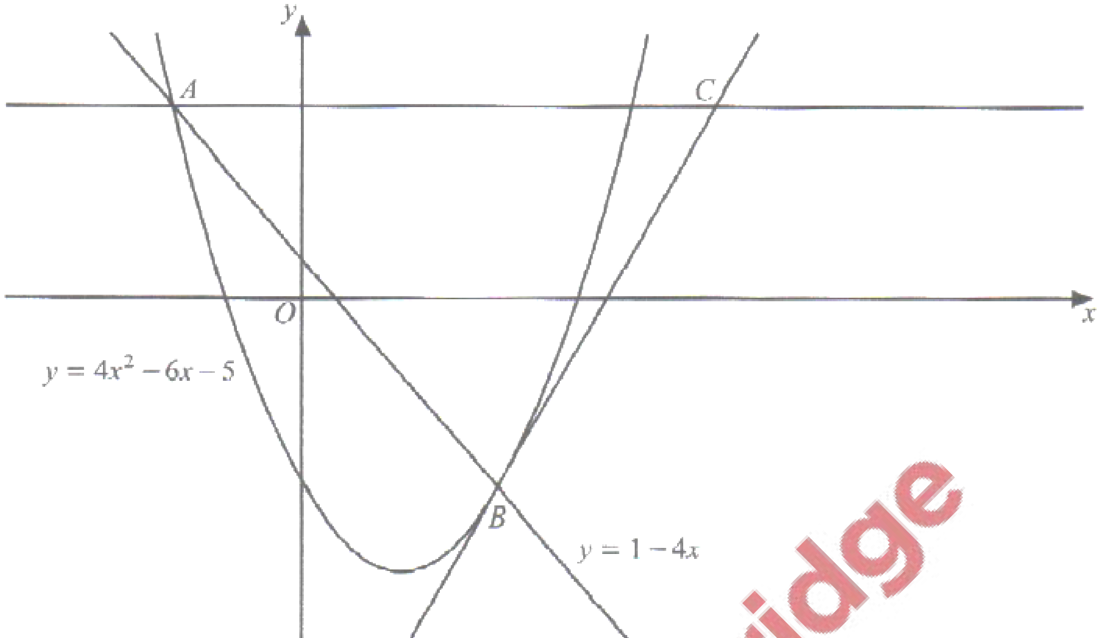
<u>**Quadratic functions – 2023 Additional Math 0606**</u>

1. Nov/2023/Paper_0606/11/No.8



The diagram shows the line $y = 1 - 4x$ meeting the curve $y = 4x^2 - 6x - 5$ at the points A and B. The tangent to the curve at B meets the horizontal line through A at the point C. Find the x-coordinate of C, giving your answer correct to 2 decimal places. [10]

Substitute equation of the line into the equation of the curve.

$$1 - 4x = 4x^2 - 6x - 5$$

$$\Rightarrow \quad 4x^2 - 6x - 5 - 1 + 4x = 0$$

$$4x^2 - 2x - 6 = 0$$

$$2(2x^2 - x - 3) = \frac{0}{2}$$

$$\Rightarrow \quad 2x^2 - x - 3 = 0$$ (solving for x using the quadratic formular)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2 \times 2}$$

$$x = \frac{1 \pm 5}{4}$$

$$x = \frac{1-5}{4} = -1, \quad x = \frac{1+5}{4} = \frac{3}{2}$$

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A is on the negative x-axis while B is on the positive x-axis.

$\therefore$ x-Coordinate of A $= -1$

x-Coordinate of B $= \frac{3}{2}$

When $x = -1$, $y = 1 - 4(-1) = 5$

When $x = \frac{3}{2}$, $y = 1 - 4\left(\frac{3}{2}\right) = -5$

$\therefore A(-1, 5)$ and $B\left(\frac{3}{2}, -5\right)$

Equation of the line passing through point A and C is $y = 5$

Gradient of the tangent $= \frac{dy}{dx} = \frac{d}{dx}(4x^2 - 6x - 5)$

$= 8x - 6$

At $B\left(\frac{3}{2}, -5\right)$, $\frac{dy}{dx} = 8\left(\frac{3}{2}\right) - 6$

$= 6$

Equation of the tangent at B:

$y - (-5) = 6\left(x - \frac{3}{2}\right)$

$y + 5 = 6x - 9$

$y = 6x - 9 - 5$

$y = 6x - 14$

The line passing through A and C meets the tangent at point C.

Substitute equation of line AC into the equation of the tangent.

$\Rightarrow 5 = 6x - 14$

$6x = 5 + 14$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{19}{6}$ $\Rightarrow x = 3.17$ (2dp)

$\therefore$ x-coordinate of $C = 3.17$

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2. Nov/2023/Paper_0606/13/No.7

Solve the equation $6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0$. Give your answers in exact form. [4]

$$\left(6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0\right) \times x^{\frac{1}{3}}$$

$$6\left(x^{\frac{1}{3}}\right)^2 - 2x^{\left(-\frac{1}{3}+\frac{1}{3}\right)} - x^{\frac{1}{3}} = 0$$

$$6\left(x^{\frac{1}{3}}\right)^2 - 2 - x^{\frac{1}{3}} = 0$$

Let $x^{\frac{1}{3}} = y$

$$\Rightarrow 6y^2 - 2 - y = 0$$

$$6y^2 - y - 2 = 0$$ (solve for y using the quadratic formular).

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2 \times 6}$$

$$y = \frac{1 \pm 7}{12}$$

$$y = \frac{1-7}{12} = -\frac{1}{2}, \quad y = \frac{1+7}{12} = \frac{2}{3}$$

$$\Rightarrow x^{\frac{1}{3}} = -\frac{1}{2}, \quad x^{\frac{1}{3}} = \frac{2}{3}$$

$$x = \left(-\frac{1}{2}\right)^3, \quad x = \left(\frac{2}{3}\right)^3$$

$$\therefore x = -\frac{1}{8}, \quad x = \frac{8}{27}$$

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3. Nov/2023/Paper_0606/21/No.1

(a) Write $19-12x-3x^2$ in the form $a(x+b)^2+c$ where a, b and c are integers. [4]

$19-12x-3x^2 = -3x^2-12x+19$

$= -3\left[x^2+4x-\frac{19}{3}\right]$

Using Completing square method:

$= -3\left[x^2+4x+\left(\frac{1}{2}(4)\right)^2-\left(\frac{1}{2}(4)\right)^2-\frac{19}{3}\right]$

$= -3\left[x^2+4x+(2)^2-(2)^2-\frac{19}{3}\right]$

$= -3\left[(x+2)^2-4-\frac{19}{3}\right]$

$= -3\left[(x+2)^2-\frac{31}{3}\right]$

$= -3(x+2)^2+\left(-3\times-\frac{31}{3}\right)$

$= -3(x+2)^2+31$

(b) Hence find the maximum value of $19-12x-3x^2$ and the value of x at which this maximum occurs. [2]

From (a) $19-12x-3x^2 = -3(x+2)^2+31$

$\therefore$ Maximum Value is 31 when $x=-2$

(c) Use your answer to part (a) to solve the equation $19-12\sqrt{u}-3u=0$. [3]

Compare $19-12\sqrt{u}-3u$ with $19-2x-3x^2$

$\Rightarrow x=\sqrt{u}$

$\Rightarrow 19-12\sqrt{u}-3u = -3(\sqrt{u}+2)^2+31=0$

$\frac{-3(\sqrt{u}+2)^2}{-3} = \frac{-31}{-3}$

$(\sqrt{u}+2)^2 = \frac{31}{3}$

$\sqrt{u}+2 = \pm\sqrt{\frac{31}{3}}$

$\sqrt{u} = -2\pm\sqrt{\frac{31}{3}}$

$u = \left(-2+\sqrt{\frac{31}{3}}\right)^2$

$\therefore u = 1.48$ (3sf)

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4. Nov/2023/Paper_0606/21/No.6

Find the value of the constant a for which the line $y = (2a+1)x - 10$ is a tangent to the curve $y = ax^2 - 5x + 2$. [6]

Substitute the equation of the tangent into the equation of the curve.

$\Rightarrow (2a+1)x - 10 = ax^2 - 5x + 2$

$2ax + x - 10 = ax^2 - 5x + 2$

$ax^2 - 5x + 2 - 2ax - x + 10 = 0$

$ax^2 - 6x - 2ax + 12 = 0$

$ax^2 - (6+2a)x + 12 = 0$

Discriminant $b^2 - 4ac = 0$ Since there is only one real root.

$b^2 - 4ac = (-(6+2a))^2 - 4(a)(12) = 0$

$6(6+2a) + 2a(6+2a) - 48a = 0$

$36 + 12a + 12a + 4a^2 - 48a = 0$

$4a^2 - 24a + 36 = 0$

$\frac{\cancel{4}(a^2 - 6a + 9)}{\cancel{4}} = \frac{0}{4}$

$a^2 - 6a + 9 = 0$

Solving for 'a' using the quadratic formula:

$a = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2 \times 1}$

$= \frac{6 \pm 0}{2}$

$\therefore a = 3$

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5. Nov/2023/Paper_0606/21/No.8

DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(2-\sqrt{10})x^2+x+(2+\sqrt{10})=0$, giving your answers in the form $a+b\sqrt{10}$, where a and b are rational. [7]

The quadratic equation is of the form $ax^2+bx+c=0$.
Solving for x using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4(2-\sqrt{10})(2+\sqrt{10})}}{2\times(2-\sqrt{10})}$$

$$(2-\sqrt{10})(2+\sqrt{10})=2(2+\sqrt{10})-\sqrt{10}(2+\sqrt{10})$$
$$=4+2\sqrt{10}-2\sqrt{10}-10$$
$$=-6$$

$$x=\frac{-1\pm\sqrt{1^2-4(-6)}}{2(2-\sqrt{10})}=\frac{-1\pm5}{2(2-\sqrt{10})}$$

$$x=\frac{-1-5}{2(2-\sqrt{10})}=\frac{-6}{2(2-\sqrt{10})}=\frac{-3}{2-\sqrt{10}},\quad x=\frac{-1+5}{2(2-\sqrt{10})}=\frac{4}{2(2-\sqrt{10})}=\frac{2}{2-\sqrt{10}}$$

For $x=\frac{-3}{2-\sqrt{10}}\times\frac{2+\sqrt{10}}{2+\sqrt{10}}$ (Rationalise the denominator)

$$x=\frac{-3(2+\sqrt{10})}{-6}=\frac{2+\sqrt{10}}{2}=1+\frac{1}{2}\sqrt{10}$$

For $x=\frac{2}{2-\sqrt{10}}\times\frac{2+\sqrt{10}}{2+\sqrt{10}}$

$$=\frac{2(2+\sqrt{10})}{-6}=-\frac{1}{3}(2+\sqrt{10})=-\frac{2}{3}-\frac{1}{3}\sqrt{10}$$

$\therefore x=1+\frac{1}{2}\sqrt{10},\quad x=-\frac{2}{3}-\frac{1}{3}\sqrt{10}$

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6. Nov/2023/Paper_0606/22/No.2

Find the non-zero value of k for which the line $y = -2x - 6k - 1$ is a tangent to the curve $y = x(x + 2k)$. [5]

Substitute equation of the line into the equation of the curve

$$-2x - 6k - 1 = x(x + 2k)$$
$$-2x - 6k - 1 = x^2 + 2xk$$
$$\Rightarrow x^2 + 2xk + 2x + 6k + 1 = 0$$
$$x^2 + (2k + 2)x + 6k + 1 = 0$$

The discriminant $b^2 - 4ac = 0$ since there is only one real root.

$$b^2 - 4ac = (2k + 2)^2 - 4(1)(6k + 1) = 0$$
$$\Rightarrow 2k(2k + 2) + 2(2k + 2) - 4(6k + 1) = 0$$
$$4k^2 + 4k + 4k + 4 - 24k - 4 = 0$$
$$4k^2 + 8k + 4 - 24k - 4 = 0$$
$$\Rightarrow 4k^2 - 16k = 0$$
$$4k(k - 4) = 0$$
$$4k = 0, \quad k - 4 = 0$$
$$k = 0, \quad k = 4$$

But k is non-zero

$\therefore k = 4$

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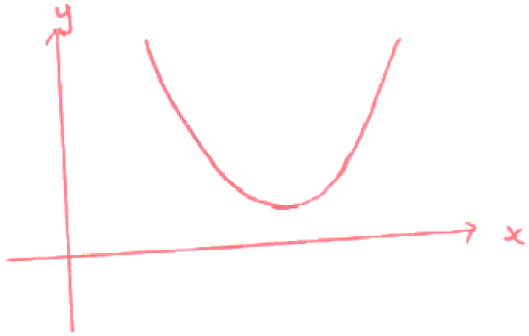
7. Nov/2023/Paper_0606/23/No.2

Find the values of k for which the curve $y = x^2 + kx + (4k - 15)$ is completely above the x-axis. [4]



If the curve is completely above the x-axis there are zero real roots, so the discriminant $b^2 - 4ac < 0$

$$b^2 - 4ac = k^2 - 4(1)(4k - 15) < 0$$

$$k^2 - 16k + 60 < 0$$

For critical values consider $k^2 - 16k + 60 = 0$

Solving for k using the quadratic formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(+60)}}{2 \times 1}$$

$$k = \frac{16 \pm \sqrt{16}}{2} = \frac{16 \pm 4}{2}$$

$$k = \frac{16 - 4}{2} = 6, \quad k = \frac{16 + 4}{2} = 10$$

$\therefore$ The critical values are 6 and 10.

$\therefore$ $k > 6$, $k < 10$

$\therefore$ $6 < k < 10$

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8. March/2023/Paper_0606/12/No.1

Find the exact values of k such that the straight line $y = 1 - k - x$ is a tangent to the curve $y = kx^2 + x + 2k$. [4]

Substitute equation of the tangent into equation of the curve.

$1 - k - x = kx^2 + x + 2k$

$kx^2 + x + 2k + x + k - 1 = 0$

$kx^2 + 2x + 3k - 1 = 0$

The quadratic equation has only one solution (one root)

So the discriminant $b^2 - 4ac = 0$.

$a = k, \quad b = 2, \quad c = 3k - 1$

$b^2 - 4ac = 2^2 - 4(k)(3k-1) = 0$

$\Rightarrow 4 - 4k(3k-1) = 0$

$4 - 12k^2 + 4k = 0$

$\frac{4(1 - 3k^2 + k)}{4} = \frac{0}{4} \Rightarrow 1 - 3k^2 + k = 0$

$3k^2 - k - 1 = 0$

Solving for k using the quadratic formular

$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$k = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2 \times 3}$

$\therefore k = \frac{1 \pm \sqrt{13}}{6}$

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9. June/2023/Paper_0606/11/No.1

(a) Write $5x^2 - 14x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be found. [3]

$5\left[x^2 - \frac{14}{5}x + \frac{8}{5}\right]$

Using Completing square method

$5\left[x^2 - \frac{14}{5}x + \left(\frac{1}{2}\left(-\frac{14}{5}\right)\right)^2 - \left(\frac{1}{2}\left(-\frac{14}{5}\right)\right)^2 + \frac{8}{5}\right]$

$= 5\left[x^2 - \frac{14}{5}x + \left(-\frac{7}{5}\right)^2 - \left(-\frac{7}{5}\right)^2 + \frac{8}{5}\right]$

$= 5\left[\left(x - \frac{7}{5}\right)^2 - \frac{49}{25} + \frac{8}{5}\right]$

$= 5\left[\left(x - \frac{7}{5}\right)^2 - \frac{9}{25}\right]$

$= 5\left(x - \frac{7}{5}\right)^2 - \left(5 \times \frac{9}{25}\right)$

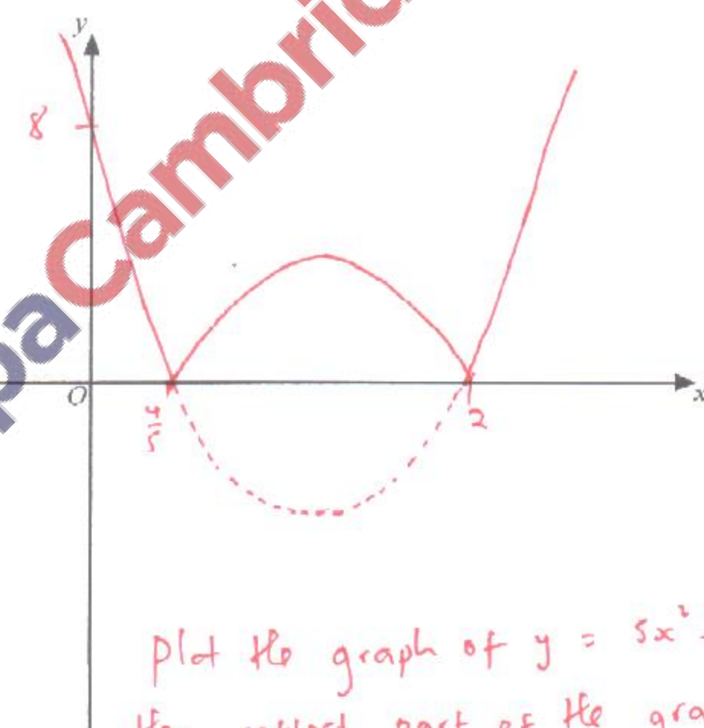
$= 5\left(x - \frac{7}{5}\right)^2 - \frac{9}{5}$

(b) Hence write down the coordinates of the stationary point on the curve $y = 5x^2 - 14x + 8$. [2]

From part (a) $y = 5\left(x - \frac{7}{5}\right)^2 - \frac{9}{5}$

$= \left(\frac{7}{5}, -\frac{9}{5}\right)$

(c) On the axes below, sketch the graph of $y = |5x^2 - 14x + 8|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



Consider $y = 5x^2 - 14x + 8$

When $x = 0$, $y = 8 = 8$

When $y = 0$, $5\left(x - \frac{7}{5}\right)^2 - \frac{9}{5} = 0$

$\frac{5\left(x - \frac{7}{5}\right)^2}{5} = \frac{\frac{9}{5}}{5}$

$\left(x - \frac{7}{5}\right)^2 = \frac{9}{25}$

$x - \frac{7}{5} = \pm\frac{3}{5}$

$x = \frac{7}{5} \pm \frac{3}{5} \Rightarrow x = \frac{4}{5}, 2$

Plot the graph of $y = 5x^2 - 14x + 8$ then reflect part of the graph below the x-axis in the x-axis.

(d) Write down the range of values of k for which the equation $|5x^2 - 14x + 8| = k$ has 4 distinct roots. [2]

From part (b) Coordinates of the stationary point are $\left(\frac{7}{5}, -\frac{9}{5}\right)$

$\therefore\ 0 < k < \frac{9}{5}$

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