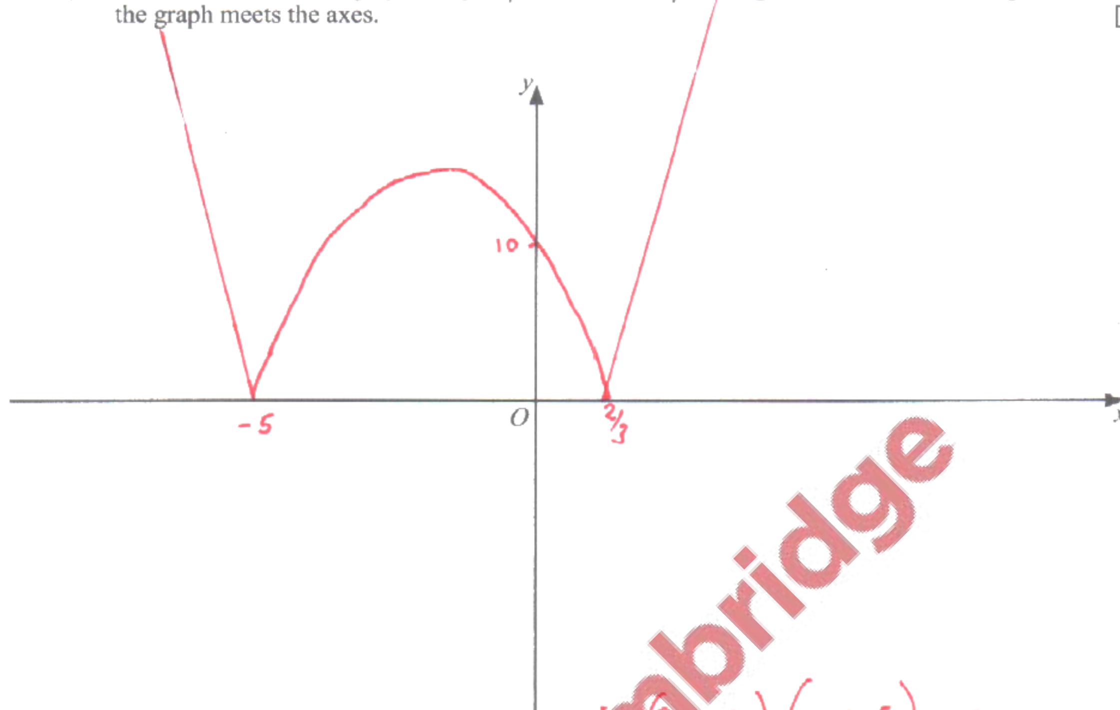
1. Nov/2022/Paper_0606_12/No.2

(a) On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes. [4]



When $x = 0$, $y = |3(0) + 13(0) - 10|$
$= |-10|$
$= 10$

When $y = 0$, $0 = 3x^2 + 13x - 10$

$\Rightarrow (3x - 2)(x + 5) = 0$
$3x - 2 = 0$, $x + 5 = 0$
$x = 2/3$, $x = -5$

(b) Find the set of values of the constant k such that the equation $k = |3x^2 + 13x - 10|$ has exactly 2 distinct roots. [4]

$K = 3x^2 + 13x - 10$

$\Rightarrow 3x^2 + 13x - 10 - K = 0$
$3x^2 + 13x - (10 + k) = 0$

If there are exactly 2 distinct roots,
the discriminant $b^2 - 4ac > 0$

$b = 13, \ a = 3, \ c = -(10 + k)$

$13^2 - 4(3)(-(10 + k)) > 0$
$169 + 12(10 + k) > 0$

Consider $169 + 12(10 + k) = 0$

$\Rightarrow 169 + 120 + 12k = 0$
$289 + 12k = 0$

$\Rightarrow \frac{\cancel{12}k}{\cancel{12}} = \frac{-289}{12}$

$K = \frac{-289}{12}$ or -24.1

$\therefore$ Critical value is $-\frac{289}{12}$

$\Rightarrow$ For the inequality
$169 + 12(10 + k) > 0$

$K > \frac{289}{12}$ or $k > 24.1$
and
$K = 0$

2. Nov/2022/Paper_0606_13/No.2

(a) Show that $2x^2+x-15$ can be written in the form $2(x+a)^2+b$, where a and b are exact constants to be found. [2]

$2x^2+x-15 = 2\left[x^2+\frac{1}{2}x-\frac{15}{2}\right]$

using completing square method

$2\left[x^2+\frac{1}{2}x+\left(\frac{1}{2}b\right)^2-\left(\frac{1}{2}b\right)^2-\frac{15}{2}\right]$

$2\left[\left(x^2+\frac{1}{2}x+\left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^2\right)-\left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^2-\frac{15}{2}\right]$

$2\left[\left(x^2+\frac{1}{2}x+\left(\frac{1}{4}\right)^2\right)-\frac{1}{16}-\frac{15}{2}\right]$

$2\left[\left(x+\frac{1}{4}\right)^2-\frac{121}{16}\right]$

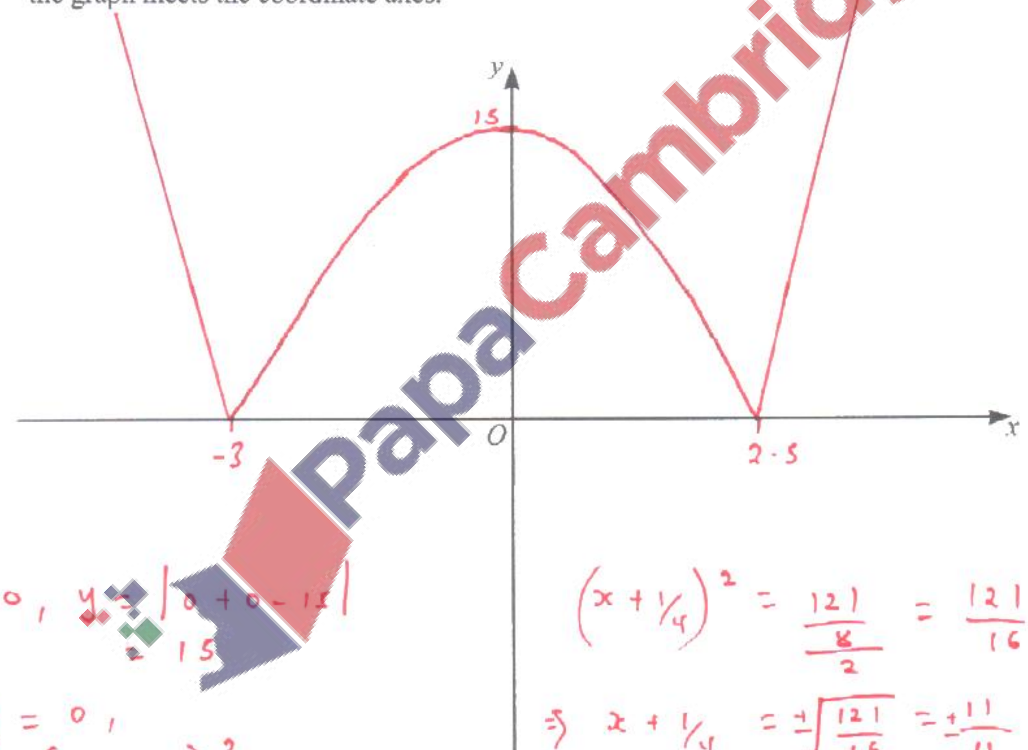
$2\left(x+\frac{1}{4}\right)^2-\left(2\times\frac{121}{16}\right)$

$=2\left(x+\frac{1}{4}\right)^2-\frac{121}{8}$

(b) Hence write down the coordinates of the stationary point on the curve $y=2x^2+x-15$. [2]

from (a) $\left(-\frac{1}{4}, -\frac{121}{8}\right)$

(c) On the axes, sketch the graph of $y=|2x^2+x-15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



When $x=0$, $y=|0+0-15|$
15

When $y=0$,

$0=2\left(x+\frac{1}{4}\right)^2-\frac{121}{8}$

$\Rightarrow 2\left(x+\frac{1}{4}\right)^2=\frac{121}{8}$

$\left(x+\frac{1}{4}\right)^2=\frac{\frac{121}{8}}{2}=\frac{121}{16}$

$\Rightarrow x+\frac{1}{4}=\pm\sqrt{\frac{121}{16}}=\pm\frac{11}{4}$

$x=-\frac{1}{4}\pm\frac{11}{4}$

$\therefore x=-3,\ 2\cdot5$

(d) Write down the value of the constant k for which the equation $|2x^2+x-15|=k$ has 3 distinct solutions. [1]

from (a) $|2x^2+x-15|=\left|2\left(x+\frac{1}{4}\right)^2-\frac{121}{8}\right|=k$

$\therefore k=\left|\frac{-121}{8}\right|=\frac{121}{8}$

(a) Write $3x^2+15x-20$ in the form $a(x+b)^2+c$ where a, b and c are rational numbers. [4]

$$3x^2+15x-20 = 3\left[x^2+\frac{15x}{3}-\frac{20}{3}\right]$$
$$= 3\left[x^2+5x-\frac{20}{3}\right]$$

Using Completing square method

$$3\left[x^2+5x+\left(\tfrac{1}{2}b\right)^2-\left(\tfrac{1}{2}b\right)^2-\frac{20}{3}\right]$$

where $b=5$

$$= 3\left[\left(x^2+5x+\left(\tfrac{5}{2}\right)^2\right)-\left(\tfrac{5}{2}\right)^2-\frac{20}{3}\right]$$
$$= 3\left[\left(x+\tfrac{5}{2}\right)^2-\frac{25}{4}-\frac{20}{3}\right]$$
$$= 3\left[\left(x+\tfrac{5}{2}\right)^2-\frac{155}{12}\right]$$
$$= 3\left(x+\tfrac{5}{2}\right)^2-\left(3\times\frac{155}{12}\right)$$
$$= 3\left(x+\frac{5}{2}\right)^2-\frac{155}{4}$$

(b) State the minimum value of $3x^2+15x-20$ and the value of x at which it occurs. [2]

From (a), the minimum value is $-\frac{155}{4}$ and it occurs when x is $-\frac{5}{2}$

(c) Use your answer to **part (a)** to solve the equation $3y^{\frac{2}{3}}+15y^{\frac{1}{3}}-20=0$, giving your answers correct to three significant figures. [3]

From (a) $3x^2+15x-20 = 3\left(x+\frac{5}{2}\right)^2-\frac{155}{4}$

$3y^{2/3}+15y^{1/3}-20=0 \Rightarrow y^{1/3}=x$

$$\Rightarrow 3\left(y^{1/3}+\tfrac{5}{2}\right)^2-\frac{155}{4}=0$$
$$3\left(y^{1/3}+\tfrac{5}{2}\right)^2=\frac{155}{4}$$
$$\left(y^{1/3}+\tfrac{5}{2}\right)^2=\frac{\frac{155}{4}}{3}=\frac{155}{12}$$
$$\Rightarrow y^{1/3}+\tfrac{5}{2}=\pm\sqrt{\frac{155}{12}}$$
$$y^{1/3}=-\frac{5}{2}\pm\sqrt{\frac{155}{12}}$$
$$\therefore y=\left(-\frac{5}{2}\pm\sqrt{\frac{155}{12}}\right)^3$$
$$\Rightarrow y=\left(-\frac{5}{2}-\sqrt{\frac{155}{12}}\right)^2=-226,\quad y=\left(-\frac{5}{2}+\sqrt{\frac{155}{12}}\right)^3=1.31$$
$$\therefore y=-226,\quad y=1.31$$