
$= \underline{\underline{7776}}$

(ii) Explain whether or not the sum to infinity of this geometric progression exists. [1]

no, $|r| > 1$ $r = 6$

Hence S_∞ does not exist.

12



The diagram shows part of the curve $y = (9 - x)(x - 3)$ and the line $y = k - 3$, where $k > 3$. The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A. The curve meets the x-axis at B, and the line $y = k - 3$ meets the curve at the point $C(k,\ k - 3)$. Find the area of the shaded region. [9]

$y = (9-x)(x-3)$

Expand. $9(x-3) - x(x-3)$

$= 9x - 27 - x^2 + 3x$

$y = 9x - 27 - x^2 + 3x$

$y = -x^2 + 12x - 27$

$\frac{dy}{dx} = -2x + 12 = 0$

$-2x + 12 = 0$

$\cancel{-2x} = \cancel{-12}$

$\frac{2x}{2} = \frac{12}{2}$

$x = \underline{6}$

$y = (9-6)(6-3)$

$y = (3)(3)$

$y = \underline{9}$

X-axis $y = 0$

$(9-x)(x-3) = 0$

$(x-3)(9-x) \Rightarrow x = 3$

$x = \underline{9}$

$B(9, 0)$.

$y = k-3$, $y = (9-x)(x-3)$

$(k, k-3)$

x, y

$k - 3 = 9k - 27 - k^2 + 3k$

$k - 3 = -k^2 + 12k - 27$

$k^2 - 11k + 24 = 0$ find possible values of k.

$P = 24$ $(-8, -3)$

$S = -11$

$(k-8)(k-3)$

$k = \underline{8}$ $k = \underline{3}$

Since the value of k has to be greater. $K = 8$.

5

Area = 2 by 5

$= 5 \times 2 = 10$

$\int_8^9 -x^2 + 12x - 27$

$= \left[-\frac{x^3}{3} + 6x^2 - 27\right] \left[8^2 - 12(8) - 27\right]$

$= [-243 + 486 - 27]$

$=$

Area of shaded part =

$$\int_8^9 -x^2+12x-27\,dx$$

$$=\left[\frac{-x^3}{3}+\frac{12x^2}{2}-27x\right]_8^9$$

Substitute $x=9$ $\left[\frac{-9^3}{3}+\frac{12(9^2)}{2}-27(9)\right]-\left[\frac{-8^3}{3}+\frac{12(8^2)}{2}-27(8)\right]$

$= \cancel{-243}+\cancel{486}-\cancel{243}$

$= 0-\left[\frac{-512}{3}+384-216\right]$

$= 0+(-2\tfrac{2}{3})$

$= 2\tfrac{2}{3}$

Shaded area $= 10+2\tfrac{2}{3}$

$= 12\tfrac{2}{3}$