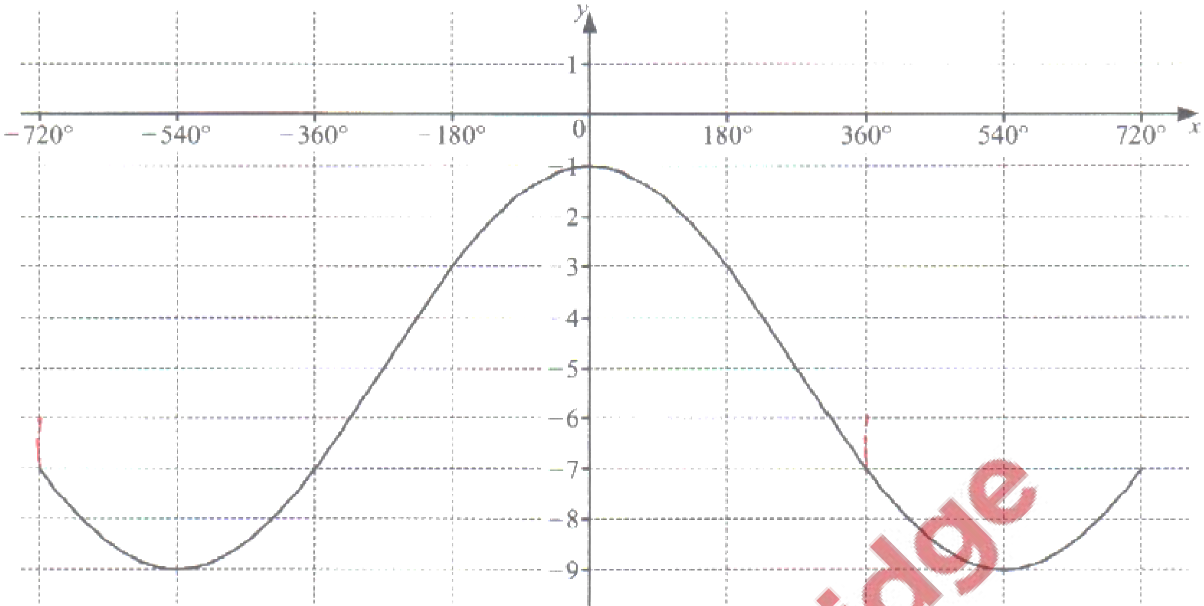
Trigonometry – 2023 Additional Math 0606

1. Nov/2023/Paper_0606/11/No.1



The diagram shows part of the graph of $y = a\cos\left(\frac{x}{b}\right) + c$, where a, b and c are integers. Find the values of a, b and c. [3]

amplitude   period $= \frac{360°}{\frac{1}{b}}$ vertical translation

$$a = \frac{-1-(-9)}{2} = \frac{8}{2} = 4$$

$$\text{Period} = 360° + 720° = 1080°$$

$$1080° = \frac{360°}{\frac{1}{b}} \Rightarrow \frac{1080°}{b} = 360°$$

$$\Rightarrow b = \frac{1080°}{360°} = 3$$

$$y = 4\cos\frac{x}{3} \longrightarrow y = 4\cos\frac{x}{3} + c$$

Translation of 5 units down

$\therefore c = -5$

$b = 3$

$a = 4$

2. Nov/2023/Paper_0606/12/No.2

The function g is defined by $g(x) = 5\sin\frac{3x}{4} - 2$ for all values of x.

(a) Write down the amplitude of g. [1]

amplitude = 5

(b) Write down the period of g in degrees. [1]

$$\text{Period} = \frac{360^\circ}{\frac{3}{4}} = 360^\circ \times \frac{4}{3} = 480^\circ$$

(c) On the axes, sketch the graph of $y = g(x)$, for $-180^\circ \leqslant x \leqslant 180^\circ$. [3]

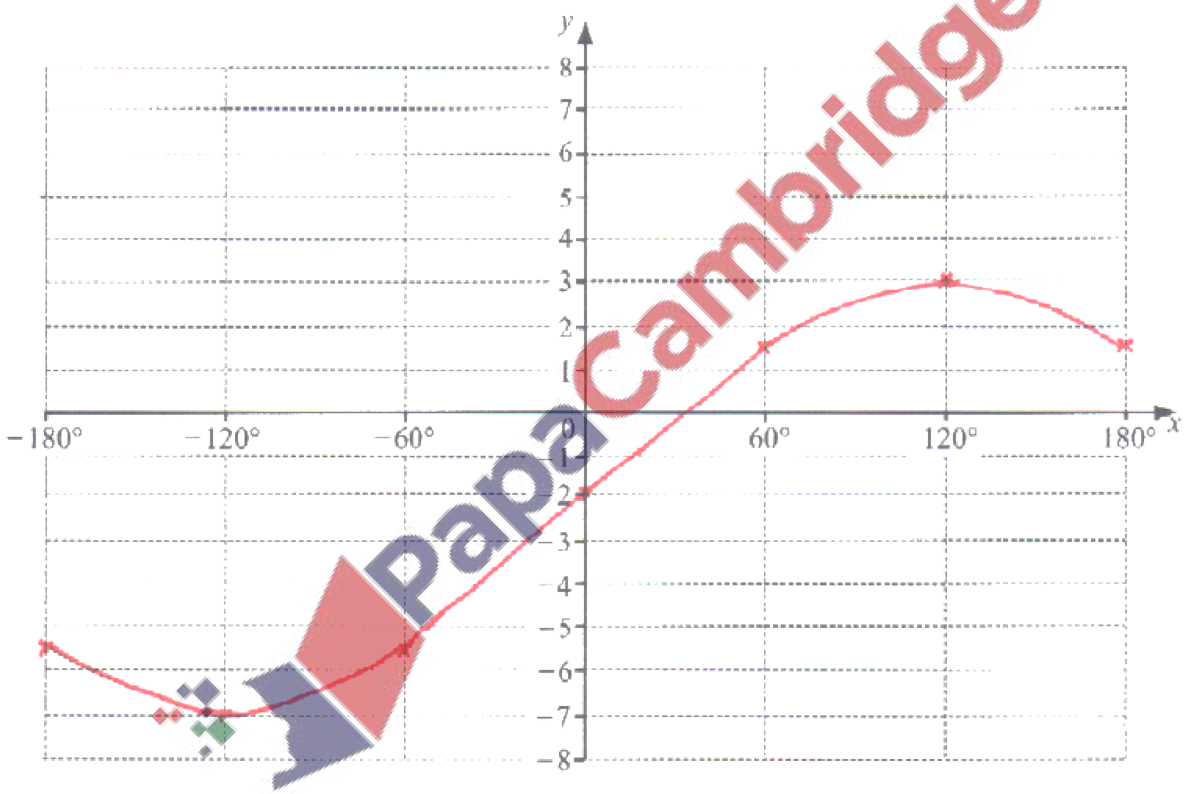


Table of Values

x	-180°	-120°	-60°	0	60°	120°	180°
y	-5.5	-7	-5.5	-2	1.5	3	1.5

3. Nov/2023/Paper_0606/12/No.5

Solve the equation $3\sec^2\left(2\theta+\frac{\pi}{6}\right)=4$ for $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, giving your answers in terms of π. [5]

$$\frac{\cancel{3}\sec^2\left(2\theta+\frac{\pi}{6}\right)}{\cancel{3}}=\frac{4}{3}$$

$$\Rightarrow \sec^2\left(2\theta+\frac{\pi}{6}\right)=\frac{4}{3}$$

$$\sec\left(2\theta+\frac{\pi}{6}\right)=\sqrt{\frac{4}{3}}$$

$$\sec\left(2\theta+\frac{\pi}{6}\right)=\pm\frac{2}{\sqrt{3}}, \text{ but } \sec\theta=\frac{1}{\cos\theta}$$

$$\Rightarrow \frac{1}{\cos\left(2\theta+\frac{\pi}{6}\right)}=\pm\frac{2}{\sqrt{3}}$$

$$\cos\left(2\theta+\frac{\pi}{6}\right)=\pm\frac{\sqrt{3}}{2}$$

$$2\theta+\frac{\pi}{6}=\cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

Cos is positive in the first and fourth quadrants only.

$$\Rightarrow 2\theta+\frac{\pi}{6}=-\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2\theta=\left(-\frac{\pi}{6}-\frac{\pi}{6}\right), \left(\frac{\pi}{6}-\frac{\pi}{6}\right), \left(\frac{5\pi}{6}-\frac{\pi}{6}\right)$$

$$2\theta=-\frac{\pi}{3}, 0, \frac{2\pi}{3}$$

$$\theta=\frac{1}{2}\left(-\frac{\pi}{3}\right), \frac{1}{2}(0), \frac{1}{2}\left(\frac{2\pi}{3}\right)$$

$$\therefore \theta=-\frac{\pi}{6}, 0, \frac{\pi}{3}$$

4. Nov/2023/Paper_0606/13/No.3

On the axes, draw the graph of $y = 2\sin\frac{x}{3} - 1$ for $-360° \leqslant x \leqslant 360°$. [4]

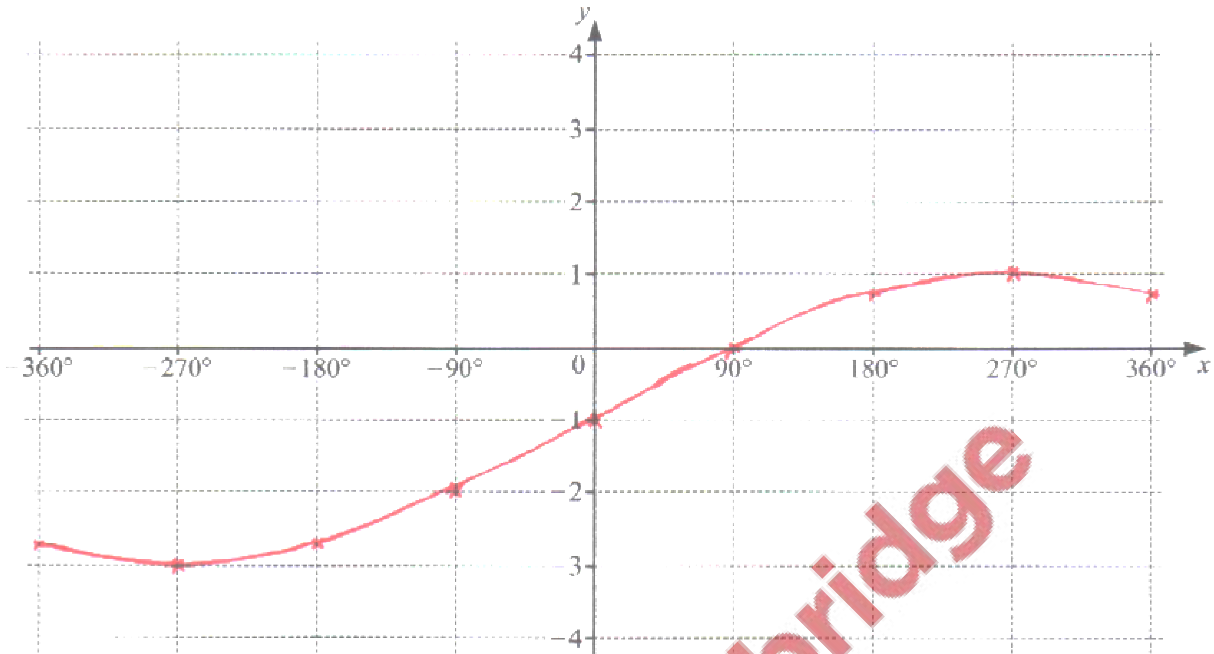


Table of Values

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	-2.7	-3	-2.7	-2	-1	0	0.7	1	0.7

5. Nov/2023/Paper_0606/13/No.12

Solve the equation $3\operatorname{cosec}^2\left(\frac{2x}{3}-\frac{\pi}{3}\right)=4$, for $0 < x \leqslant 3\pi$. Give your answers in terms of π. [5]

$$\frac{\cancel{3}}{\cancel{3}}\operatorname{cosec}^2\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\frac{4}{3}$$

$$\operatorname{cosec}^2\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\frac{4}{3}$$

$$\operatorname{cosec}\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\sqrt{\frac{4}{3}}$$

$$\operatorname{cosec}\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\frac{2}{\sqrt{3}}, \quad \text{but } \operatorname{cosec}\theta=\frac{1}{\sin\theta}$$

$$\Rightarrow \frac{1}{\sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)}=\pm\frac{2}{\sqrt{3}} \Rightarrow \sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\frac{\sqrt{3}}{2}$$

$$\frac{2x}{3}-\frac{\pi}{3}=\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$$\frac{2x}{3}-\frac{\pi}{3}=\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\frac{2x}{3}=\left(\frac{\pi}{3}+\frac{\pi}{3}\right), \left(\frac{2\pi}{3}+\frac{\pi}{3}\right), \left(\frac{4\pi}{3}+\frac{\pi}{3}\right), \left(\frac{5\pi}{3}+\frac{\pi}{3}\right)$$

$$\frac{2x}{3}=\frac{2\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$$

$$x=\frac{3}{2}\left(\frac{2\pi}{3}\right), \frac{3}{2}(\pi), \frac{3}{2}\left(\frac{5\pi}{3}\right), \frac{3}{2}(2\pi)$$

$$\therefore x=\pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$$

6. Nov/2023/Paper_0606/21/No.11

(a) Show that $\dfrac{1}{\sec x - \operatorname{cosec} x} + \dfrac{1}{\sec x + \operatorname{cosec} x} = \dfrac{2\cos x}{1 - \cot^2 x}$. [5]

Consider the Left Hand Side: $\dfrac{1}{\sec x - \operatorname{cosec} x} + \dfrac{1}{\sec x + \operatorname{cosec} x}$

But $\dfrac{1}{\sec x} = \cos x$ and $\dfrac{1}{\operatorname{cosec} x} = \sin x$

$$\frac{1}{\sec x - \operatorname{cosec} x} + \frac{1}{\sec x + \operatorname{cosec} x} = \frac{1}{\frac{1}{\cos x} - \frac{1}{\sin x}} + \frac{1}{\frac{1}{\cos x} + \frac{1}{\sin x}}$$

$$= \frac{1}{\frac{\sin x - \cos x}{\sin x \cos x}} + \frac{1}{\frac{\sin x + \cos x}{\sin x \cos x}}$$

$$= \frac{\sin x \cos x}{\sin x - \cos x} + \frac{\sin x \cos x}{\sin x + \cos x}$$

$$= \frac{\sin x \cos x(\sin x + \cos x) + \sin x \cos x(\sin x - \cos x)}{(\sin x - \cos x)(\sin x + \cos x)}$$

$$= \frac{\sin^2 x \cos x + \sin x \cos^2 x + \sin^2 x \cos x - \sin x \cos^2 x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\sin^2 x \cos x}{\sin^2 x - \cos^2 x}$$

$$= \frac{\cancel{\sin^2 x}(2\cos x)}{\cancel{\sin^2 x}\left(1 - \frac{\cos^2 x}{\sin^2 x}\right)}$$

$$= \frac{2\cos x}{1 - \cot^2 x}$$ As required

(b) Solve the equation $3\tan^2(y+\frac{\pi}{4}) = 1$ for $-2\pi < y < 0$. [4]

$$\tan^2\left(y+\frac{\pi}{4}\right) = \frac{1}{3}$$

$$\tan\left(y+\frac{\pi}{4}\right) = \sqrt{\frac{1}{3}}$$

$$\tan\left(y+\frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{3}}$$

$$y+\frac{\pi}{4} = \tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$$

(tan is positive in the first and third quadrant only)

$$y+\frac{\pi}{4} = -\frac{\pi}{6},\ \frac{\pi}{6},\ \left(-\pi+\frac{\pi}{6}\right),\ \left(-\pi-\frac{\pi}{6}\right)$$

$$y+\frac{\pi}{4} = -\frac{\pi}{6},\ \frac{\pi}{6},\ -\frac{5\pi}{6},\ -\frac{7\pi}{6}$$

$$y = \left(-\frac{\pi}{6}-\frac{\pi}{4}\right),\ \left(\frac{\pi}{6}-\frac{\pi}{4}\right),\ \left(-\frac{5\pi}{6}-\frac{\pi}{4}\right),\ \left(-\frac{7\pi}{6}-\frac{\pi}{4}\right)$$

$$\therefore\ y = -\frac{5}{12}\pi,\ -\frac{\pi}{12},\ -\frac{13}{12}\pi,\ -\frac{17}{12}\pi$$

7. Nov/2023/Paper_0606/22/No.10

(a) By writing $\cot x$ and $\tan x$ in terms of $\cos x$ and $\sin x$, show that

$$\frac{\sin x}{1-\cot x}+\frac{\cos x}{1-\tan x}=\sin x+\cos x.$$ [5]

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

Consider the LHS : $\frac{\sin x}{1-\cot x} + \frac{\cos x}{1-\tan x}$

$$\frac{\sin x}{1-\cot x} + \frac{\cos x}{1-\tan x} = \frac{\sin x}{1-\frac{\cos x}{\sin x}} + \frac{\cos x}{1-\frac{\sin x}{\cos x}}$$

$$= \frac{\sin x}{\frac{\sin x-\cos x}{\sin x}} + \frac{\cos x}{\frac{\cos x-\sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\sin x-\cos x} + \frac{\cos^2 x}{\cos x-\sin x}$$

$$= \frac{\sin^2 x}{\sin x-\cos x} + \frac{\cos^2 x}{-(\sin x-\cos x)}$$

$$= \frac{\sin^2 x}{\sin x-\cos x} + \frac{\cos^2 x}{\sin x-\cos x}$$

$$= \frac{\sin^2 x-\cos^2 x}{\sin x-\cos x}$$

But $\sin^2 x-\cos^2 x$ is a difference of two squares

$\sin^2 x-\cos^2 x = (\sin x-\cos x)(\sin x+\cos x)$

$$= \frac{(\sin x-\cancel{\cos x})(\sin x+\cos x)}{\sin x \cancel{-\cos x}}$$

$= \sin x + \cos x$ As required

(b) Solve the equation $9\cot x + 3\operatorname{cosec} x = \tan x$, for $0° < x < 360°$. [5]

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$, $\tan x = \frac{\sin x}{\cos x}$

$\left(9\frac{\cos x}{\sin x} + \frac{3}{\sin x} = \frac{\sin x}{\cos x}\right) \times \sin x \cos x$

$9\cos^2 x + 3\cos x = \sin^2 x$

But $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

$9\cos^2 x + 3\cos x = 1 - \cos^2 x$

$9\cos^2 x + 3\cos x - 1 + \cos^2 x = 0$

$10\cos^2 x + 3\cos x - 1 = 0$

Let $\cos x = y$ $(ay^2 + by + c = 0)$

$10y^2 + 3y - 1 = 0$

$y = \frac{-3 \pm \sqrt{3^2 - 4(10)(-1)}}{2 \times 10}$

$y = \frac{-3 \pm 7}{20}$

$y = \frac{-3-7}{20} = -\frac{1}{2}$, $y = \frac{-3+7}{20} = \frac{1}{5}$

$\cos x = -\frac{1}{2}, \frac{1}{5}$ (cosine is positive in the first and fourth quadrant)

$x = \cos^{-1}\left(-\frac{1}{2}\right), \cos^{-1}\left(\frac{1}{5}\right)$

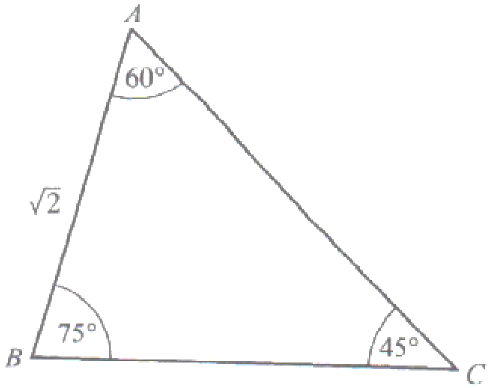
$x = 120°, (360° - 120°), 78.5, (360° - 78.5°)$

$= 120°, 240°, 78.5°, 281.5°$

$\therefore x = 78.5°, 120°, 240°, 281.5°$

8. Nov/2023/Paper_0606/23/No.7

DO NOT USE A CALCULATOR IN THIS QUESTION.



You may use the following trigonometrical ratios.

$\sin 60° = \dfrac{\sqrt{3}}{2}$, $\sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos 60° = \dfrac{1}{2}$, $\cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 60° = \sqrt{3}$, $\tan 45° = 1$

(a) Given that the area of triangle ABC is $\dfrac{3+\sqrt{3}}{4}$, show that $\sin 75° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$. [5]

Using sine rule

$$\frac{BC}{\sin 60°} = \frac{AB}{\sin 45°}$$

$$\frac{BC}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\frac{\sqrt{2}}{2}}$$

$$\frac{BC}{\frac{\sqrt{3}}{2}} = 2$$

$$BC = \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2} = \sqrt{3}$$

$$\text{Area} = \frac{1}{2} \times AB \times BC \times \sin 75°$$

$$\frac{1}{2} \times \sqrt{2} \times \sqrt{3} \times \sin 75° = \frac{3+\sqrt{3}}{4}$$

$$\sin 75° = \frac{3+\sqrt{3}}{4} \times \frac{2}{\sqrt{6}}$$

$$= \frac{3+\sqrt{3}}{2\sqrt{6}} \times \frac{2\sqrt{6}}{2\sqrt{6}}$$

$$= \frac{6\sqrt{6}+2\sqrt{18}}{24} = \frac{6\sqrt{6}+6\sqrt{2}}{24}$$

$$= \frac{6(\sqrt{6}+\sqrt{2})}{24} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

(b) Hence find the exact length of AC. [2]

$$\frac{AC}{\sin 75°} = \frac{BC}{\sin 60°}$$

$$\frac{AC}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$\frac{AC}{\frac{\sqrt{6}+\sqrt{2}}{4}} = 2$$

$$AC = \frac{\sqrt{6}+\sqrt{2}}{4} \times 2$$

$$\therefore AC = \frac{\sqrt{6}+\sqrt{2}}{2}$$

9. Nov/2023/Paper_0606/23/No.8

(a) Show that $\dfrac{\sin x}{\tan x - 1} - \dfrac{\cos x}{\tan x + 1} = \dfrac{\cos x}{\sin^2 x - \cos^2 x}$. [5]

Consider the Left hand side, $\dfrac{\sin x}{\tan x - 1} - \dfrac{\cos x}{\tan x + 1}$

But $\tan x = \dfrac{\sin x}{\cos x}$

$$\Rightarrow \frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = \frac{\sin x}{\frac{\sin x}{\cos x} - 1} - \frac{\cos x}{\frac{\sin x}{\cos x} + 1}$$

$$= \frac{\sin x}{\frac{\sin x - \cos x}{\cos x}} - \frac{\cos x}{\frac{\sin x + \cos x}{\cos x}}$$

$$= \frac{\sin x \cos x}{\sin x - \cos x} - \frac{\cos^2 x}{\sin x + \cos x}$$

$$= \frac{\sin x \cos x(\sin x + \cos x) - \cos^2 x(\sin x - \cos x)}{(\sin x - \cos x)(\sin x + \cos x)}$$

$$= \frac{\sin^2 x \cos x + \sin x \cos^2 x - \sin x \cos^2 x + \cos^3 x}{\sin x(\sin x + \cos x) - \cos x(\sin x + \cos x)}$$

$$= \frac{\sin^2 x \cos x + \cos^3 x}{\sin^2 x + \sin x \cos x - \sin x \cos x - \cos^2 x}$$

$$= \frac{\cos x(\sin^2 x + \cos^2 x)}{\sin^2 x - \cos^2 x}$$

But $\sin^2 x + \cos^2 x = 1$

$$= \frac{\cos x}{\sin^2 x - \cos^2 x}$$

(b) Hence solve the equation $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = 1$ for $0° < x < 360°$. [5]

From (a) $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = \frac{\cos x}{\sin^2 x - \cos^2 x}$

$\Rightarrow \frac{\cos x}{\sin^2 x - \cos^2 x} = 1$

$\cos x = \sin^2 x - \cos^2 x$, but $\sin^2 x = 1 - \cos^2 x$

$\cos x = 1 - \cos^2 x - \cos^2 x$

$\cos x = 1 - 2\cos^2 x$

$\Rightarrow \cos x - 1 + 2\cos^2 x = 0$

$2\cos^2 x + \cos x - 1 = 0$

Solving for $\cos x$ using the quadratic formula

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2 \times 2} = \frac{-1 \pm 3}{4}$

$\cos x = \frac{-1 - 3}{4} = -1$, $\cos x = \frac{-1 + 3}{4} = \frac{1}{2}$

$\cos x = -1, \frac{1}{2}$

$x = \cos^{-1}(-1)$, $\cos^{-1}\left(\frac{1}{2}\right)$

$x = 180°, 60°, (360° - 60°)$

$\therefore x = 60°, 180°, 300°$

10. March/2023/Paper_0606/12/No.10

(a) It is given that $2+\cos\theta = x$ for $1 < x < 3$ and $2\operatorname{cosec}\theta = y$ for $y > 2$. Find y in terms of x. [4]

$2 + \cos\theta = x$

$\Rightarrow \cos\theta = x - 2$

$2\operatorname{cosec}\theta = y$, but $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \frac{2}{\sin\theta} = y$

$\Rightarrow \sin\theta = \frac{2}{y}$

Recall $\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow (x-2)^2 + \left(\frac{2}{y}\right)^2 = 1$

$(x-2)^2 + \frac{4}{y^2} = 1$

$\Rightarrow \frac{4}{y^2} = 1 - (x-2)^2$

$y^2 = \frac{4}{1-(x-2)^2}$

But $y > 2$

$y = \sqrt{\frac{4}{1-(x-2)^2}}$

$\therefore y = \frac{2}{\sqrt{1-(x-2)^2}}$

(b) Solve the equation $3\cos\frac{\phi}{2} = \sqrt{3}\sin\frac{\phi}{2}$ for $-4\pi < \phi < 4\pi$. [5]

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$3\cos\frac{\phi}{2} = \sqrt{3}\sin\frac{\phi}{2}$

$\Rightarrow \frac{\sin\frac{\phi}{2}}{\cos\frac{\phi}{2}} = \frac{3}{\sqrt{3}}$

$\tan\frac{\phi}{2} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\frac{\phi}{2} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \tan\frac{\phi}{2} = \sqrt{3}$

$\frac{\phi}{2} = \tan^{-1}(\sqrt{3})$

$\frac{\phi}{2} = \frac{\pi}{3}, \left(\pi + \frac{\pi}{3}\right), \left(-\pi + \frac{\pi}{3}\right), \left(-2\pi + \frac{\pi}{3}\right)$

$\frac{\phi}{2} = \frac{\pi}{3}, \frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{5\pi}{3}$

$\phi = 2\left(\frac{\pi}{3}, \frac{4\pi}{3}, \frac{-2\pi}{3}, \frac{-5\pi}{3}\right)$

$\therefore \phi = \frac{2\pi}{3}, \frac{8\pi}{3}, -\frac{4\pi}{3}, \frac{-10\pi}{3}$

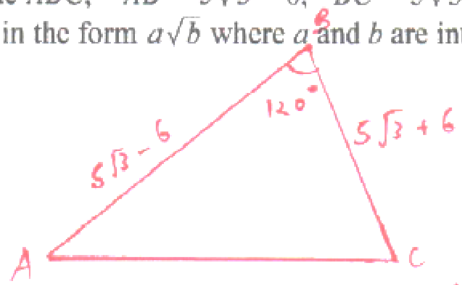
11. June/2023/Paper_0606/11/No.5

DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.

(a) You are given that $\cos 120° = -\frac{1}{2}$, $\sin 120° = \frac{\sqrt{3}}{2}$ and $\tan 120° = -\sqrt{3}$.

In the triangle ABC, $AB = 5\sqrt{3} - 6$, $BC = 5\sqrt{3} + 6$ and angle $ABC = 120°$. Find AC, giving your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1. [4]



Using cosine rule: $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$

$AC^2 = (5\sqrt{3}-6)^2 + (5\sqrt{3}+6)^2 - 2(5\sqrt{3}-6)(5\sqrt{3}+6)\cos 120°$

$AC^2 = (5\sqrt{3}-6)^2 + (5\sqrt{3}+6)^2 - \cancel{2}(5\sqrt{3}-6)(5\sqrt{3}+6)\left(-\frac{1}{\cancel{2}}\right)$

$AC^2 = (5\sqrt{3}-6)^2 + (5\sqrt{3}+6)^2 + (5\sqrt{3}-6)(5\sqrt{3}+6)$

$(5\sqrt{3}-6)^2 = 5\sqrt{3}(5\sqrt{3}-6) - 6(5\sqrt{3}-6)$

$= 75 - 30\sqrt{3} - 30\sqrt{3} + 36 = 111 - 60\sqrt{3}$

$(5\sqrt{3}+6)^2 = 5\sqrt{3}(5\sqrt{3}+6) + 6(5\sqrt{3}+6)$

$= 75 + 30\sqrt{3} + 30\sqrt{3} + 36 = 111 + 60\sqrt{3}$

$(5\sqrt{3}-6)(5\sqrt{3}+6) = 5\sqrt{3}(5\sqrt{3}+6) - 6(5\sqrt{3}+6)$

$= 75 + 30\sqrt{3} - \cancel{30\sqrt{3}} - 36 = 39$

$\Rightarrow AC^2 = 111 - 60\sqrt{3} + 111 + 60\sqrt{3} + 39$

$AC^2 = 261$

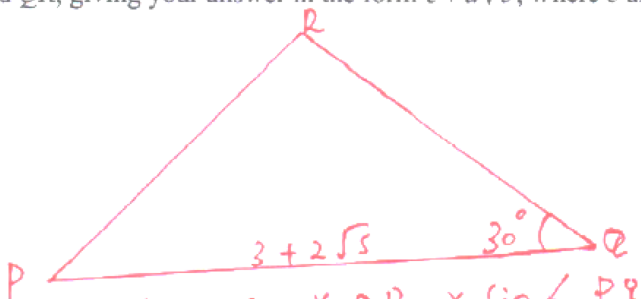
$AC = \sqrt{261} = \sqrt{9 \times 29}$

$AC = \sqrt{9}\sqrt{29}$

$\therefore AC = 3\sqrt{29}$

(b) You are given that $\cos 30° = \dfrac{\sqrt{3}}{2}$, $\sin 30° = \dfrac{1}{2}$ and $\tan 30° = \dfrac{1}{\sqrt{3}}$.

In the triangle PQR, $PQ = 3+2\sqrt{5}$ and angle $PQR = 30°$. Given that the area of this triangle is $\dfrac{2+5\sqrt{5}}{4}$, find QR, giving your answer in the form $c+d\sqrt{5}$, where c and d are integers. [4]



Area of $PQR = \frac{1}{2} \times PQ \times QR \times \sin\angle PQR$

$\Rightarrow \frac{1}{2} \times (3+2\sqrt{5}) \times QR \times \sin 30° = \frac{2+5\sqrt{5}}{4}$

$\frac{1}{2} \times (3+2\sqrt{5}) \times QR \times \frac{1}{2} = \frac{2+5\sqrt{5}}{4}$

$QR\left(\frac{3+2\sqrt{5}}{4}\right) = \frac{2+5\sqrt{5}}{4}$

$\Rightarrow QR = \frac{2+5\sqrt{5}}{\cancel{4}} \times \frac{\cancel{4}}{3+2\sqrt{5}}$

$QR = \frac{2+5\sqrt{5}}{3+2\sqrt{5}}$ Rationalise the denominator.

$QR = \frac{2+5\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{2(3-2\sqrt{5})+5\sqrt{5}(3-2\sqrt{5})}{3(3-2\sqrt{5})+2\sqrt{5}(3-2\sqrt{5})} = \frac{6-4\sqrt{5}+15\sqrt{5}-50}{9-6\sqrt{5}+6\sqrt{5}-20}$

$QR = \frac{-44+11\sqrt{5}}{-11} = \frac{-44}{-11} + \frac{11\sqrt{5}}{-11}$

$\therefore QR = 4-\sqrt{5}$

12. June/2023/Paper_0606/11/No.6

(a) Show that $\dfrac{\cot\theta+\tan\theta}{\sec\theta} = \operatorname{cosec}\theta$. [4]

Consider the Left Hand Side : $\dfrac{\cot\theta + \tan\theta}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$

$$\Rightarrow \frac{\cot\theta + \tan\theta}{\sec\theta} = \frac{\frac{1}{\tan\theta} + \tan\theta}{\frac{1}{\cos\theta}} = \left(\frac{1}{\tan\theta} + \tan\theta\right) \times \cos\theta$$

$$= \cos\theta\left(\frac{1}{\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \cos\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin\theta(\sin\theta)}{\sin\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

But $\cos^2\theta + \sin^2\theta = 1$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

(b) Hence solve the equation $\left(\dfrac{\cot\frac{\phi}{3}+\tan\frac{\phi}{3}}{\sec\frac{\phi}{3}}\right)^2 = 2$, for $-540° < \phi < 540°$. [6]

From part (a) $\dfrac{\cot\theta + \tan\theta}{\sec\theta} = \operatorname{cosec}\theta$

$$\Rightarrow \left(\frac{\cot\frac{\phi}{3} + \tan\frac{\phi}{3}}{\sec\frac{\phi}{3}}\right)^2 = \operatorname{cosec}^2\frac{\phi}{3} = 2$$

$$\Rightarrow \left(\frac{1}{\sin\frac{\phi}{3}}\right)^2 = 2 \quad \Rightarrow \frac{1}{\sin\frac{\phi}{3}} = \pm\sqrt{2}$$

$$\sin\frac{\phi}{3} = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\phi}{3} = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\phi = 3\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

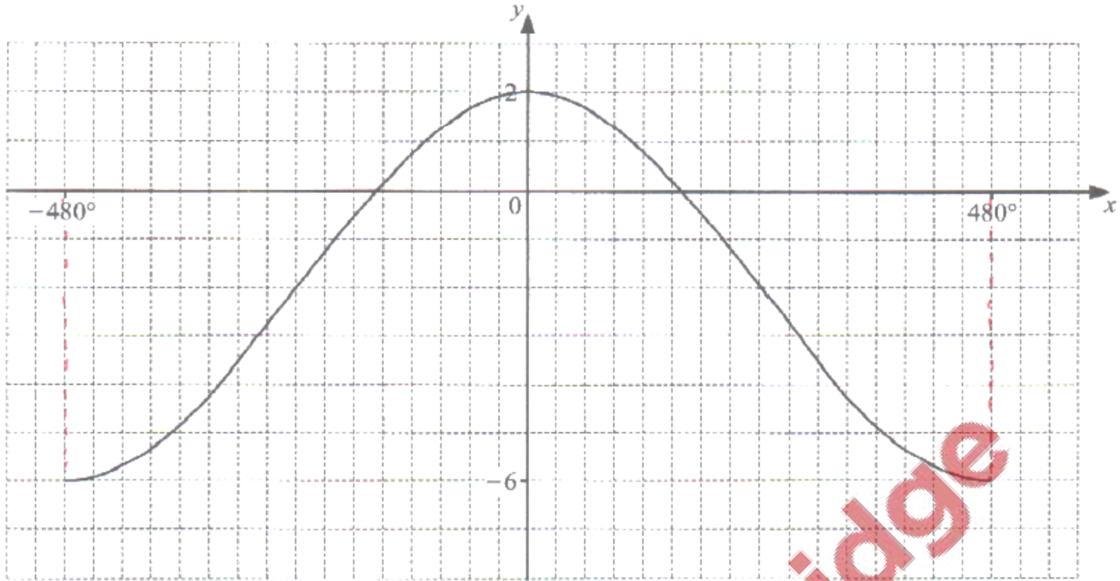
Sine is positive in the first and fourth quadrant only.

$$\Rightarrow \phi = 3\left(-45°, 45°, -135°, 135°\right)$$

$$\therefore \phi = -135°, 135°, -405°, 405°$$

13. June/2023/Paper_0606/12/No.1

The diagram shows the graph of $y = a\cos bx + c$. Find the values of the constants a, b and c. [3]



$y = a\cos bx + c$

amplitude $= a$

Period $= \dfrac{360°}{b}$

c = vertical translation $\begin{pmatrix} 0 \\ c \end{pmatrix}$

From the graph: $a = \dfrac{2-(-6)}{2} = \dfrac{8}{2}$

amplitude $= 4$

$\therefore a = 4$

The period of a function is the length of one cycle.

Period $= 480° + 480° = 960°$

$\Rightarrow 960° = \dfrac{360°}{b} \Rightarrow b = \dfrac{360°}{960°}$

$\therefore b = \dfrac{3}{8}$

For $y = 4\cos\frac{3}{8}x$, When $x = 0$, $y = 4$ (From 4 to 2 is a translation of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$)

$\therefore c = -2$

$\therefore a = 4$, $b = \dfrac{3}{8}$ and $c = -2$

 0606/12/M/J/23 [Turn over

14. June/2023/Paper_0606/12/No.6

(a) Given that $\cot^2\theta = \dfrac{1}{y+2}$ and $\sec\theta = x-4$, find y in terms of x. [2]

$\cot\theta = \dfrac{1}{\tan\theta} \Rightarrow \cot^2\theta = \dfrac{1}{\tan^2\theta}$

$\cot^2\theta = \dfrac{1}{y+2}$

$\Rightarrow \dfrac{1}{\tan^2\theta} = \dfrac{1}{y+2}$

$\Rightarrow \tan^2\theta = y+2$

But $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow (x-4)^2 = 1 + y + 2$

$(x-4)^2 = y+3$

$\therefore y = (x-4)^2 - 3$

(b) Solve the equation $\sqrt{3}\,\mathrm{cosec}\left(2\phi + \dfrac{3\pi}{4}\right) = 2$, for $-\pi < \phi < \pi$, giving your answers in terms of π. [5]

$\mathrm{Cosec}\left(2\phi + \dfrac{3\pi}{4}\right) = \dfrac{2}{\sqrt{3}}$

But $\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\Rightarrow \dfrac{1}{\sin\left(2\phi + \frac{3\pi}{4}\right)} = \dfrac{2}{\sqrt{3}}$

$\Rightarrow \sin\left(2\phi + \dfrac{3\pi}{4}\right) = \dfrac{\sqrt{3}}{2}$

$2\phi + \dfrac{3\pi}{4} = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$2\phi + \dfrac{3\pi}{4} = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{7\pi}{3}, \dfrac{8\pi}{3}$

$2\phi = \dfrac{\pi}{3} - \dfrac{3\pi}{4}, \dfrac{2\pi}{3} - \dfrac{3\pi}{4}, \dfrac{7\pi}{3} - \dfrac{3\pi}{4}, \dfrac{8\pi}{3} - \dfrac{3\pi}{4}$

$\phi = \dfrac{\frac{\pi}{3} - \frac{3\pi}{4}}{2}, \dfrac{\frac{2\pi}{3} - \frac{3\pi}{4}}{2}, \dfrac{\frac{7\pi}{3} - \frac{3\pi}{4}}{2}, \dfrac{\frac{8\pi}{3} - \frac{3\pi}{4}}{2}$

$\therefore \phi = -\dfrac{5\pi}{24}, -\dfrac{\pi}{24}, \dfrac{19\pi}{24}, \dfrac{23\pi}{24}$

15. June/2023/Paper_0606/13/No.1

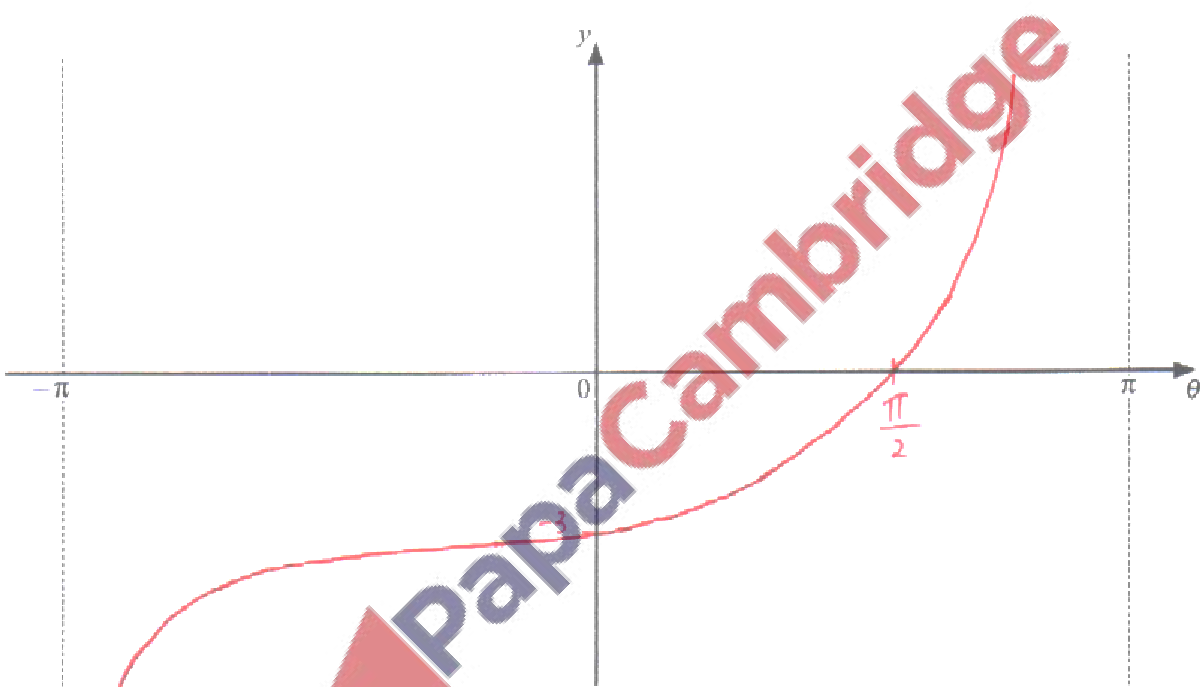
(a) Write down the period, in radians, of $3\tan\frac{\theta}{2}-3$. [1]

Given $y = a\tan b\theta + c$, Period $= \frac{\pi}{b}$

let $y = 3\tan\frac{1}{2}\theta - 3$

Period $= \frac{\pi}{\frac{1}{2}} = 2\pi$

(b) On the axes, sketch the graph of $y = 3\tan\frac{\theta}{2}-3$ for $-\pi \leqslant \theta \leqslant \pi$, stating the coordinates of the points where the graph meets the axes. [3]



When $\theta = 0$, $y = 3\tan 0 - 3 = -3 \Rightarrow (0, -3)$

When $y = 0$, $3\tan\frac{\theta}{2} - 3 = 0$

$\frac{\cancel{3}\tan\frac{\theta}{2}}{\cancel{3}} = \frac{3}{3}$

$\tan\frac{\theta}{2} = 1$

$\frac{\theta}{2} = \tan^{-1}(1) \Rightarrow \theta = 2\tan^{-1}(1)$

$\theta = \frac{\pi}{2}$,

$\Rightarrow \left(\frac{\pi}{2}, 0\right)$

16. June/2023/Paper_0606/13/No.10

(a) Show that $\cos^4\theta - \sin^4\theta + 1 = 2\cos^2\theta$. [3]

Consider the LHS :
$\cos^4\theta - \sin^4\theta + 1$

$\cos^4\theta - \sin^4\theta$ is a difference of two squares.

$\Rightarrow \cos^4\theta - \sin^4\theta = (\cos^2\theta)^2 - (\sin^2\theta)^2$
$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

But $\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \cos^4 - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$\cos^4\theta - \sin^4\theta + 1 = \cos^2\theta - \sin^2\theta + 1$
$= \cos^2\theta - (1 - \cos^2\theta) + 1$
$= \cos^2\theta - 1 + \cos^2\theta + 1$
$= 2\cos^2\theta$

(b) Solve the equation $\cos^4\frac{\phi}{3} - \sin^4\frac{\phi}{3} + 1 = \frac{1}{2}$, for $-3\pi < \phi < 3\pi$, giving your answers in terms of π. [5]

Comparing with the equation from part (a)
$\cos^4\frac{\phi}{3} - \sin^4\frac{\phi}{3} + 1 = 2\cos^2\frac{\phi}{3} = \frac{1}{2}$

$\Rightarrow \cos^2\frac{\phi}{3} = \frac{\frac{1}{2}}{2} \Rightarrow \cos^2\frac{\phi}{3} = \frac{1}{4}$

$\cos\frac{\phi}{3} = \pm\sqrt{\frac{1}{4}}$

$\cos\frac{\phi}{3} = \pm\frac{1}{2} \Rightarrow \phi = 3\cos^{-1}\left(\pm\frac{1}{2}\right)$

$\phi = 3\left(\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}\right)$

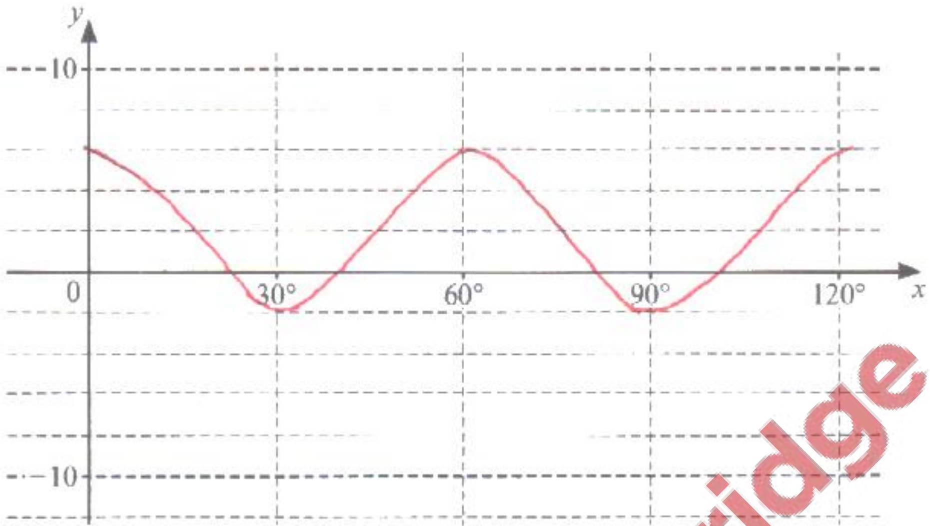
$\phi = \pm\pi, \pm 2\pi$

$\therefore \phi = -2\pi, -\pi, \pi, 2\pi$

17. June/2023/Paper_0606/21/No.2

The function g is defined for $0° \leqslant x \leqslant 120°$ by $g(x) = 2 + 4\cos 6x$.

(a) On the axes, sketch the graph of $y = g(x)$. [3]



x	0	30°	60°	90°	120°
$y = g(x)$	6	−2	6	−2	6

(b) State the amplitude of g. [1]

Given $y = a\cos bx + c$, amplitude $= a$

$\therefore$ amplitude of $g = 4$

(c) State the period of g. [1]

$\text{Period} = \dfrac{360°}{b}$, but $b = 6$

$= \dfrac{360°}{6}$

$= 60°$