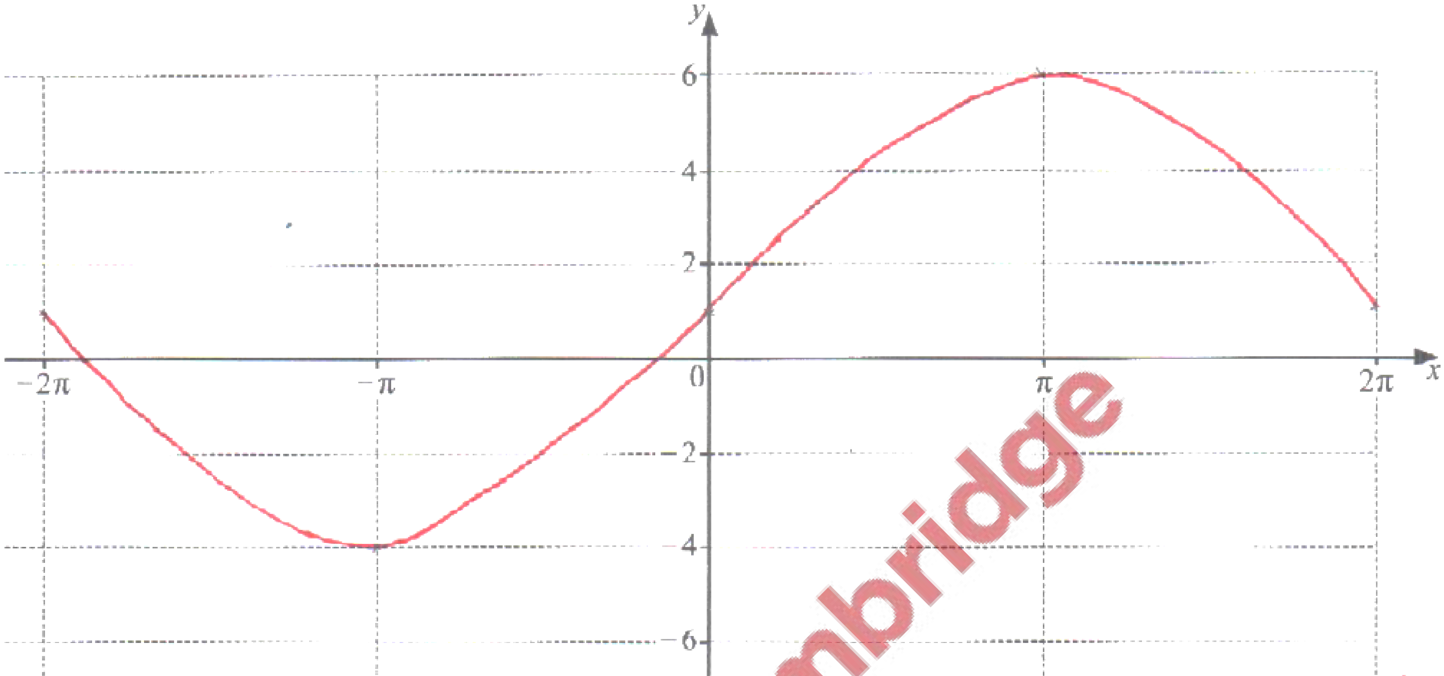
**1.** Nov/2022/Paper_0606_11/No.2

(a) On the axes, sketch the graph of $y = 5\sin\frac{x}{2} + 1$ for $-2\pi \leqslant x \leqslant 2\pi$. [3]




Maximum $(\pi, 6)$

Minimum $(-\pi, -4)$

| $x$ | $-2\pi$ | $-\pi$ | 0 | $\pi$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | 1 | $-4$ | 1 | 6 | 1 |

(b) Write down the amplitude of $5\sin\frac{x}{2} + 1$. [1]

The amplitude represents half the distance between the maximum and minimum values of the function. Given $y = a\sin x$, amplitude $= |a|$

$\Rightarrow$ amplitude $= |5| = 5$

(c) Write down the period of $5\sin\frac{x}{2} + 1$. [1]

Given the function $y = 5\sin\left(\frac{1}{2}x\right) + 1$,

Period $: \dfrac{2\pi}{\left|\frac{1}{2}\right|} = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

**2.** Nov/2022/Paper_0606_11/No.12

**(a)** Show that $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} = 2\sin\theta\sec^2\theta$. [3]

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} = \frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}\theta(\operatorname{cosec}\theta + 1) - 1(\operatorname{cosec}\theta + 1)}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta + \operatorname{cosec}\theta - \operatorname{cosec}\theta - 1}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$$

But $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta - 1 = \cot^2\theta$

$$\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}\theta}{\cot^2\theta}$$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$

$$= \frac{2}{\sin\theta} \times \frac{1}{\cot^2\theta} = \frac{2}{\sin\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos^2\theta} = 2\sin\theta\sec^2\theta$$

**(b)** Hence solve the equation $\dfrac{1}{\operatorname{cosec}2\phi - 1} + \dfrac{1}{\operatorname{cosec}2\phi + 1} = 4\sin 2\phi$, for $-90° \leqslant \phi \leqslant 90°$. [6]

From (a) $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} = 2\sin\theta\sec^2\theta$

$\Rightarrow \dfrac{1}{\operatorname{cosec}2\phi - 1} + \dfrac{1}{\operatorname{cosec}2\phi + 1} = 2\sin 2\phi\sec^2 2\phi$

$\Rightarrow 2\sin 2\phi\sec^2 2\phi = 4\sin 2\phi$

$2\sin 2\phi\sec^2 2\phi - 4\sin 2\phi = 0$

$2\sin 2\phi(\sec^2 2\phi - 2) = 0$

$\Rightarrow 2\sin\phi = 0$, $\sec^2 2\phi - 2 = 0$

$\sin 2\phi = 0$

$2\phi = \sin^{-1}(0) = 0°, \pm 180°$

$\Rightarrow \phi = 0°, \pm 90°$

$\sec^2 2\phi - 2 = 0$

$\Rightarrow \dfrac{1}{\cos^2 2\phi} = 2$

$\Rightarrow \cos^2 2\phi = \frac{1}{2}$

$\cos 2\phi = \pm\sqrt{\frac{1}{2}}$

$2\phi = \cos^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$

$2\phi = \pm 45°, \pm 135°$

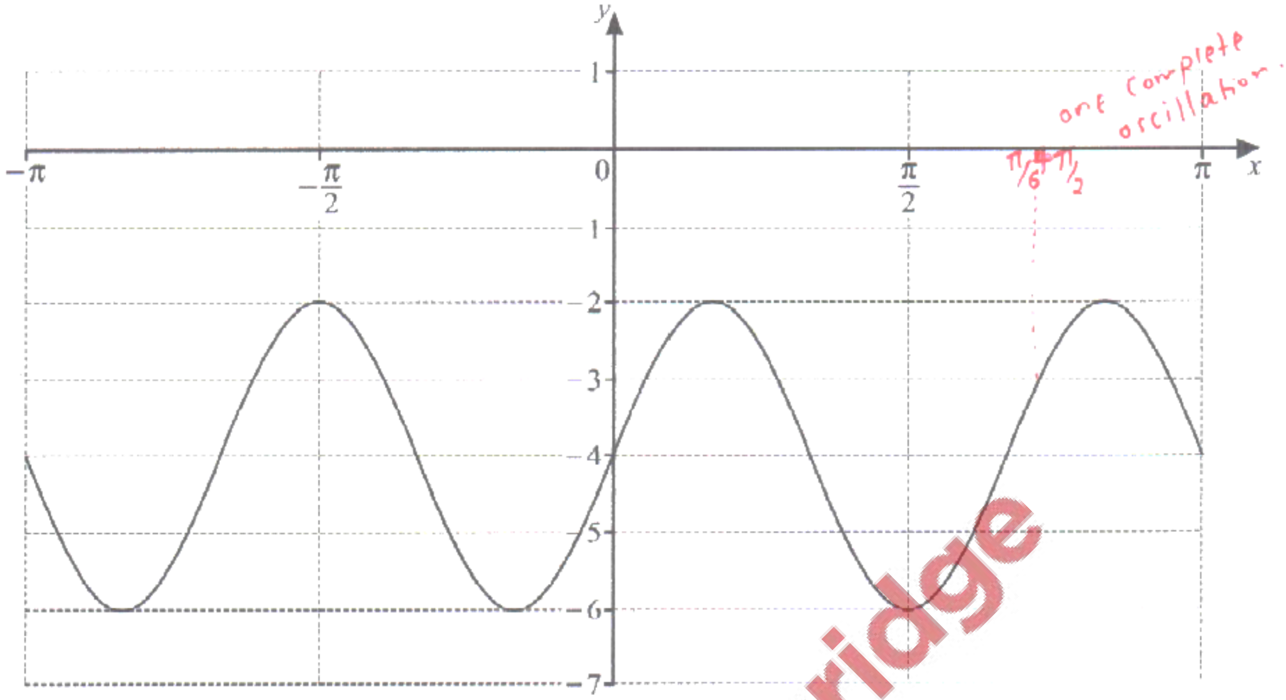
$\phi = \dfrac{\pm 45°}{2}, \dfrac{\pm 135°}{2}$

$\phi = \pm 22.5°, \pm 67.5°$

$\therefore$ In the interval $-90° \leq \phi \leq 90°$

$\phi = \pm 22.5°, 0°, \pm 67.5°, \pm 90°$

**3.** Nov/2022/Paper_0606_12/No.1




The diagram shows the graph of $y = a\sin bx + c$, where $a$, $b$ and $c$ are integers. Find the values of $a$, $b$ and $c$. [3]

$a$ = amplitude = $\dfrac{\text{Maximum Value} - \text{minimum Value}}{2}$

$= \dfrac{-2-(-6)}{2} = \dfrac{-2+6}{2} = \dfrac{4}{2} = 2$

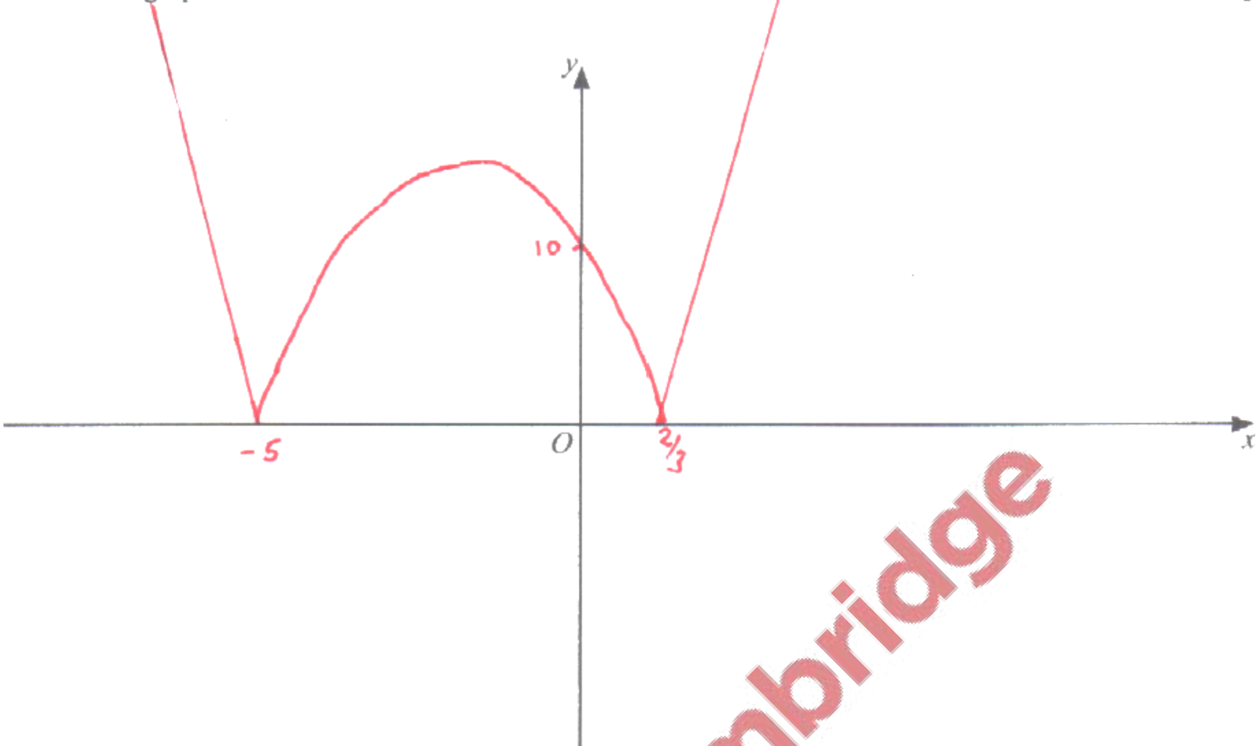
$\therefore a = 2$

$b = \dfrac{2\pi}{\text{Period}} = \dfrac{2\pi}{\frac{\pi}{2}+\frac{\pi}{6}} = \dfrac{2\pi}{\frac{2}{3}\pi} = 3$

$c$ = $y$-intercept $= -4$

**4.** Nov/2022/Paper_0606_12/No.4

**(a)** On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes. [4]






$\Rightarrow \; 3x^2 + 13x - 10 - K = 0$

$3x^2 + 13x - (10 + K) = 0$

If there are exactly 2 distinct roots, the discriminant $b^2 - 4ac > 0$

$b = 13, \; a = 3, \; c = -(10+k)$

$13^2 - 4(3)(-(10+k)) > 0$

$169 + 12(10+k) > 0$

Consider $169 + 12(10+k) = 0$

$\Rightarrow \; \frac{\cancel{12}k}{\cancel{12}} = \frac{-289}{12}$

$K = \frac{-289}{12}$ or $-24.1$

$\therefore$ Critical value is $-\frac{289}{12}$

$\Rightarrow$ For the inequality $169 + 12(10+k) > 0$

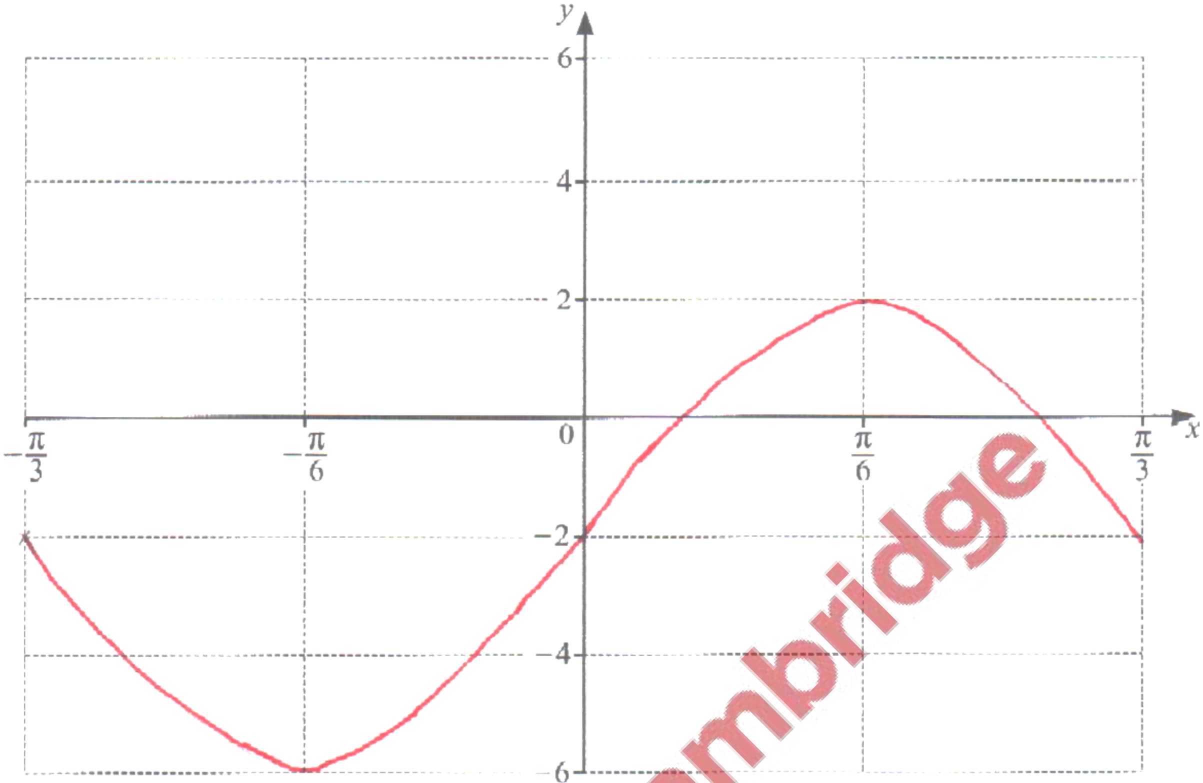
$K > \frac{289}{12}$ or $k > 24.1$

and

$K = 0$

**5.** Nov/2022/Paper_0606_13/No.1

On the axes, sketch the graph of $y = 4\sin 3x - 2$ for $-\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$. [3]




When $x = -\pi/3$, $y = 4\sin\left(3 \times -\pi/3\right) - 2$

$y = 4\sin(-\pi) - 2$

$= -2$

$x = -\pi/6$, $y = 4\sin\left(3 \times -\pi/6\right) - 2$

$= 4\sin\left(-\frac{\pi}{2}\right) - 2$

$= -4 - 2$

$= -6$

When $x = 0$, $y = 4\sin 0 - 2$

$= -2$

When $x = \pi/6$, $y = 4\sin\left(3 \times \pi/6\right) - 2$

$= 4\sin\left(\frac{\pi}{2}\right) - 2$

$= 4 - 2$

$= 2$

When $x = \pi/3$, $y = \left(4\sin\left(3 \times \pi/3\right)\right) - 2$

$= 4\sin(\pi) - 2$

$= 0 - 2$

$= -2$

| x | $-\frac{\pi}{3}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ |
|---|---|---|---|---|---|
| y | -2 | -6 | -2 | 2 | -2 |

**6.** Nov/2022/Paper_0606_13/No.10

Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where $x$ is in radians, for $-1.5 \leqslant x \leqslant 1.5$. [5]

$$\frac{\sqrt{2}}{2}\cos(3x+1.2) = \sin(3x+1.2)$$

$$\Rightarrow \frac{\sqrt{2}}{2} = \frac{\sin(3x+1.2)}{\cos(3x+1.2)}$$

$$\frac{\sqrt{2}}{2} = \tan(3x+1.2)$$

$$\Rightarrow 3x + 1.2 = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

tan is positive in the first and third quadrant.

$$\Rightarrow 3x + 1.2 = 0.615,\ -\pi + 0.615 = -2.526,\ \pi + 0.615 = 3.757$$

$$3x + 1.2 = 0.615,\ -2.526,\ 3.757$$

$$3x = 0.615 - 1.2,\ -2.526 - 1.2,\ 3.757 - 1.2$$

$$3x = -0.585,\ -3.726,\ 2.557$$

$$x = -\frac{0.585}{3},\ \frac{-3.726}{3},\ \frac{2.557}{3}$$

$$x = -0.195,\ -1.24,\ 0.852$$

In the interval $-1.5 \leq x \leq 1.5$,

$$x = -0.195,\ -1.25,\ 0.852$$

**7.** Nov/2022/Paper_0606_22/No.7

(a) Show that $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=2\operatorname{cosec} x$. [4]

Consider LHS

$$\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}$$

$$\frac{\sin x(\sin x)+(1-\cos x)(1-\cos x)}{(1-\cos x)\sin x}$$

$$\frac{\sin^2 x+1(1-\cos x)-\cos x(1-\cos x)}{(1-\cos x)\sin x}$$

$$\frac{\sin^2 x+1-\cos x-\cos x+\cos^2 x}{(1-\cos x)\sin x}$$

$$\frac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$$

$$\frac{(\sin^2 x+\cos^2 x)+1-2\cos x}{(1-\cos x)\sin x}$$

But $\sin^2 x+\cos^2 x=1$

$$\Rightarrow \frac{1+1-2\cos x}{(1-\cos x)\sin x}$$

$$=\frac{2-2\cos x}{(1-\cos x)\sin x}$$

$$=\frac{2\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}\sin x}$$

$$=\frac{2}{\sin x}$$ but $\frac{1}{\sin x}=\operatorname{cosec} x$

$$=2\operatorname{cosec} x$$ As required

(b) Hence solve the equation $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=3\sin x-1$ for $0°<x<360°$. [4]

From (a) $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=2\operatorname{cosec} x=\frac{2}{\sin x}$

$$\Rightarrow \frac{2}{\sin x}=3\sin x-1$$

$$\sin x(3\sin x-1)=2$$

$$3(\sin x)^2-\sin x-2=0$$

Let $\sin x=y$

$$3y^2-y-2=0$$

$$3y^2-3y+2y-2=0$$

$$3y(y-1)+2(y-1)=0$$

$$(y-1)(3y+2)=0$$

$y-1=0$, $3y+2=0$

$\Rightarrow y=1$ $\quad y=-2/3$

$\Rightarrow \sin x=1,\ -2/3$

When $\sin x=1$

$x=\sin^{-1}(1)$

$=90°$

When $\sin x=-2/3$

$x=\sin^{-1}(-2/3)$

$x=-41.8°,\ 180°+41.8°=221.8°$

$360°-41.8°=318.2°$

$\therefore$ In the interval $0°<x<360°$

$x=90°,\ 221.8°,\ 318.2°$

**8.** Nov/2022/Paper_0606_23/No.5

(a) Show that $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=2\sec x$. [4]

Consider the LHS:

$$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}$$

$$\frac{\cos x(\cos x)+(1-\sin x)(1-\sin x)}{(1-\sin x)\cos x}$$

$$\frac{\cos^2 x+1(1-\sin x)-\sin x(1-\sin x)}{(1-\sin x)\cos x}$$

$$\frac{\cos^2 x+1-\sin x-\sin x+\sin^2 x}{(1-\sin x)\cos x}$$

$$\frac{(\cos^2 x+\sin^2 x)+1-2\sin x}{(1-\sin x)\cos x}$$

But $\cos^2 x+\sin^2 x=1$

$$\Rightarrow \frac{1+1-2\sin x}{(1-\sin x)\cos x}$$

$$\frac{2-2\sin x}{(1-\sin x)\cos x}=\frac{2\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}\cos x}$$

$$=\frac{2}{\cos x}=2\sec x \qquad \boxed{\frac{1}{\cos x}=\sec x}$$

(b) Hence solve the equation $\dfrac{\cos\frac{\theta}{2}}{1-\sin\frac{\theta}{2}}+\dfrac{1-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=8\cos^2\frac{\theta}{2}$ for $-360^\circ<\theta<360^\circ$. [4]

From (a) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=2\sec x$

Comparing the two equation $x=\dfrac{\theta}{2}$

$$\Rightarrow \frac{\cos\frac{\theta}{2}}{1-\sin\frac{\theta}{2}}+\frac{1-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=2\sec\frac{\theta}{2}=8\cos^2\frac{\theta}{2}$$

$$\Rightarrow \frac{2}{\cos\frac{\theta}{2}}=8\cos^2\frac{\theta}{2} \qquad \left(\text{from } \sec\frac{\theta}{2}=\frac{1}{\cos\frac{\theta}{2}}\right)$$

$$\left(\cos\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}\right)=\frac{2}{8}$$

$$\cos^3\frac{\theta}{2}=\frac{1}{4} \Rightarrow \cos\frac{\theta}{2}=\sqrt[3]{\frac{1}{4}}$$

$$\frac{\theta}{2}=\cos^{-1}\left(\sqrt[3]{\frac{1}{4}}\right)$$

$$\frac{\theta}{2}=\pm 50.95^\circ$$

$$\Rightarrow \theta=\pm 50.95^\circ\times 2=\pm 101.9^\circ$$

$$\therefore \theta=\pm 101.9^\circ$$