

# MATHEMATICS (US)

---

Paper 0444/21  
Paper 21 (Extended)

## Key messages

To succeed in this paper candidates need to have completed full syllabus coverage, remember necessary formulae, show all necessary working clearly and use a suitable level of accuracy.

## General comments

This examination appeared to give the candidates plenty of opportunity to display their skills, although some found the demand of the paper rather challenging. There was a larger number of questions than on some previous papers but there was no evidence that candidates had time issues reaching the last few questions. There were candidates who omitted some questions or parts of questions, but this usually appeared due to being unsure of the skill they were being asked to demonstrate.

Candidates showed most success with the basic skills assessed in **Questions 1, 2, 3, 6, and 9a**. The most challenging questions were **Questions 7b, 11 to 13, 15b, 16, 17, 20, and 23 to 28**. Candidates were quite good at showing their working and so, in the majority of cases, it was easier to award method marks when answers were not correct or were inaccurate. Some candidate lost marks due to misreading or not following the demands of a question, this was particularly evident in **Questions 5, 7b, 16, 19b and 26**.

Greater clarity of numbers and letters in candidates' writing would have been helpful as often these were difficult to distinguish, in particular figures 1 and 7 and letters *a* and *q*. Marks were sometimes hard to award due to candidates not making it clear how they were approaching a solution. **Question 12** was an example where calculations were often seen piecemeal without an obvious strategy.

## Comments on specific questions

### **Question 1**

This offered candidates a comfortable start to the paper with most managing to offer one of the two answers (31 or 37), some giving both. There was a small number with answers out of range or a non-prime in range (e.g. 33, 35 or 39).

### **Question 2**

The successful candidates on this question usually evaluated  $3^4$  and  $2^3$  first before subtracting. Other than arithmetic slips, errors included summing four 3s or an answer of  $1^1$ ,  $1^7$  or  $1^{12}$  (subtracting the base numbers before a calculation involving the powers).

### **Question 3**

Usually question was well attempted, although a few candidates seemed to use 1 hour = 100 minutes. Another common error was being 1 hour out. A variety of methods was seen (e.g. approximating to 3 hours then adjusting by 3 minutes incorrectly, or lining up 13:02 – 10:05 but subtracting each smaller value from the larger one) leading to the answer 3 hours 3 minutes. A small number offered 3 hours 7 minutes.

### **Question 4**

Many were correct here but there were a number of candidates who did not appreciate the correct way round to convert between the currencies, so whilst many correctly divided 384 by 1.2, sometimes a multiplication

was attempted. Many who adopted the correct approach had difficulties performing the division accurately, but clarity of their intent enabled a method mark to be awarded.

### Question 5

Those with the correct factors usually gave their answer as a product as required. There was a variety of approaches, most commonly using factor trees, but sometimes with incomplete decomposition. A small number of candidates did not understand the demand, either listing any factors or simply giving a calculation resulting in 180. A few left the question blank.

### Question 6

Many correct answers were seen here with most showing working, usually with a denominator of 21, but others using 147 making extra work. Not all converted to the simplest form for their final answer (e.g. leaving it as  $\frac{7}{21}$ ) and consequently did not score the final mark.

### Question 7

Whilst many candidates navigated the calculation in **part (a)** correctly, arithmetic slips were most common among those using less efficient routes (such as  $0.45 \times 16$ ). When finding a product of three terms by hand candidates would be advised to consider the simplest order (here  $\frac{1}{2} \times 16$  first followed by  $\times 0.9$ ). A small number of attempts had the order of operation incorrect, e.g. trying to square 3.6.

In **part (b)** many were able to take correct steps to rearrange the formula, but often leaving answers with incomplete steps such as  $\sqrt{\frac{s+a}{0.5}}$  or  $\sqrt{\frac{s}{\frac{1}{2}a}}$ . Unsuccessful candidates usually took wrong first steps such as

subtracting  $\frac{1}{2}$  or had incorrect attempts to square root too early. Some took an unhelpful first step of dividing by  $t^2$ . A minority did not understand the demand, producing numerical answers (sometimes 'undoing' their work in **part (a)** ending back at  $t = 4$ ).

### Question 8

There were two factors to be extracted, 7 and  $y$ . A small minority took out only one common factor to give  $7(2xy - y^2)$  or  $y(14x - 7y)$ , scoring 1 mark, but others attempted to 'solve' the expression as an equation, or managed to introduce  $x^2$  into the situation. A further common error was to see an incorrect  $7y(7x - y)$ .

### Question 9

**Part (a)** gave candidates a comfortable start to the question with most spotting the pattern and obtaining the correct next term. A very small number instead stated the difference of  $-5$ . Working with linear sequences seems to be familiar to the majority, with the correct answer to **part (b)** being the most common response. Nearly all candidates recognised the term difference of  $-5$  with most then correctly using it as  $-5n$  in their answer. Commonly these also correctly found the constant term 27, with only a small number incorrectly giving  $22 - 5n$ . Other examples of incorrect answers seen included  $n - 5$  and  $27(n - 5)$ . Candidates should be encouraged to use correct notation and avoid answers starting with  $n = \dots$ , or with poor notation such as  $27 - n \cdot 5$ . A few did not understand the demand of the question, either leaving it blank or giving numerical answers.

### Question 10

Whilst many candidates were aware they needed to divide the angle sum by 30, correct answers were in a minority, as often the correct angle sum for a pentagon of  $540^\circ$  was not found. Most gained credit for knowing a multiple of  $180^\circ$  was needed for the division but this was very commonly  $360^\circ$ , leading to an incorrect answer of  $(12 \times 9 =) 108^\circ$ . Those with calculations finding the interior angle of a regular pentagon of  $108^\circ$  (by first finding  $360 \div 5$  for the exterior angle) did not score as this did not help to answer the question. Other common incorrect attempts found 30 but then divided by 5, or divided 180 by 30.

### Question 11

Few candidates could tackle this correctly; most were not able to convert the two numbers to a common form to allow subtraction leading to a valid '198' in their answer. A very few started well but left the answer as  $198 \times 10^{98}$ . Not uncommon errors were subtraction of powers with an answer of e.g.  $2 \times 10^2$ , or adding powers giving an answer including  $10^{198}$ .

### Question 12

Candidates found this question very challenging with many unsure how to start, and some left it blank. Only a small number managed a fully correct calculation. Often they were able to gain some credit either for correctly converting the speed to 20 m/s or for an attempt to multiply the speed by time 7 (either before or after attempts at conversion). It was not uncommon to see unsuccessful attempts at conversion, either using a factor of 60 just once, multiplying instead of dividing, or sometimes using a factor of 100 rather than 1000 for kilometres. A minority reached 140 metres for the distance travelled in 7 seconds, but this was often then seen as the final answer. Common incorrect calculations included speed 72 divided by time 7, or station length 100 divided by time 7.

### Question 13

Only a small number of candidates knew they had to simplify each surd using square number factors. Some came close but for example incorrectly simplified  $\sqrt{250}$  to  $10\sqrt{5}$  rather than  $5\sqrt{10}$ . Incorrect answers seen often resulted from either attempting  $\sqrt{250+810}$  or  $\frac{250+810}{2}$ . Others made ill-fated attempts to evaluate each surd rather than simplify.

### Question 14

Only a minority of candidates seemed to know that  $\frac{1}{64}$  was equal to  $4^{-3}$ , with 3 or 64 often offered as the answer. Amongst those that did understand,  $4^{-3}$  was sometimes offered as their answer, but as  $x$  was asked for  $-3$  was the only acceptable answer.

### Question 15

Similar triangles appeared to be a less familiar topic with many erroneously subtracting 2 in **part (a)**, although a good sized minority managed correct answers using a scale factor.

Correct answers in **(b)** did not usually follow an incorrect **part (a)**. For those with a scale factor approach this was often then used as a linear factor in **(b)** when attempting the new area, with the common wrong answer of 480. The other most common incorrect answer came from halving the surface area to 320.

### Question 16

Fully correct descriptions here were in a minority. In order, the most common aspects to score marks were for 'enlargement' (or 'dilation'), then the correct centre, with the scale factor being least commonly correct (often instead given as  $\frac{1}{2}$ , 2 or  $-2$ ). Despite the instruction to give a single transformation it was very common to see an attempt at a succession of transformations (which scores no marks), commonly involving rotation and translation and/or dilation. A number of candidates seemed to lack the familiarity with the vocabulary of transformations expected by the specification (instead using e.g. 'shrink', 'magnify', 'turn', 'slide').

### Question 17

For questions such as this candidates should be encouraged to start by showing substitution into the formula, in this case for the volume of a sphere given in the formula list, to form an equation. Many spent time attempting to evaluate  $\frac{9}{2} \times 3.14\dots$ , when had they equated the given volume to  $\frac{4}{3}\pi r^3$  they would have

realised this evaluation was unnecessary. Many struggled to make their method clear and the manipulation of the fractions was not always good. Of the few that reached  $\frac{27}{8}$  some failed to cube root to find the radius.

### Question 18

In **part (a)** candidates should have noted that PA and PB are tangents to the circle from the same external point, so they must be equal. A minority were able to use this reasoning to conclude that triangle PAB was isosceles. The most common incorrect answers were 'equilateral' and 'scalene'.

The most common answer to **(b)(i)** was the correct angle of  $65^\circ$ . A not uncommon wrong answer was  $50^\circ$ . Correct follow through however meant that the mark was often given in **part (ii)**, where candidates recognised the right-angle between radius and tangent. An answer of  $130^\circ$  was sometimes given, perhaps as being angle *AOB* rather than the required *OAB*. Some stated the answer as  $90^\circ$ .

**Part (c)** was often answered correctly but a common error was triangles *PAB* and *OAB*, which are both isosceles but clearly not the same. Others did not seem to realise that naming a triangle needed three vertices stated.

### Question 19

Although a number of candidates left **part (a)** blank, many knew the correct answer of 4, with the most common incorrect answer being 2. In **part (b)**, whilst many correctly recognised the rotation, only a small number fully described it with angle and centre of rotation. The most common incorrect answers involved reflection, whilst others simply described where individual vertices would move to.

### Question 20

Whilst some candidates were able to give the correct answer of  $70^\circ$  in **part (a)**, few were able to give a correct geometrical reason as asked. Often they were let down by poor vocabulary or simply referring to being 'based on the same arc', without reference to the relevant angles being formed at the centre and at the circumference. Common incorrect answers were  $140^\circ$ , or  $40^\circ$  (perhaps thinking it had a sum of  $180^\circ$  with the given  $140^\circ$ ).

A variety of answers were given to **part (b)**, some correct. There were candidates who were able to gain a follow through mark, possibly recognising and using the angle sum of opposite angles in the cyclic quadrilateral.

### Question 21

The most effective first step here taken by candidates was by those who multiplied by  $(3t - 2)$ , although some forgot the brackets which made the step incorrect. Some also multiplied by 5 in the same step ('cross multiplying') completely removing fractions from the equation, making it a more familiar problem. The least successful candidates made no attempt at solving and just wrote down an answer or took a 'trial and error' approach.

### Question 22

Whilst collecting the constant terms was often achieved there was more difficulty dealing with the  $\sqrt{x}$  terms. The more successful candidates were able to see the problem as 'find  $\sqrt{x}$ ' first. Some tried to deal with  $\sqrt{x}$  by squaring first, which was an unsuccessful approach. Some thought they could 'cancel' the  $\sqrt{x}$  from each side which left them without a variable. A handful of good attempts were spoilt by confusion with signs whilst manipulating the equation.

### Question 23

Candidates had difficulty with this problem, with only a minority scoring full marks. Some were unaware that brackets were needed at all, whilst those not quite earning 1 mark did not have two brackets the same, a key first step, (e.g. instead reaching an unhelpful  $1 - q + a(q - 1)$ ). It was also not unusual to see errors dealing

with the signs for the second bracket (i.e. having the incorrect  $1 - q - a(1 + q)$ ). Of those that did correctly reach  $1 - q - a(1 - q)$  not all then treated the first two terms as  $1(1 - q)$  which leads to the final step.

#### Question 24

Whilst only a minority were able to get a fully correct answer many were able to either obtain the correct coefficient 36 or the correct power 144. Some candidates had both of these as 36 or both as 144.

#### Question 25

In **part (a)**, only a minority of candidates knew the quick way to identify the value of  $a$  (halving the coefficient of  $x$  in the expression), and then finding  $b$ . A larger number attempted to expand  $(x + p)^2$ , sometimes correctly to score 1 mark but, commonly not then proceeding to the correct answers. A fair number of candidates choosing this route were unable to square the bracket correctly. Often  $p$  and  $q$  were 'found' by a rearrangement of the question with  $p$  and  $q$  offered in terms of  $x$  and  $q$  or  $x$  and  $p$ , which did not address the problem.

Whilst **part (b)** was much more successful for candidates than **part (a)**, with a number of correct answers seen, it was extremely rare to see any use their answer to **part (a)**. Instead most chose to rearrange the equation and use the quadratic formula, or in a very few cases factorise. Some of these ignored the RHS and worked with  $x^2 + 8x + 10$  only, which was not acceptable. Some clearly could not spot either approach to the question and merely tested a few possibilities, but unless they found both  $x = 2$  and  $x = -10$  they earned no marks as they had no clear method.

#### Question 26

Only a minority of candidates were able to score marks for at least one of the two relationships stated as an equation, but much less common was being able to combine these two into a correct final answer. Poor use of notation meant that many used  $k$  to stand for two different constants of proportionality, sometimes both found correctly, which then usually caused confusion. Only a minority of candidates were able to combine the two relationships, with a correct answer often given as  $2\sqrt{\frac{64}{x}}$ . Those missing out the step of writing as equations using a constant of proportionality were least successful. Some candidates seemed to think that a numerical answer was expected.

#### Question 27

Candidates who realised that factorisation of the denominator was required were very much in a minority, but those gaining credit for this step sometimes cancelled to leave their answer as  $(x + 1)$  rather than  $\frac{1}{x + 1}$ . Other common misconceptions were thinking it was possible to 'cancel' the  $-3$  (or sometimes an  $x$ ) from top and bottom.

#### Question 28

Candidates commonly find vectors tricky and this question was no exception, with a number of blank responses and some with answers that were not vectors or were given as numerical column vectors. Whilst correct answers to **part (a)** were rare some managed to gain a mark here for  $(t - p)$  or an incorrect multiple. Clear labelling of intermediate vectors candidates are trying to find would most likely lead to more marks being earned in many cases. If unlabelled, the Examiner does not know the candidate's intention. Other common errors observed included for example finding vector  $\overline{TP}$  but thinking it is  $\overline{PT}$ .

After an unsuccessful attempt at **part (a)** few were able to score in **part (b)**. Candidates should be encouraged to start vector questions by considering a route using vertices (and hence vectors) from the diagram, which can often, as here, gain credit. Although its absence was not penalised, candidates should be encouraged to use correct notation for vectors with underlining of single lower case letters or an arrow above a pair of upper case letters.

# MATHEMATICS (US)

Paper 0444/41  
Paper 41 (Extended)

## Key messages

Candidates sitting this paper need to ensure that they have a good understanding and knowledge of all the topics on the extended syllabus. A significant number of scripts did not offer any responses to whole questions, the most common being **Questions 4**, functions, and **Question 7**, trigonometry.

Candidates generally showed a good level of working but there were a significant number of scripts with incorrect answers written on the answer line but with no method or working. Had some of these answers had working it may have been possible to award method marks to the candidate.

When working through the longer questions, candidates should retain the accuracy of their calculations through the stages. Candidates are prematurely rounding within the working, and as a consequence their final answers are frequently not within the required range. Unless directed otherwise, candidates should give answers to at least 3 significant figure accuracy.

## General comments

There were some excellent scores on this paper with a good number of candidates demonstrating that they had a clear understanding across the wide range of topics examined.

Candidates should make sure they read the detail of the questions carefully. For example, in **Question 3(a)**, Chris received 12 strawberries, but some candidates had Alex receiving the 12 strawberries. In **Question 6(a)(iv)** many candidates overlooked the axes having different scales. In **8(a)(i)** a possibility space was required, not the probabilities

Candidates should be aware of when a question connects to the next part. In **Question 1, (a)(ii)** used the answer to **(a)(i)**, **part (b)(ii)** was much easier if **(b)(i)** was used and in **Question 10, part (b)** was much easier if **(a)** was used.

Questions that ask candidates to 'show' results require candidates to start with the given information and arrive at the value or result that is asked to be shown. For example, in **Question 5(a)(i)(a)**, candidates should not start with the  $67.5^\circ$  but start with either  $\frac{360}{8}$  or  $(8-2)\times 180$  and show all the steps needed to reach the  $67.5^\circ$ . In **Question 7**, the length of  $BC$  needed to be arrived at, not started with. In **9(b)(i)** candidates needed to arrive at the quadratic equation rather than try and solve the equation. The next **part 9(b)(ii)** was where the solving needed to be completed.

In probability questions, if the question is given with fractions then it is usually best to stay working in fractions rather than working in decimals which will not necessarily be exact. For example, **Question 8(a)** and **(b)**. Probabilities should not be given as ratios.

Logarithms are not part of the extended syllabus. Some candidates used logarithms in both **1(b)(ii)** and **3(d)(ii)** but this was not the expected way to solve these questions. In **1(b)(ii)** respondents were expected to recognise that  $2^{-y} = 8$  gives  $y = 3$  and in **3(d)(ii)** candidates were expected to complete trials.

Questions with algebra in the stem will normally be expected to be solved by setting up an algebraic equation.

For example, **Question 1(a)(i)** is easier to solve from the linear equation than from trials. **Question 9(a)** involving quadratics, and asked for working to be shown, requiring an algebraic method to gain full marks. In

this, some candidates worked out that  $x = 6$  from trials, but without a rigorous algebraic method they have not shown convincingly that this is the only solution.

### Question 1

- (a) (i) The most successful candidates started by writing down a linear equation to solve. Common errors included omitting the  $x$  and hence finding Geeta buys 8 apples, writing the terms as a product rather than a sum of expressions, and errors when collecting like terms and errors with division. Some did not set up an equation but tried to find  $x$  by trials. This is not an efficient method and only those that found the correct answer scored.
- (ii) Many candidates answered this correctly. Most scored at least one mark by using their (a)(i) to work out the amount spent on apples and/or oranges. The most successful converted the \$5.55 to 555 cents at this stage. Others worked in dollars, and some then gave the cost of a banana as 0.21 rather than 21 cents. A common error was to calculate  $5.55 - 0.15 - 0.18$ .
- (b) (i) A fair number of candidates answered this question correctly. The most effective method was to add the 1 to both sides and then multiply by 16. Those choosing to multiply by 16 first, frequently forgot to multiply the  $-1$  by 16 and arrived at  $x = 3$ . If candidates had checked their answer by substituting it back into the original equation, they may have realised when they had made an error.
- (ii) It was rare for candidates to use the previous part to answer this part and start with  $2^{-y} = \text{their } w$ . Most candidates started again and usually made the same rearrangement errors as in the previous part. Others started by incorrectly replacing  $3(2^{-y})$  by  $6^{-y}$ . Those that correctly arrived at  $2^{-y} = 8$  could not always work out the value of  $y$  with candidates giving incorrect answers such as  $-4$ ,  $3$ ,  $-\frac{1}{3}$  or resorting to logs, sometimes successfully.

### Question 2

- (a) Only a minority of candidates were able to give an example of continuous data. These examples usually included length, volume, mass or temperature. Some candidates gave specific examples such as height of children. Common incorrect answers included: population, 1, 2, 3, 4, 5, ..., 0.33333...,  $\pi$ , or an equation.
- (b) (i)(a) A good proportion of candidates were able to correctly give the range of 4 by recognising that the marks went from 6 to 10. The most common incorrect answer was 8 which came from subtracting the maximum and minimum frequencies, namely  $9 - 1$ . Other incorrect answers seen included 24, the sum of the frequencies.
- (b) Many scripts answered this correctly. Some weaker ones gave the answer 9, possibly relating it to the highest number of marks, 10. A common misconception was to consider the numbers in both lines of the table and give one or more of 3, 8 and 9.
- (c) Some candidates answered this correctly by working out they needed the mean of the 12<sup>th</sup> and 13<sup>th</sup> highest marks, namely  $\frac{8+9}{2}$ . Some candidates were able to select the 8 and 9 by recognising that  $1 + 3 + 8 = 12$ , so the 12<sup>th</sup> number was an 8 and hence the 13<sup>th</sup> a 9. Others took the longer approach writing out 6, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10 but not all were successful in either writing this out accurately or finding the middle terms. Common errors included finding the median of the five frequencies as 3 or finding the median of the five available marks as 8 or finding the median of the 10 numbers in the table as 7.5.
- (ii) Only the strongest scripts answered this correctly. Common errors included giving the number of candidates who scored 10 marks as 9 and going no further. Others gave the fraction  $\frac{9}{24}$ , sometimes converting it to 37.5 per cent but not calculating the number of degrees of the sector. Incorrect methods included  $10 \times 9 = 90^\circ$ ,  $\frac{10}{24} \times 360 = 150^\circ$  or calculations involving the mean marks or using  $\pi$ .

- (c) (i) This part was answered well by a good proportion of candidates. The most common errors were usually from misreading the scale or answers of 110 (the middle of the LQ and the UQ) or 100 (the middle of 0 to 200) or 105 (the middle of the lowest and highest values, 40 and 170) or  $65 \left( \frac{170 - 40}{2} \right)$ . Some candidates gave the range or the interquartile range.
- (ii) Most candidates gave the correct answer for the range. The most common errors included writing 40 and 170, but not calculating the difference, finding the interquartile range, giving the median, inaccuracies with reading the values or errors with arithmetic.
- (iii) Many candidates successfully found the interquartile range. As before, the most common errors were giving the UQ and LQ values but not their difference, finding the whole range or errors with reading the scale or giving the median.
- (d) (i) Candidates generally set their working out clearly and methodically and many produced fully correct answers. With clear working, those making slips with arithmetic often scored 3 marks. Some used the upper or lower boundaries of the classes or other numbers such as 50.5, 70.5, 85.5, 95.5 and 125.5 rather than the mid-interval values, but often their working enabled them to score two of the marks. Those that used the class widths rather than the mid-interval values did not score. Other errors included finding the mean of the mid-interval values, errors with the mid-interval values, finding the cumulative frequencies, dividing by 5 rather than 200 or incorrect answers with no working.
- (ii) This question proved challenging for most. Some candidates scored one mark for either  $\frac{86}{50}$  or  $\frac{114}{60}$ . Most did not use the principle of  $\text{height} = \frac{\text{frequency}}{\text{class width}}$  however. A very common incorrect answer of 22.8 from  $\frac{86}{114} = \frac{17.2}{x}$  took no account of the class widths being 50 and 60. Other attempts showed various sums, differences, products and divisions of the various numbers given in the question.

### Question 3

- (a) Many candidates answered this correctly. The most common method was to work out that 1 part was 6 strawberries so  $3 + 2 + 2$  parts is 42. Other successful methods included recognising that Alex had 1.5 as many strawberries as Chris and to sum  $18 + 12 + 12 = 42$ . The most common error was a misreading of the question with Alex having 12 strawberries and a total of 28 from  $\frac{12}{3} \times 7$ .
- (b) (i) A good number of candidates answered this correctly. A number of errors were made due to some not reading the question carefully. Common incorrect answers seen included \$0.78 (finding the reduction but not the new price), \$6.38 (subtracting 12 cents rather than 12 per cent), \$7.28 (increasing \$6.50 by 12 per cent), \$5.80 (dividing \$6.50 by 1.12), \$7.39 (dividing \$6.50 by 0.88).
- (ii) A fair number of candidates recognised this as a reverse percentage question and answered it correctly. A very common error was to increase \$11 by 12 per cent to \$12.32 without candidates realising that 12 per cent of \$11 is not the same as 12 per cent of the original price. Other errors included \$11.12 (from \$11 add 12 cents) and \$11.78 (\$11 + the reduction of \$0.78 found in the previous part)
- (c) Only the strongest scripts answered this correctly, recognising that the investment would receive interest on interest. Those that did, usually set up an equation such as  $1.025 \times X = 1.066$  to successfully find X. The most common wrong answer seen was  $6.6 - 2.5 = 4.1$ . Other wrong answers included  $2.5 + 6.6 = 9.1$ , and  $\frac{6.6}{2.5} = 2.64$ .
- (d) (i) Some candidates answered this question correctly. A common mistake was to work out that 1.6 g was lost on the first day and then wrongly assumed the mass lost 1.6 g on each of the next 4 days,



giving  $80 - 5 \times 1.6 = 72$ . Other errors seen included using exponential growth rather than decay or working out  $\frac{80}{1.02^5}$ . Of those employing the correct method, many worked out the value day by day rather than using  $0.98^5$  and, as a consequence, errors were seen in arithmetic, numbers within the process were rounded prematurely and often the mass was found after 4 days and not 5.

- (ii) Candidates almost always used the same method in this part as the previous part and hence a similar number answered the question correctly. The most successful candidates either took their answer to the previous part or used 80 and multiplied it by  $0.98^k$  for various integer values of  $k$  until 67 was straddled. Some set up an equation  $80 \times 0.98^k = 67$  and solved it, often successfully, by trials. Some candidates overlooked the need to give the extra whole days and gave 9 as their answer.

#### Question 4

- (a) (i) This question was answered correctly by almost all candidates.
- (ii) This question was answered well. Common incorrect answers included working out  $g(2) = 4$ , working out  $g(2)f(2) = 4 \times 3 = 12$  and  $g(x)f(2) = (3x - 2) \times 3 = 9x - 6$ .
- (b) Many candidates were able to correctly find the inverse function. Most candidates started by swapping the  $x$  and  $y$  in the function to  $x = 3y - 2$  and then rearranging. The common rearranging errors seen included either not dividing every term by 3 or moving the  $-2$  to the other side with the wrong sign. Other errors included just reversing the signs in  $g(x)$  as  $g^{-1}(x) = -3x + 2$  or confusing the inverse function with reciprocal as  $g^{-1}(x) = \frac{1}{3x - 2}$ .
- (c) This question was generally answered correctly with most candidates showing  $\frac{1}{x} = 5^{-2}$  before  $x = 25$ . Common wrong answers included  $\frac{1}{25}$ . A few candidates thought that  $x \neq 0$  was part of the function and tried to solve  $\frac{1}{x} + x \neq 0 = 5^{-2}$ .
- (d) Most candidates were able to set up the combined function correctly. Common errors seen included  $2x - 1 - \frac{1}{x} = \frac{2x - 1 - 1}{x}$  or attempts at solving the equation  $2x - 1 = \frac{1}{x}$ . In addition, a significant number of candidates thought that  $x \neq 0$  was part of the expression and included it in their fraction.
- (e) Few responses were awarded the mark in this question, as  $5^{25}$  was required to be evaluated. Others evaluated it but wrote it down in calculator language such as 2.98 E17 rather than in clear standard form. A very common incorrect answer was 625, found from  $j(2)j(2)$  rather than the  $jj(2)$  required.
- (f) This question also proved to be challenging to the cohort, with the key to this question being able to rewrite  $j^{-1}(x) = 4$  as  $x = j(4)$  and hence evaluate  $5^4$  as 625. Common incorrect methods included  $x^5 = 4$  and  $5 = 4^x$ .

#### Question 5

- (a) (i)(a) Many candidates were able to convincingly show that angle OAM is  $67.5^\circ$ . Some started from angle  $AOB = \frac{360}{8}$  and used either exterior + interior angles = 180 or angles in a triangle, namely OAM or OAB. Others started from  $\frac{(8 - 2) \times 180}{8}$  with equal success. Candidates who did not score

either started with the  $67.5^\circ$  or stated that angles in an octagon total  $1080^\circ$  without evidencing  $(8 - 2) \times 180$ .

- (b) There were a good number of clear and accurate solutions to this question. All of the methods involved using trigonometry and finding either or both of  $OA$  and  $OM$  followed by the area of a triangle using  $\frac{1}{2}bh$  or  $\frac{1}{2}ab\sin C$ . The most common errors included premature approximation or forgetting to multiply by  $\frac{1}{2}$  or 8 or 16. Many candidates mistakenly multiplied their triangle area by 6 instead of 8. A noticeable number of scripts did not attempt this question.
- (ii) Again, a good number of accurate solutions were seen in this part with many candidates using the correct formula for the area of a circle with  $OA$  (or  $OB$ ) as the radius. Common errors seen again included premature approximation or using  $OM$  as the radius. Many of the candidates who did not attempt the previous part also did not attempt this part.
- (b) (i) Many candidates correctly found the volume of the container. The common errors included using the wrong formula for the area of a circle, forgetting to find half the area of a circle, or halving twice or not multiplying by 4. Some candidates attempted to find the volume in cm but few of these were able to correctly convert back to  $m^3$ .
- (ii) Only a few candidates scored full marks on this multi-step and challenging problem-solving question. Most candidates treated the shaded area as a semi-circle with radius 0.3m and therefore usually scored no marks. Only the most astute candidates recognised that the container still had a radius of 0.45m and that trigonometry was required to find appropriate angles in order to be able to work out useful areas of triangles and sectors within the whole shape.

### Question 6

- (a) (i) Most candidates were able to read off the graph accurately at  $x = 2$ . A noticeable minority of candidates incorrectly read the graph at  $y = 2$  giving  $x = 1.3$  as their answer. In addition, a number of candidates did not attempt this question.
- (ii) Almost all of the candidates who answered the question were able to read off the 3 solutions to the equations accurately. Many did not offer a response at all though.
- (iii) A good number of candidates were able to give the correct integer value for  $k$ . Most of the many different errors came from not interpreting the question correctly. Common errors included,  $k = -8.2$ , the smallest non-integer value,  $k = -9$ , the lowest point on the graph within the range,  $k = -1.5$  and  $k = -1$ , the lowest value and the lowest integer value within the range of the inequality  $-1.5 \leq x \leq 5$ .
- (iv) Many candidates attempted to draw  $y = 10 - 2x$ . Most drew a ruled line through the point (0, 10). A common error was to not appreciate that the scales on the axes were different and consequently many drew the line  $y = 10 - 4x$ . In addition, another common error was to read off the  $x$  value where the line crossed the  $x$  axis rather than to give the  $x$  value where the line intersected the curve. Other errors included not drawing the line accurately or drawing the line without a ruler.
- (v) Quite a few scripts gave no response to this question. Of those who drew a line on the graph, some satisfied both criteria but others either intersected the graph of  $f(x)$  once or passed through the origin. The majority of lines were ruled and extended to the edges of the grid.
- (b) Only a few candidates were able to find the correct equation of line  $L$ . Most candidates were unable to show knowledge that the products of the gradients of a pair of perpendicular lines is  $-1$ . Common errors seen included using the gradient  $-2$  or  $\frac{1}{2}$ . Others reached  $y = -\frac{1}{2}x + c$  but did not substitute (4,  $-1$ ) correctly to find  $c$ .

### Question 7

- (a) Candidates needed to recognise that the cosine rule was needed to find angle  $ACD$ . Those that selected it were usually successful, provided they set it up to find angle  $ACD$  and not  $CAD$  or  $CDA$ , which was sometimes the case. Some made errors with the formula with  $+2ab\cos C$  or  $-4ab\cos C$  or  $-2ab\sin C$  seen within it. Common errors included multiplying  $\cos C$  by the whole of  $12^2 + 14^2 - 2 \times 12 \times 14$  or introducing an extra negative sign when rearranging. Some candidates tried to find angle  $ACD$  by using the sine rule and this often used a wrong assumption such as angle  $ABC + \text{angle } ADC = 180$  possibly thinking quadrilateral  $ABCD$  was a cyclic quadrilateral. Some candidates wrongly assumed angle  $ACD = 25^\circ$  as alternate to angle  $ACD$  or that angle  $ACD = ACB = 32^\circ$ .
- (b) Candidates often recognised the sine rule needed to be used and there were many correct rearrangements seen. The best solutions were those where the candidate arrived at  $\frac{14\sin 25}{\sin 123}$  and only then used their calculator. Those giving rounded calculator values at each step risked being inaccurate at the end. The rigour of the question involved meant that respondents needed to obtain the length of  $BC$  to more than 2 decimal place accuracy in order to confirm that its value is indeed 7.05 when rounded. As a result, many responses lost the final mark.
- (c) There was generally good understanding that the perpendicular from  $B$  to  $AC$  was the required length and some indicated this clearly on their diagram. The most efficient method was using right angled trigonometry in triangle  $BCN$ , where  $N$  is the foot of the perpendicular from  $B$  to  $AC$ , although some slightly prolonged their work by using the sine rule and  $\sin 90^\circ$  term in their solution. Other longer correct solutions were common such as finding the length of  $AB$  before using triangle  $ABN$  or using the area of triangle  $ABC$  with  $\frac{1}{2} \times 14 \times 7.05 \times \sin 32 = \frac{1}{2} \times 14 \times h$ . A common misconception seen assumed that the shortest distance would be obtained by halving either angle  $ABC$  or the line  $AC$ .
- (d) Many candidates recognised the need to use the cosine rule again, this time in triangle  $BCD$ , with most using *their* angle found in **part (a)**. Again, there were many who felt the need to use triangle  $BAD$  and, although perfectly correct, created extra work for themselves and often the many stages lost some accuracy. Similar errors with the cosine rule were seen as in **part (a)**. Despite having evidence that angle  $BCD \neq 90^\circ$ , some candidates wrongly chose to use Pythagoras, calculating  $7.05^2 + 12^2$ .
- (e) A minority of candidates showed understanding of how bearings are measured. Those with understanding simply added their answer from **part (a)** to  $270^\circ$ . Answers linked to the points of the compass such as due NE were common and some candidates gave a distance.

### Question 8

- (a) (i) The best approach was to use a 2 by 2 possibility table to show systematically the 20 combinations and their totals. A minority of candidates used a table approach, but these were usually successful. Most candidates tried to write down a list of the combinations, attempting to do so systematically. The most common errors included not being systematic, omitting some combinations, omitting to add the scores or having 5 numbers on both spinners and thus 25 combinations. Others just gave the possible different totals, namely 2,3,4,5,6,7,8,9.
- (ii)(a) Candidates with the correct possibility diagram and totals were usually able to give the correct answer. Some candidates gave the number of possibilities rather than the probability. Candidates who gave their answer as a ratio did not score.
- (b) Candidates who answered (a)(ii)(a) correctly usually answered this part correctly. Again, some candidates gave the number of possibilities rather than probabilities and, again, answers as a ratio were not accepted.

- (c) Candidates were more successful on this part with a good number recognising, with or without a possibility diagram, that all the combinations gave a total less than 10 and thus the probability was 1. Answers as a ratio were not accepted.
- (iii) A good number of candidates demonstrated an understanding of expectation and were able to score the mark whether using the correct probability or their probability from (a)(ii)(a).
- (b)(i) Most candidates answered this correctly. The best approach is to give the answer as a fraction and those candidates giving the probability to 2 significant figures as 17 per cent, with no prior working, did not score. Again, answers as a ratio were not accepted.
- (ii)(a) A good number of candidates answered this correctly. The most common incorrect answer was  $\frac{2}{3}$  from adding the two fractions together rather than multiplying them. As previously, answers as a ratio were not accepted.
- (b) This question proved very challenging with candidates not understanding that the word 'or' means: a tail or a 4 or both. Candidates who were successful had often written out the 12 possible outcomes and picked out the 7 from these. The two most common incorrect methods were to find  $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$  without realising that this included (T, 4) twice or  $\frac{6}{12} = \frac{1}{2}$  which did not include (T, 4) once. Again, answers as a ratio were not accepted.
- (c) Many candidates scored one mark for working out the probability of it being fine and Jodie going swimming, namely  $\frac{4}{5} \times \frac{2}{3}$  but not considering the probability of Jodie going swimming if it is not fine. Common errors seen included candidates adding rather than multiplying the fractions. Answers as a ratio were not accepted.

### Question 9

- (a) This question was also a challenging question, requiring working to be shown. A number of candidates produced exceptionally clear and detailed solutions, scoring full marks. A number of stages were required, and most candidates were able to earn one or more of the marks. Candidates needed to firstly set up an equation connecting the information, and many were successful in doing so. Common errors included using perimeter rather than area or having the 29 on the wrong side of the equation. Many candidates correctly gave the area of the larger rectangle as the quadratic expression,  $2x^2 - x - 1$ . Candidates then needed to solve their quadratic equation either evidencing their factorising or using the quadratic formula. Those candidates who found  $x = 6$  by trials or inspection lost the method mark because they had not proved that this was the only possible positive solution, which the use of the quadratic formula or factorisation showed. Candidates who had a value for  $x$  were often able to arrive at a perimeter with a value equal to  $2 \times \text{their } x$ .
- (b)(i) A few candidates were able to start the question by writing an algebraic equation involving the areas of the two triangles and the difference between their areas. These candidates usually worked through the steps showing clear algebraic skills, often being precise with their use of brackets and expanding carefully. Because this was a show question, all brackets, signs, and numbers need to be absolutely perfect to score full marks. The majority of candidates who did not score on this question either left it blank or attempted to solve the equation by either rearranging or using the quadratic formula, which was not required in this part.
- (ii) Some candidates did realise that they needed to solve the quadratic equation given in part (i) to find  $y$ . Those who used the quadratic formula usually showed precise substitution into the formula to find the values of  $y$ . Some stopped there but others recognised the need to go back to the diagram, or their original algebraic expression for the area of the small triangle, to complete the question successfully. The most common error was finding the roots of the equation to not enough accuracy which led to the area of the little triangle being out of range. Those who attempted to solve the quadratic algebraically, did not know how to isolate the  $y$  by using the completing the square method and so were almost always unsuccessful.

**Question 10**

- (a) Candidates answered this question well with most realising that you could add the two equations to eliminate  $q$  and find the required values. The most common errors included slips with arithmetic and signs when finding the second variable. Candidates who attempted this question, and who did not score full marks, frequently picked up one mark for one of the values, or, more often for two values satisfying one of the given equations.
- (b) Very few candidates recognised that this part connected to the previous part, with most attempting to solve the equations again. Many candidates who scored full marks in the previous part were unable to solve these, often being unsure how to deal with the words. Most candidates who either wrote down or reached  $\sin u = 0.5$  and/or  $\cos v = 1$  were able to find the principal values but the secondary values proved more elusive, with those drawing sketches of the sine and cosine graphs being the most successful.