UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS (US) **0459/01**

Paper 1 **For Examination from 2013**

SPECIMEN MARK SCHEME

2 hours

MAXIMUM MARK: 80

This document consists of **9** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mark Scheme Notes

- Marks are of the following three types:

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark, and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note. B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2. –1 each error. A mark is deducted from the total mark available up to the maximum mark available for that question. The minimum mark awarded is zero e.g., if a candidate makes 3 errors in a question worth 2 marks they score zero.

- The following abbreviations may be used in a mark scheme.

AG	'Answer given' on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
cao	'Correct answer only' (emphasizing that no "follow through" from a previous error is allowed).
isw	'Ignore subsequent working'.
oe	'Or equivalent'.
sc	'Special case'. Awarded for some questions where e.g., the candidate has not used the method specified but a different, correct, method leading to the correct answer.
soi	'Seen or implied'.

Question	Answer	Mark	Guidance
1	$(x+2)^2+(y-8)^2=7^2$ or $x^2+y^2+4x-16y+19=0$	**B2** **[2]**	**B1** for $(x+2)^2+(y-8)^2$ or x^2+4x+y^2+16y oe **B1** for $=7^2$ or $+2^2+8^2-7^2=0$ oe
2 **(a)**	Any two valid reasons e.g. Size of population may make selection of every item impossible Gathering information may necessitate destruction of items e.g. life of a battery	**B1 + B1**	
(b)	Every member of population has the same chance of being selected at every stage if random and choice of 1st item immediately rules out approximately 90% of the remaining population oe	**B2, 1, 0** **[4]**	Explanation must incorporate the essential idea of random sampling.
3	$(z_1=)\dfrac{3+\sqrt{7i^2}}{2}$	**M1**	allow $z=$; allow $\dfrac{3\pm\sqrt{7i^2}}{2}$
	$(z_1=)\dfrac{3+i\sqrt{7}}{2}$	**A1**	
	$(z_2=)\dfrac{3-i\sqrt{7}}{2}$	**B1ft** **[3]**	ft the complex conjugate of their z_1 $\dfrac{3\pm i\sqrt{7}}{2}$ scores 3 marks

4	$(PS)^2 = (x-6)^2 + (y-1)^2$ $(x-6)^2 + (y-1)^2 = (x+1)^2$ $x^2 - 12x + 36 + y^2 - 2y + 1 = x^2 + 2x + 1$ $y(y-2) = 14x - 36$ AG	**B1** **M1** **M1** **A1** **[4]**	
5	$\left(5+2\sqrt{3}\right)^2 = 37 + 20\sqrt{3}$ $\dfrac{\left(37+20\sqrt{3}\right)}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $14 + 3\sqrt{3}$	**B1** **M1** **A1+A1** **[4]**	Seen anywhere Or **B1** for a correct pair of simultaneous equations $37 = 2p + 3q$ and $20 = p + 2q$ and **M1** for attempting to solve their equations either by elimination or substitution, condone one error. Answer only scores zero.
6	Proving triangle *AED* congruent to triangle *CFB* *AD* = *BC* (parallelogram) *ED* = *FB* (given) $\angle ADE = \angle CBF$ (alternate angles are equal) $\Delta AED \equiv \Delta CFB$ (SAS) $\angle AED = \angle CFB$ (corresponding angles of congruent triangles) $\angle AEF = \angle CFE$ (each equal to $180 - \angle AED$) Thus alternate angles are equal *AE* = *FC* (corresponding sides of congruent triangles)	**B3, 2, 1, 0** **DB1** **DB1** **[5]**	Or triangle *DEC* congruent to *BFA* *AB* = *DC* (parallelogram) $\angle ABF = \angle CDE$ (alternate angles are equal) $\Delta ABF \equiv \Delta CDE$ (SAS) Must have reasons $\angle AFB = \angle ECD$ (corresponding angles of congruent triangles) $\angle AFE = \angle FEC$ (each equal to $180 - \angle AFB$) Thus alternate angles are equal *AF* = *EC* (corresponding sides of congruent triangles) Other valid proofs should be awarded appropriate credit

7	$\mathbf{A}^{-1} = k\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$	**B1**	
	$k = \frac{1}{5}$	**B1**	
	$\mathbf{A}^2 = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	**M1** **A1**	attempt to multiply with at least two elements correct correct
	$\mathbf{B} = = 2 \times$ their $\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} -$ their $\begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	**M1**	
	$\begin{pmatrix} 1 & 15 \\ -5 & 6 \end{pmatrix}$	**A1** **[6]**	
8 (i)	0.97×0.04 0.05×0.96 Summing their products 0.0868	**M1** **M1** **M1** **A1**	
(ii)	their $\frac{0.0388}{0.0868}$ 0.447(00…) A.G.	**M1** **A1** **[6]**	

9	Eliminate x or y	**M1**	
	$4x^2 + 4x - 15 = 0$ or $4y^2 - 28y + 33 = 0$	**A1**	
	Factorise 3 term quadratic	**M1**	
	$x = \frac{3}{2}$ and $-\frac{5}{2}$	**A1**	
	$y = \frac{11}{2}$ and $\frac{3}{2}$	**A1**	
	$\sqrt{4^2 + 4^2}$	**M1**	
	$\sqrt{32}$ or $4\sqrt{2}$ or 5.66	**A1** **[7]**	
10 (i)	$m_{AB} = \frac{1}{5}$	**B1**	
	Uses $m_1 m_2 = -1$ $(= m_{BC} = -5)$	**M1**	
	BC: $y - 5 = -5(x - 6)$ or $5x + y = 35$	**M1**	or gradient $BC = \frac{5}{6 - x_c} = -5$
	C (7,0)	**A1**	
	CD: $y - 0 = \frac{1}{5}(x - 7)$ oe	**A1ft**	ft their C and m_{AB}
(ii)	D $(1, -1.2)$	**B1ft** **[6]**	ft their equation of CD

11 **(a)**	$\sin x = 2\cos x$ $\tan x = 2$ 63.4 243.4	**M1** **M1** **A1** **A1**	
(b)	$2(1 - \cos^2 y) + 3\cos y = 0$ $2\cos^2 y - 3\cos y - 2 = 0$ $(2\cos y + 1)(\cos y - 2) = 0$ $\cos y = -\frac{1}{2}$ 120 240	**M1** **M1** **A1** **A1** **[8]**	or correct use of quadratic formula or completing the square extra solutions within range –1 (once each part)
12	(1200**i** + 240**j**) ÷ 4 their (300**i** + 60**j**) – (260**i** + 156**j**) 40**i** – 96**j** $\sqrt{40^2 + 96^2}$ 104 $\tan^{-1}\left(\frac{96}{40}\right)$ or $\tan^{-1}\left(\frac{96}{40}\right)$ 157(.4)	**M1** **M1** **A1** **M1** **A1** **M1** **A1** **[7]**	clear indication of direction

<table>
<tr><td>13 (i)</td><td>4π, 16π, 36π</td><td>B1</td><td></td></tr>
<tr><td></td><td>4π, 16π – 4π , 36π – 16π
4π, 12π, 20π</td><td>M1</td><td></td></tr>
<tr><td>(ii)</td><td>$\frac{1}{9}$ soi</td><td>B1</td><td></td></tr>
<tr><td></td><td>$\frac{1}{12}$</td><td>B1</td><td></td></tr>
<tr><td>(iii)</td><td>
<table>
<tr><td><i>S</i></td><td>0</td><td>3</td><td>6</td><td>12</td></tr>
<tr><td>$P(S = s)$</td><td>$\frac{1}{4}$</td><td>$\frac{5}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table>
</td><td>B2, 1, 0</td><td></td></tr>
<tr><td>(iv)</td><td>their $0\times\frac{1}{4}+3\times\frac{5}{12}+6\times\frac{3}{12}+12\times\frac{1}{12}$ soi</td><td>M1</td><td></td></tr>
<tr><td></td><td>3.75</td><td>A1</td><td>ft their E(<i>S</i>)</td></tr>
<tr><td></td><td>75</td><td>A1 ft
[9]</td><td></td></tr>
</table>

14 (a) (i)	$\text{fg}(x) = 3 - \frac{x}{x+2}$	**B1**	
(ii)	$3 - \frac{x}{x+2} = 10$		
	$3(x + 2) - x = 10\ (x + 2)$ or better	**M1**	for dealing with fraction appropriately state this mathematically
	leading to $x = -1.75$	**A1**	
(b) (i)	$\text{h}(x) > 4$	**B1**	for attempting to obtain inverse function
(ii)	$\text{h}^{-1}(x) = \text{e}^{x-4}$	**M1**	or **M1** for $4 + \ln x = 9$ and
	$\text{h}^{-1}(9) = \text{e}^5\ (\approx 148)$	**A1**	**A1** for $x = \text{e}^5\ (\approx 148)$
(iii)	correct graphs	**B1 + B1**	B1 for each curve
	idea of symmetry	**B1** **[9]**	
		[80]	

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