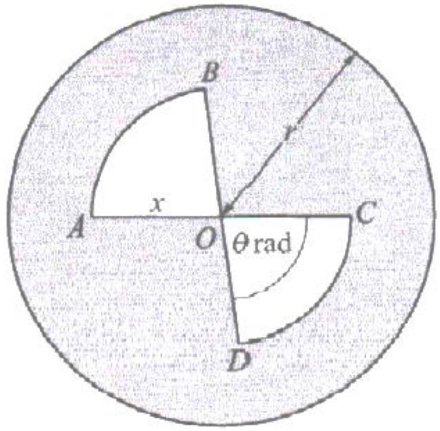
Circular measure – 2022 Nov IGCSE 0606 Additional Math

1. Nov/2022/Paper_0606_11/No.7



The diagram shows a circle with centre O and radius r. OAB and OCD are sectors of a circle with centre O and radius x, where $0 < x \leqslant r$. Angle AOB = angle COD = θ radians, where $0 < \theta < \pi$.

(a) Find, in terms of r, x and θ, the perimeter of the shaded region. [3]

Perimeter = Perimeter of the Circle + 2 Perimeter of the sector OAB

$= 2\pi r + 2(\text{Arc length} + x + x)$

$= 2\pi r + 2(x\theta + 2x)$

$= 2\pi r + 2x\theta + 4x$

$\boxed{\text{Arc Length} = r\theta}$

(b) Find, in terms of r, x and θ, the area of the shaded region. [1]

Area = Area of the Circle − 2 Area of the Sector OAB

$= \pi r^2 - 2\left(\frac{1}{2} \times x^2 \times \theta\right)$

$= \pi r^2 - x^2\theta$

$\boxed{\text{Area of a Sector} = \frac{1}{2} r^2\theta}$

It is given that x can vary and that r and θ are constant.

(c) Write down the least possible area of the shaded region in terms of r and θ. [2]

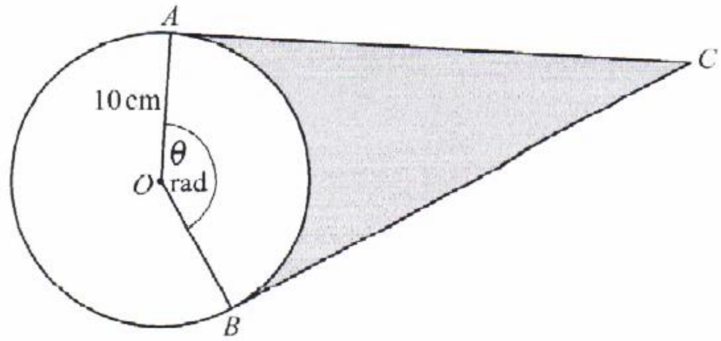
The least possible area is achieved when $x = r$

From (b) Area $= \pi r^2 - x^2\theta$

Least Area $= \pi r^2 - r^2\theta$

$= r^2(\pi - \theta)$

2. Nov/2022/Paper_0606_12/No.7



The diagram shows a circle, centre O, radius 10 cm. The points A and B lie on the circumference of the circle. The tangent at A and the tangent at B meet at the point C. The angle AOB is θ radians. The length of the minor arc AB is 28 cm.

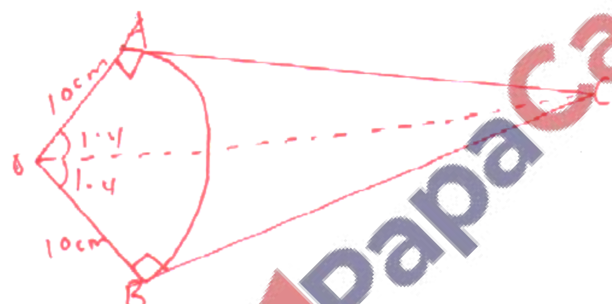
(a) Find the value of θ. [1]

Arc Length $= r\theta$

$$\Rightarrow \frac{10\theta}{10} = \frac{28}{10}$$

$$\theta = 2.8$$

(b) Find the perimeter of the shaded region. [3]



Using trigonometric ratios

$\tan 1.4 = \frac{AC}{10}$ or $\frac{BC}{10}$

$\Rightarrow AC = BC = 10 \tan 1.4$

$= 57.98$ cm

$\simeq 58$ cm

Perimeter = Length of AC + Length of BC + Minor arc length AB

Perimeter of the shaded region $= 58 + 58 + 28$

$= 144$ cm

(c) Find the area of the shaded region. [3]

Area of the shaded region = 2 Area of triangle OAC − Area of the minor Sector OAB.

$$= 2\left(\frac{1}{2} \times OA \times AC\right) - \left(\frac{1}{2} r^2 \theta\right)$$

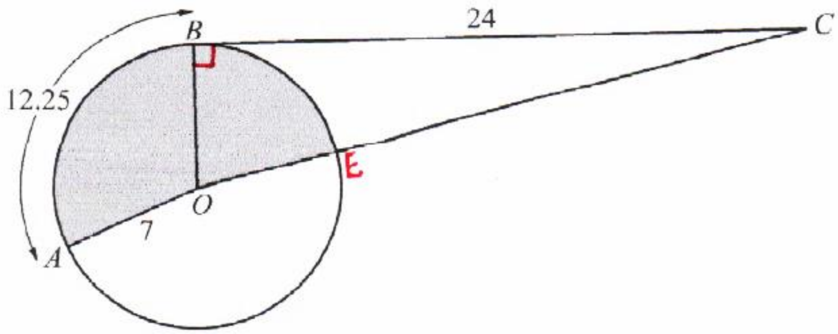
$$= 2\left(\frac{1}{2} \times 10 \times 58\right) - \left(\frac{1}{2} \times 10^2 \times 2.8\right)$$

$$= 2(290) - 140$$

$$= 580 - 140$$

$$= 440 \text{ cm}^2$$

In this question all lengths are in metres.



The diagram shows a circle, centre O, radius 7. The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24. The length of the minor arc AB is 12.25.

(a) Find the obtuse angle AOB, giving your answer in radians. [1]

Arc length $= r\theta$

$\Rightarrow \quad \dfrac{12.25}{7} = \dfrac{7\theta}{7}$

$\therefore \theta = 1.75$

$\angle AOB = 1.75$

(b) Find the perimeter of the shaded region. [4]

Extracting triangle BOC

Using trigonometric ratios

$\tan \angle BOC = \dfrac{24}{7}$

$\Rightarrow \quad \angle BOC = \tan^{-1}\left(\dfrac{24}{7}\right)$

$\angle BOC = 1.287$ (radians)

Arc Length $BE = r\theta$

$= 7 \times 1.287$

$= 9.009$

Perimeter = Arc length AB + Arc length BE + Radius + Radius

Perimeter $= 12.25 + 9.009 + 7 + 7$

$= 35.259$

$= 35.3$ m (3sf)

(c) Find the area of the shaded region. [2]

Area = Area of the sector AOB + Area of the sector BOE

$= \left(\frac{1}{2} \times 7^2 \times 1.75\right) + \left(\frac{1}{2} \times 7^2 \times 1.287\right)$

$= 42.875 + 31.5315$

$= 74.4065$

$= 74.4 \ m^2$ (3sf)

$\boxed{\text{Area of a sector} = \frac{1}{2} r^2 \theta}$