Differentiation and integration – 2023 Additional Math 0606

1. Nov/2023/Paper_0606/11/No.7

A curve has equation $y = f(x)$, where $f(x) = (2x+1)(3x-2)^2$.

(a) Show that $f'(x)$ can be written in the form $2(3x-2)(px+q)$, where p and q are integers. [3]

(b) Hence find the coordinates of the stationary points on the curve. [2]

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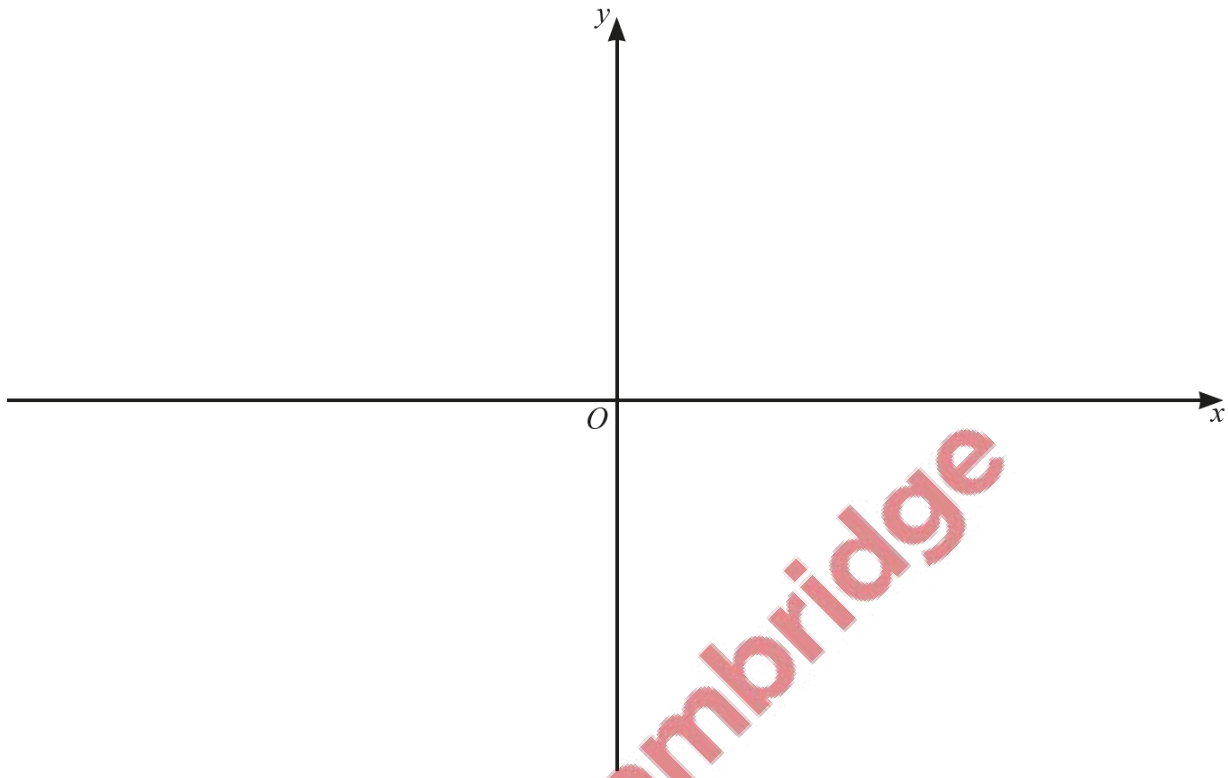
(c) On the axes below, sketch the graph of $y = \mathrm{f}(x)$, stating the intercepts with the coordinate axes. [3]



(d) Find the values of k such that the equation $\mathrm{f}(x) = k$ has 3 distinct solutions. [2]

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2. Nov/2023/Paper_0606/11/No.10

(a) Given that $y = \dfrac{\sqrt{3x^2 - 2}}{x - 4}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{Ax + B}{(x-4)^2\sqrt{3x^2 - 2}}$, where A and B are integers to be found. [5]

(b) Hence find, in terms of h, the approximate change in y when x increases from 3 to $3 + h$, where h is small. [3]

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3. Nov/2023/Paper_0606/12/No.12

A curve has equation $y = \dfrac{\sqrt{5x-2}}{x-3}$.

(a) Explain why the curve does not exist when $x < \frac{2}{5}$. [1]

(b) Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$, where A and B are positive integers. [5]

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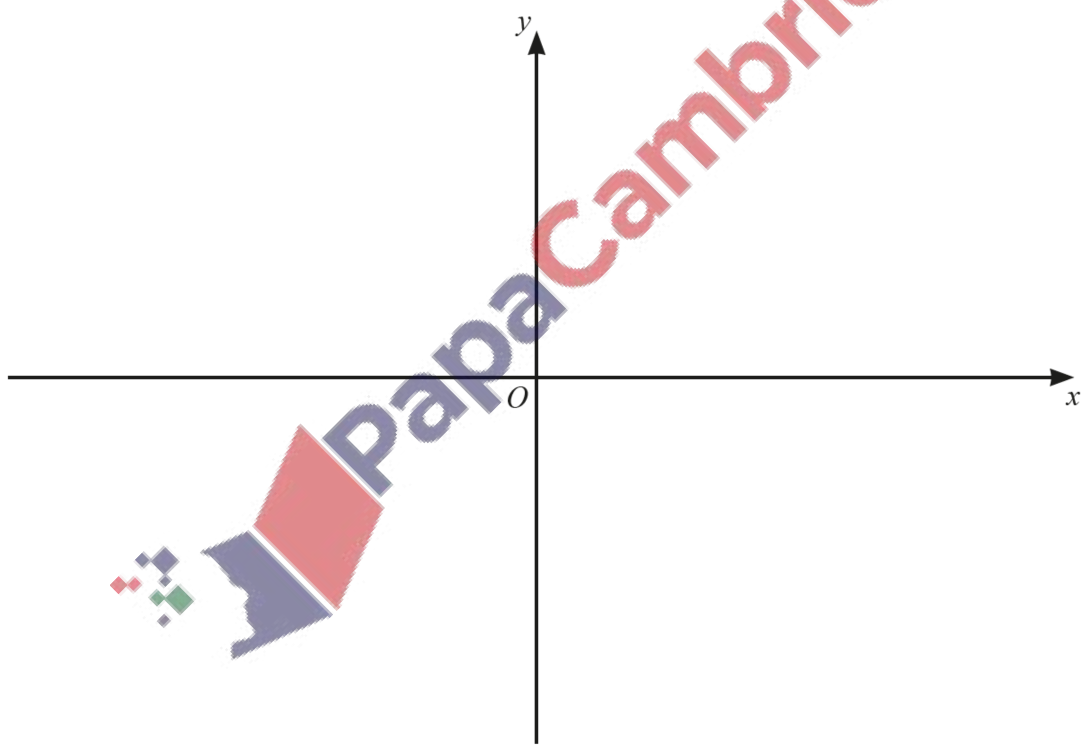
4. Nov/2023/Paper_0606/13/No.6

The polynomial $q(x)$ is given by $q(x) = -\frac{1}{3}(2x-1)(x+3)^2$.

(a) Find the x-coordinates of the stationary points on the curve $y = q(x)$. [4]

(b) On the axes, sketch the graph of $y = q(x)$ stating the intercepts with the coordinate axes. [3]



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(c) Find the values of k such that $q(x) = k$ has exactly one solution. [3]

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5. Nov/2023/Paper_0606/13/No.9

Given that $y = \dfrac{(5x+2)^{\frac{1}{3}}}{(x-1)^2}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{-(Ax+B)}{3(5x+2)^{\frac{2}{3}}(x-1)^3}$, where A and B are integers. [5]

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6. Nov/2023/Paper_0606/21/No.3

(a) Find $\int\left(4x+5-\frac{1}{2x+3}\right)\mathrm{d}x$. [3]

(b) Hence find the exact value of $\int_{1}^{3}\left(4x+5-\frac{1}{2x+3}\right)\mathrm{d}x$, simplifying your answer. [3]

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7. Nov/2023/Paper_0606/21/No.7

A particle moves in a straight line. At time t seconds after passing through a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = 10\sin 2t - 6\cos 2t$.

(a) Find an expression for the acceleration of the particle. [2]

(b) Find the acceleration when $t = \frac{\pi}{4}$. [1]

(c) Find the first time at which the acceleration is zero. [3]

(d) Find the displacement of the particle between $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$. [4]

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8. Nov/2023/Paper_0606/22/No.5

(a) Find the equation of the normal to the curve $y = x^3 - 7x^2 + 12x - 5$ at the point (1, 1). [5]

(b) Find the x-coordinates of the two points where the normal cuts the curve again. Give your answers in the form $x = a \pm \sqrt{b}$ where a and b are integers. [5]

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9. Nov/2023/Paper_0606/22/No.6

Find the exact value of $\int_2^3 \frac{(x+2)^2}{x} dx$. [6]

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10. Nov/2023/Paper_0606/22/No.7

A particle is travelling in a straight line. Its displacement, s metres, from the origin at time t seconds is given by $s = 1.5e^{2t} + 2e^{-2t} - t$.

(a) Find expressions for the velocity, $v\,\text{ms}^{-1}$, and acceleration, $a\,\text{ms}^{-2}$, of the particle. [3]

(b) Find the time, T seconds, when the particle is at rest. [4]

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(c) Find the acceleration of the particle at time T seconds. [2]

11. Nov/2023/Paper_0606/22/No.8

A curve has equation $y = x\sin 2x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at $x = \frac{\pi}{4}$. [3]

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(c) Use your answer to **part (a)** to find the exact value of $\int_0^{\frac{\pi}{6}} 2x\cos 2x\,dx$. [5]

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12. Nov/2023/Paper_0606/23/No.4

Find the exact value of $\int_3^5 \frac{(x-1)^2}{x^3} \mathrm{d}x$. [6]

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13. Nov/2023/Paper_0606/23/No.5

The curved surface area of a cylinder with radius r and height h is $2\pi rh$.

A closed cylinder has radius r cm and volume $1000\,\text{cm}^3$.

(a) Show that the total surface area of the cylinder is $2\pi r^2 + \dfrac{2000}{r}\,\text{cm}^2$. [3]

(b) Find the value of r which makes this area a minimum. You should show that your value of r gives a minimum for this area. [5]

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14. Nov/2023/Paper_0606/23/No.6

A particle travels in a straight line. Its displacement, s metres, from the origin, at time t seconds, where $t > 2$, is given by $s = \ln(4t^2 - 5) - t$.

(a) Find expressions for the velocity, $v\,\text{ms}^{-1}$, and acceleration, $a\,\text{ms}^{-2}$, of the particle. [4]

(b) Find the time when the particle is at rest. [3]

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(c) Find the acceleration at this time. [2]

15. Nov/2023/Paper_0606/23/No.9

A curve has equation $y = xe^{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the normal to the curve at $x = 1$. [4]

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(c) Use your answer to **part (a)** to find the exact value of $\int_0^2 2xe^{2x}\,dx$. [5]

16. March/2023/Paper_0606/12/No.2

A curve has equation $y = (5-x)(x+2)^2$.

(a) Find the x-coordinates of the stationary points on the curve. [4]

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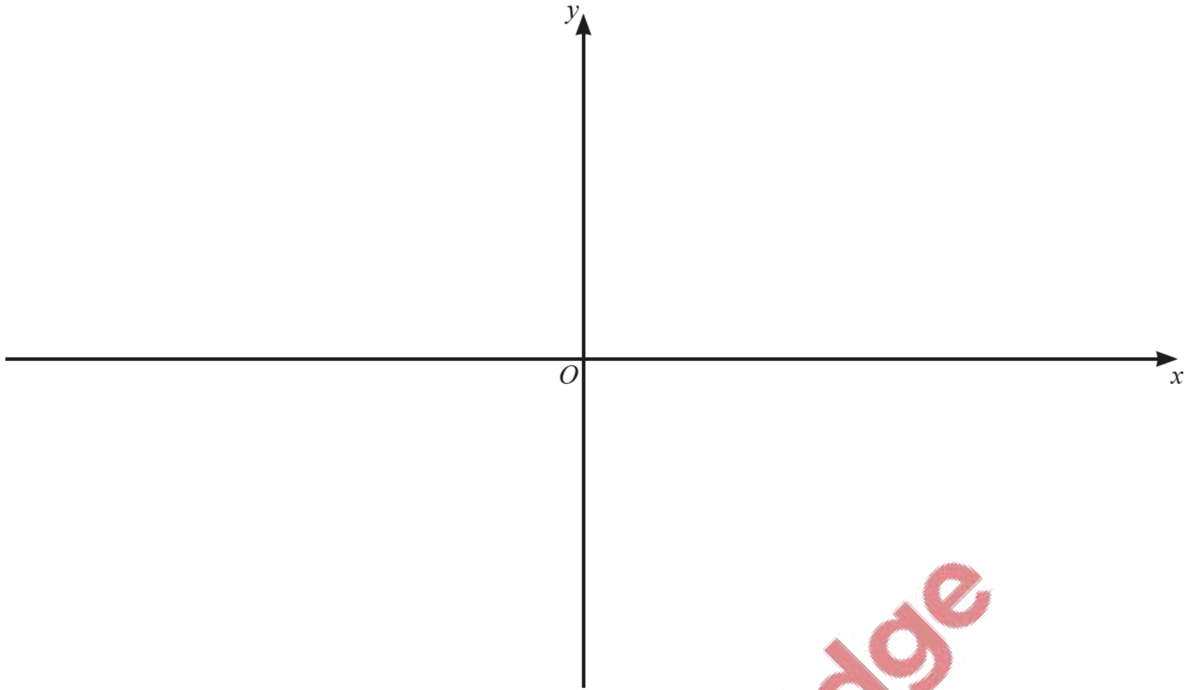
(b) On the axes below, sketch the graph of $y = (5-x)(x+2)^2$, stating the coordinates of the points where the curve meets the axes. [3]


y
O
x

(c) Find the values of k for which the equation $k=(5-x)(x+2)^2$ has one distinct root only. [3]

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17. March/2023/Paper_0606/12/No.6

Given that $f''(x) = (5x+2)^{-\frac{2}{5}}$, $f'(6) = \frac{17}{3}$ and $f(6) = \frac{26}{3}$, find an expression for $f(x)$. [8]

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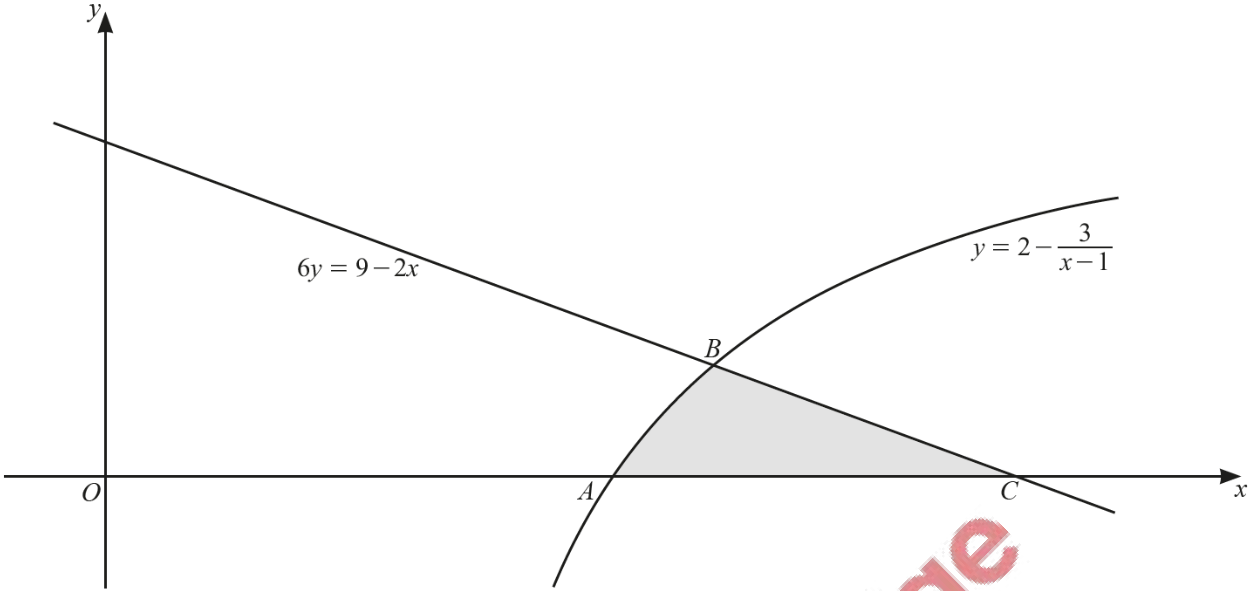
18. March/2023/Paper_0606/12/No.8



The diagram shows part of the curve $y = 2 - \frac{3}{x-1}$ and the straight line $6y = 9 - 2x$. The curve intersects the x-axis at point A and the line at point B. The line intersects the x-axis at point C. Find the area of the shaded region ABC, giving your answer in the form $p + \ln q$, where p and q are rational numbers. [11]

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19. March/2023/Paper_0606/22/No.4

$$y = \frac{\sec^2 5x - \tan^2 5x}{\operatorname{cosec} 5x}$$

Show that $y = a\sin bx$, where a and b are integers, and hence find the value of $\int_0^{\frac{\pi}{5}} y\,dx$. [4]

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20. March/2023/Paper_0606/22/No.7

(a) Variables x and y are such that $y = \dfrac{1+\cos^2 x}{\tan x}$. Use differentiation to find the approximate change in y as x increases from $\dfrac{\pi}{4}$ to $\dfrac{\pi}{4}+h$, where h is small. [5]

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(b) Given that $y = \dfrac{1}{(x-3)^3}$ show that $y - \dfrac{dy}{dx} - \dfrac{1}{3}\left(\dfrac{d^2y}{dx^2}\right)$ can be written as $\dfrac{(x+1)(x-4)}{(x-3)^5}$. [4]

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21. March/2023/Paper_0606/22/No.10

A particle P moves in a straight line such that, t seconds after passing a fixed point O, its acceleration, $a\,\text{ms}^{-2}$, is given by

$$a = 6t \qquad \text{for } 0 \leqslant t \leqslant 3,$$
$$a = \frac{18e^3}{e^t} \qquad \text{for } t \geqslant 3.$$

When $t = 1$, the velocity of P is $2\,\text{ms}^{-1}$ and its displacement from O is $-4\,\text{m}$.

(a) **(i)** Find the velocity of P when $t = 3$. [3]

(ii) Find the displacement of P from O when $t = 3$. [3]

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(b) Find an expression in terms of t for the displacement of P from O when $t \geqslant 3$. [4]

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22. March/2023/Paper_0606/22/No.11

The normal to the curve $y = \sin(4x - \pi)$ at the point $A(a, 0)$, where $\frac{\pi}{2} < a < \pi$, meets the y-axis at the point B. Find the exact area of triangle OAB, where O is the origin. [9]

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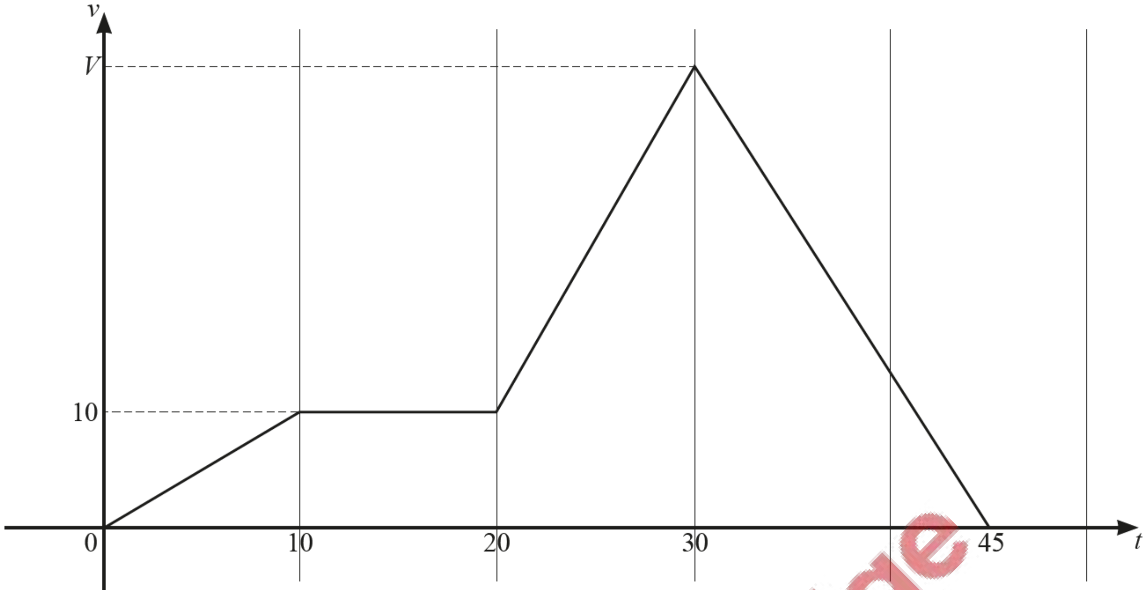
23. June/2023/Paper_0606/11/No.4



The diagram shows the velocity–time graph for a particle travelling in a straight line with velocity, $v\,\text{ms}^{-1}$, at time t seconds. When $t = 30$ the velocity of the particle is $V\,\text{ms}^{-1}$. The particle travels 800 metres in 45 seconds.

(a) Find the value of V. [2]

(b) Find the acceleration of the particle when $t = 35$. [2]

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24. June/2023/Paper_0606/11/No.8

A curve has the equation $y = \dfrac{(3x-4)^{\frac{1}{3}}}{2x+1}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$, where A and B are integers to be found. [5]

(b) Find the coordinates of the stationary point on the curve. [2]

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25. June/2023/Paper_0606/11/No.10

It is given that $y = (3x+1)^2 \ln(3x+1)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence find $\int (3x+1)\ln(3x+1)\,dx$. [4]

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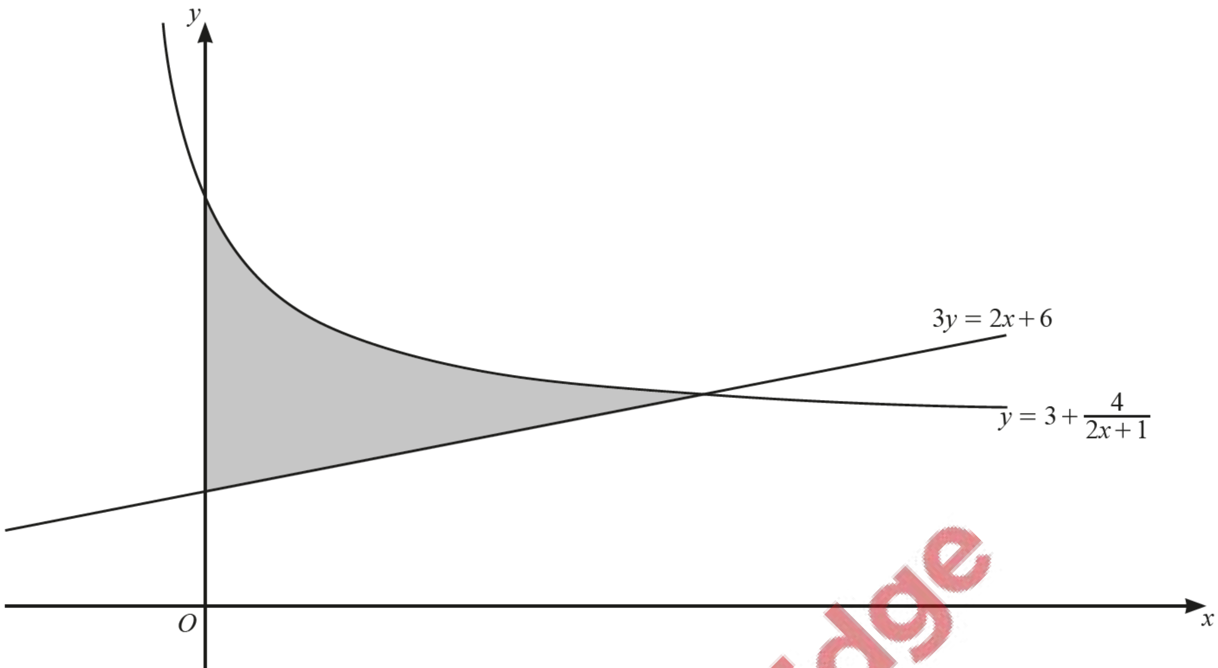
26. June/2023/Paper_0606/12/No.9



The diagram shows part of the curve $y = 3 + \frac{4}{2x+1}$ and the straight line $3y = 2x + 6$. Find the area of the shaded region, giving your answer in exact form. [10]

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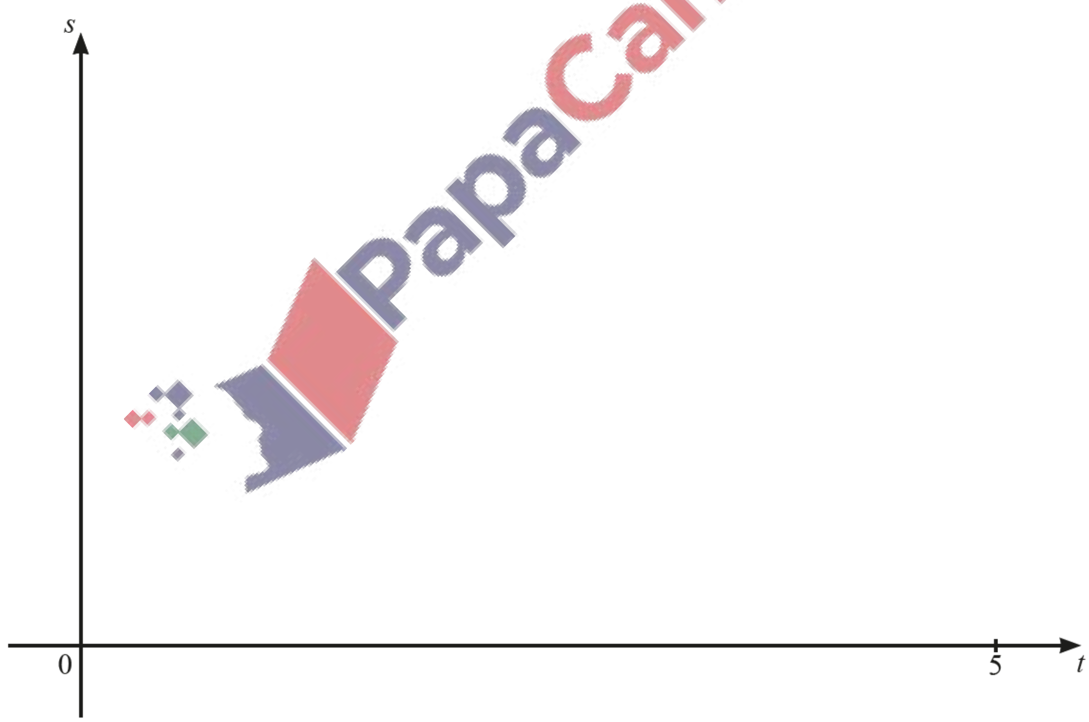
27. June/2023/Paper_0606/13/No.9

In this question lengths are in centimetres and time is in seconds.

A particle P moves in a straight line such that its displacement s, from a fixed point at a time t, is given by $s = 3(t+2)(t-4)^2$ for $0 \leqslant t \leqslant 5$.

(a) Find the values of t for which the velocity, v, of P is zero. [4]

(b) On the axes below, sketch the displacement–time graph of P, stating the intercepts with the axes. [3]



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(c) On the axes below, sketch the velocity–time graph of P, stating the intercepts with the axes. [2]

(d) **(i)** Find an expression for the acceleration of P at time t. [1]

(ii) Hence, on the axes below, sketch the acceleration–time graph of P, stating the intercepts with the axes. [2]

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28. June/2023/Paper_0606/21/No.5

The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume of a sphere is increasing at a constant rate of $24\,\text{cm}^3\,\text{s}^{-1}$. Find the rate of increase of the radius when the radius is 6 cm. [4]

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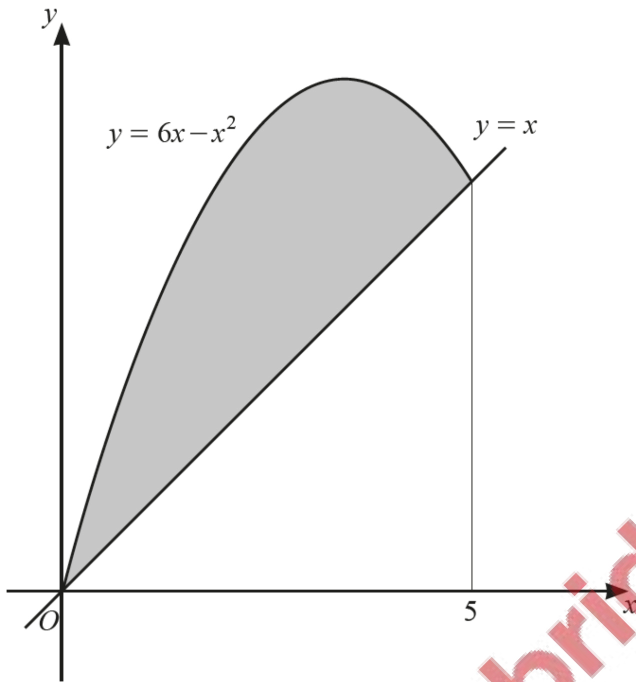
29. June/2023/Paper_0606/21/No.7

(a)



The diagram shows the curve $y = 6x - x^2$ for $0 \leqslant x \leqslant 5$ and the line $y = x$. Find the area of the shaded region. [4]

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(b) **(i)** Find $\int \left(\frac{1}{(2x-6)^3} + \cos x \right) \mathrm{d}x$. [3]

(ii) Find $\int \frac{(x^4+1)^2}{2x} \mathrm{d}x$. [3]

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30. June/2023/Paper_0606/21/No.11

The line with equation $x+3y=k$, where k is a positive constant, is a tangent to the curve with equation $x^2+y^2+2y-9=0$. Find the value of k and hence find the coordinates of the point where the line touches the curve. [9]

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31. June/2023/Paper_0606/22/No.2

A curve has equation $y = 32x^2 + \frac{1}{8x^2}$ where $x \neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

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32. June/2023/Paper_0606/22/No.4

Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leqslant x \leqslant \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

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33. June/2023/Paper_0606/22/No.7

(a) $f(x) = \sqrt{3 + (4x - 2)^5}$ where $x > 1$.

Find an expression for $f'(x)$, giving your answer as a simplified algebraic fraction. [3]

(b) Variables x and y are related by the equation $y = \frac{5x}{3x+2}$. Using differentiation, find the approximate change in x when y increases from 10 by the small amount 0.01. [4]

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(c) **(i)** Differentiate $y = x^3 \ln x$ with respect to x. [2]

(ii) Hence find $\int \left(\frac{x^2}{6}(2 + 3\ln x) \right) dx$. [3]

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34. June/2023/Paper_0606/22/No.8

A curve has equation $y = \cos\frac{x}{4}$ where x is in radians. The normal to the curve at the point where $x = \frac{4\pi}{3}$ cuts the x-axis at the point P. Find the exact coordinates of P. [7]

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35. June/2023/Paper_0606/22/No.9

A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v\,\text{ms}^{-1}$, is given by

$$v = e^{\frac{t}{4}} \quad \text{for } 0 \leqslant t \leqslant 4,$$

$$v = \frac{16e}{t^2} \quad \text{for } 4 \leqslant t \leqslant k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k. [6]

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36. June/2023/Paper_0606/23/No.3

(a) Differentiate $\ln(x^3+3x^2)$ with respect to x, simplifying your answer. [2]

(b) Hence find $\int \frac{x+2}{x(x+3)}\mathrm{d}x$. [2]

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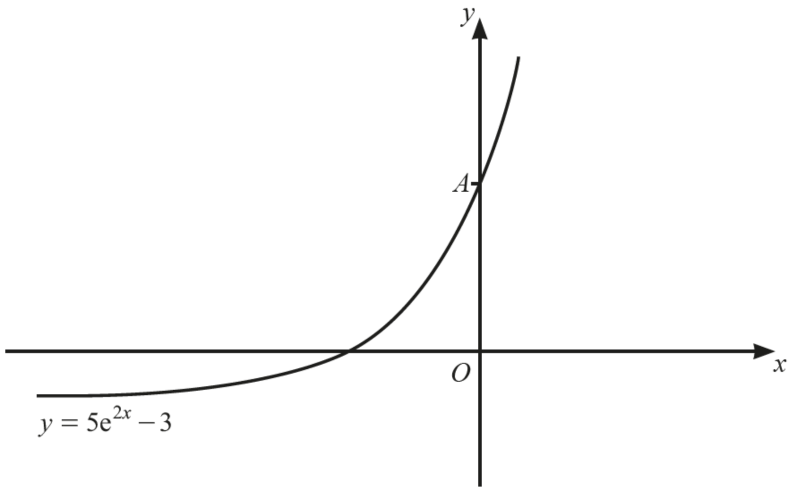
37. June/2023/Paper_0606/23/No.6



The diagram shows the curve $y = 5e^{2x} - 3$. The curve meets the y-axis at the point A. The tangent to the curve at A meets the x-axis at the point B. Find the length of AB. [6]

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38. June/2023/Paper_0606/23/No.7

Variables x and y are such that $y = \frac{4x^3 + 2\sin 8x}{1-x}$. Use differentiation to find the approximate change in y as x increases from 0.1 to $0.1 + h$, where h is small. [6]

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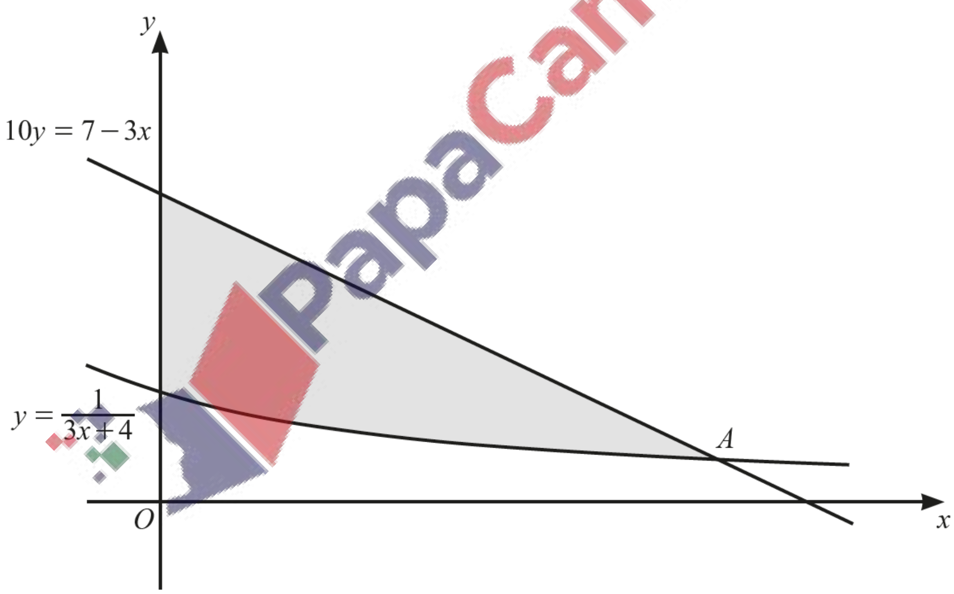
39. June/2023/Paper_0606/23/No.9

(a) Show that $\int_1^8 \frac{x+4}{\sqrt[3]{x}}\,dx = 36.6$. [3]

(b)



The diagram shows part of the line $10y = 7 - 3x$ and part of the curve $y = \frac{1}{3x+4}$.
The line and curve intersect at the point A. Verify that the y-coordinate of A is 0.1 and calculate the area of the shaded region. [8]