Factors of polynomials – 2023 Additional Math 0606

1. Nov/2023/Paper_0606/11/No.2

The polynomial $\mathrm{P}(x)$ is such that $\mathrm{P}(x) = ax^3 - 11x^2 + bx + c$, where a, b and c are integers. $\mathrm{P}(x)$ is divisible by x and has a remainder of $\frac{3}{2}$ when divided by $2x+1$. It is also given that $\mathrm{P}'(2) = 18$.

(a) Find the values of a, b and c. [6]

(b) Hence write $\mathrm{P}(x)$ as a product of three linear factors. [2]

2. Nov/2023/Paper_0606/12/No.6

The polynomial $\mathrm{p}(x)$ is such that $\mathrm{p}(x) = ax^3 + bx^2 + cx - 5$, where a, b and c are integers. It is given that $\mathrm{p}'(0) = 12$. It is also given that $\mathrm{p}(x)$ has a factor of $3x - 1$ and a remainder of 95 when divided by $x - 2$.

(a) Find the values of a, b and c. [7]

(b) Show that the equation $\mathrm{p}(x) = 0$ has only one real root. [3]

3. Nov/2023/Paper_0606/13/No.4

The polynomial P is given by $P(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $P(x)$ has a factor of $2x + 1$. $P(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b. [5]

(b) Show that the equation $P(x) = 0$ has only one real root. [3]

4. March/2023/Paper_0606/22/No.5

DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Show that $x-1$ is a factor of the expression $x^3-2x^2-19x+20$. [1]

(b) Hence write $x^3-2x^2-19x+20$ as a product of its linear factors. [3]

(c) Hence find the exact solutions of the equation $e^{3y}-2e^{2y}-19e^{y}+20=0$. [2]

5. June/2023/Paper_0606/11/No.2

The polynomial p is such that $p(x) = ax^3 + 7x^2 + bx + c$, where a, b and c are integers.

(a) Given that $p''\left(\frac{1}{2}\right) = 32$, show that $a = 6$. [2]

(b) Given that $p(x)$ has a factor of $3x - 4$ and a remainder of 7 when divided by $x + 1$, find the values of b and c. [4]

(c) Write $\mathrm{p}(x)$ in the form $(3x-4)\mathrm{q}(x)$, where $\mathrm{q}(x)$ is a quadratic factor. [2]

(d) Hence write $\mathrm{p}(x)$ as a product of linear factors with integer coefficients. [1]

6. June/2023/Paper_0606/22/No.3

DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Show that $x+3$ is a factor of $-12+23x+3x^2-2x^3$. [1]

(b) The curve $y=-5+33x+3x^2-2x^3$ and the line $y=10x+7$ intersect at three points, A, B and C. These points are such that the x-coordinate of A has the least value and the x-coordinate of C has the greatest value. Show that B is the mid-point of AC. [7]

7. June/2023/Paper_0606/23/No.4

The polynomial p is such that $p(x) = 2x^3 + 11x^2 + 22x + 40$.

(a) Show that $x = -4$ is a root of the equation $p(x) = 0$. [1]

(b) Factorise $p(x)$ and hence show that $p(x) = 0$ has no other real roots. [4]