Differentiation and integration – 2022 Nov IGCSE 0606 Additional Math

1. Nov/2022/Paper_0606_11/No.8

Find $\int_0^a \left(\frac{2}{x+1} - \frac{1}{x+2}\right)\mathrm{d}x$, where a is a positive constant. Give your answer, as a single logarithm, in terms of a. [5]

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2. Nov/2022/Paper_0606_12/No.9

(a) Show that $\frac{1}{2x+1} - \frac{1}{(2x+1)^2} + \frac{4}{4x-1} = \frac{24x^2+14x+4}{(2x+1)^2(4x-1)}$. [2]

(b) Hence find $\int_{\frac{1}{2}}^{1} \frac{24x^2+14x+4}{(2x+1)^2(4x-1)}\mathrm{d}x$, giving your answer in the form $\frac{1}{2}\ln p + q$, where p and q are rational numbers. [7]

3. Nov/2022/Paper_0606_12/No.11

A particle P moves in a straight line such that, t seconds after passing through a fixed point O, its displacement, s metres, is given by $s = \frac{(2t+1)^{\frac{3}{2}}}{t+1} - 1$.

(a) Show that the velocity of P at time t can be written in the form $\frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(a+bt)$, where a and b are integers to be found. [5]

(b) Show that P is never at instantaneous rest after passing through O. [1]

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4. Nov/2022/Paper_0606_13/No.7

Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4\sin 2x + 1)\,dx$. [5]

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5. Nov/2022/Paper_0606_13/No.11

It is given that $\int_1^a \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x}\right) \mathrm{d}x = \ln \frac{1}{5}$, where $a > 1$. Find the exact value of a. [6]

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6. Nov/2022/Paper_0606_13/No.12

It is given that $y = \dfrac{(3x^2-2)^{\frac{2}{3}}}{x-1}$, for $x > 1$.

(a) Write $\dfrac{dy}{dx}$ in the form $\dfrac{(3x^2-2)^{-\frac{1}{3}}}{(x-1)^2}(x^2+Ax+B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to $2+p$, where p is small. [2]

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7. Nov/2022/Paper_0606_21/No.3

(a) Find the coordinates of the point on the curve $y = \sqrt{1+3x}$ where the gradient of the normal is $-\frac{8}{3}$. [5]

(b) Find the equation of the normal to the curve $y = \sqrt{1+3x}$ at the point (8, 5) in the form $y = mx + c$. [3]

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8. Nov/2022/Paper_0606_21/No.5

You are given that $y = \dfrac{1}{\cos 2x}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k\sin 2x}{\cos^2 2x}$ where k is a constant to be found. [2]

(b) Find the values of x such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{5}{\sin 2x}$ for $0 < x < \dfrac{\pi}{2}$. [4]

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9. Nov/2022/Paper_0606_21/No.8

The equation of a curve is $y = x \sin x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Find the equation of the tangent to the curve at $x = \frac{\pi}{2}$ in the form $y = mx + c$. [3]

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(c) Use your answer to **part (a)** to find $\int x \cos x \, dx$. [3]

(d) Evaluate $\int_0^{\frac{\pi}{4}} x \cos x \, dx$, giving your answer correct to 2 significant figures. [2]

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10. Nov/2022/Paper_0606_22/No.3

In this question a and b are constants.

The normal to the curve $y = \frac{a}{x} + 3x - 2$ at the point where $x = 1$ has equation $y = -\frac{1}{4}x + b$.
Find the values of a and b. [6]

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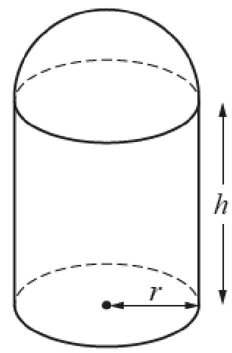
11. Nov/2022/Paper_0606_22/No.8

In this question all lengths are in centimetres.

The volume of a cylinder with radius r and height h is $\pi r^2 h$ and its curved surface area is $2\pi rh$.
The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.



The diagram shows a solid object in the shape of a cylinder of base radius r and height h, with a hemisphere of radius r on top. The total surface area of the object is $300\,\text{cm}^2$.

(a) Find an expression for h in terms of r. [2]

(b) Show that the volume, V, of the object is $150r - \frac{5}{6}\pi r^3$. [3]

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12. Nov/2022/Paper_0606_23/No.2

The tangent to the curve $y = ax^2 - 5x + 2$ at the point where $x = 2$ has equation $y = 7x + b$. Find the values of the constants a and b. [5]

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13. Nov/2022/Paper_0606_23/No.9

The equation of a curve is $y = kxe^{-2x}$, where k is a constant.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the coordinates of the stationary point on the curve $y = 10xe^{-2x}$. [3]

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(c) Use your answer to **part (a)** to find $\int 4xe^{-2x}dx$. [3]

(d) Find the exact value of $\int_0^1 4xe^{-2x}dx$. [2]

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