

# Cambridge IGCSE<sup>™</sup>

CANDIDATE NAME					
CENTRE NUMBER				CANDIDATE NUMBER	
CAMBRIDGE	INTERNAT	IONAL MATHEM	IATICS		0607/62
Paper 6 Investigation and Modelling (Extended)		Oct	ober/November 2023		
					1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

#### INSTRUCTIONS

- Answer both part A (Questions 1 to 4) and part B (Questions 5 to 7).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

#### INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

The investigation starts on page 3.

#### Answer both parts A and B.

3

# A INVESTIGATION (QUESTIONS 1 TO 4)

# **CLOCK HANDS (30 marks)**

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the angle between the hands of a clock at different times of day.

You should **not** measure angles from the clock diagrams.

In this investigation:

- the hour hand is labelled H
- the minute hand is labelled M
- the hands of the clock rotate clockwise in the direction shown
- the *clockwise angle* between the two hands is shown on the clock.



 (a) The clock shows the time 1.00 am. In one hour, hand *H* rotates clockwise from one number to the next number. For example, from 1.00 am to 2.00 am hand *H* rotates from 1 to 2.

Show that hand *H* rotates  $0.5^{\circ}$  in one minute.

(b) In one hour, hand *M* rotates through a full circle.

Show that hand M rotates  $6^{\circ}$  in one minute.

[2]

[1]

2 m is the number of minutes after the last hour. In this question the last hour is 1.00 am.

<u>Examples</u> At 1.10 am, m = 10. At 1.45 am, m = 45.

(a) This clock shows the time 1.10 am.

Show that the clockwise angle from hand H to hand M at 1.10 am is 25°.



[2]

## (b) Complete the table. You may use the clock diagrams to help you.

Number of minutes	Angle rotated since	Clockwise angle	
( <i>m</i> )	Hand <i>H</i> angle	Hand <i>M</i> angle	in degrees
6			
7			8.5
8			
9			
10			25



(c) Find an expression, in terms of m, for the clockwise angle between the hands when the last hour is 1.00 am.

(d) Find how many minutes and seconds after 1.00 am the clockwise angle is 270°. Give your answer correct to the nearest second.

..... minutes ..... seconds [4]

- 3 In this question the last hour is 2.00 am.
  - (a) This clock shows the time 2.15 am.

Show that the clockwise angle between the hands is 22.5°.



(b) Complete the table.You may use the clock diagrams to help you.

Number of minutes after the last hour (m)	Clockwise angle between the hands in degrees
15	22.5
16	
17	
18	
19	



[2]

(c) Find an expression, in terms of m, for the clockwise angle between the hands when the last hour is 2.00 am.

......[2]

4 (a) *h* is the number of hours in the time. *m* is the number of minutes after the last hour.

> Examples At 1.30 am, h = 1 and m = 30. At 8.45 am, h = 8 and m = 45.

Complete the table using expressions of the form am + b.

Use your expressions from Question 2(c) and Question 3(c).

	Number of hours in the time $(h)$	Clockwise angle in degrees between the hands $m$ minutes after $h$
Question 2(c)	1	
Question 3(c)	2	
	3	
	4	

[1]

(b) Find an expression, in terms of *m* and *h*, for the clockwise angle between the hands.

- (c) You may use the clock diagrams to help you in this part.
  - (i) Use your expression from **part** (b) to find the two angles between the hands at 10.12 am.

..... and ...... [3]

(ii) There are two times between 7.00 am and 8.00 am when an angle between the hands is 100°.Find these times correct to the nearest minute.

..... and ..... [4]



#### **B** MODELLING (QUESTIONS 5 TO 7)

## **ARCHES (30 marks)**

You are advised to spend no more than 50 minutes on this part.



This task looks at models for the lengths of arches. An arch is the curved part of a tunnel or bridge.

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Engineers use the dimensions of an arch to calculate its strength.

In this task, each arch has:

- width *w* metres
- height *h* metres
- curved length *l* metres.
- 5 <u>Semicircular arch model</u>



This arch is a semicircle. The width is 8 m.

(a) Write down the height of the arch.

......[2]

(b) Find the curved length of the arch. Give your answer correct to the nearest centimetre.

6 <u>Segmental arch model</u>



This arch is an arc of a circle with radius *r* metres. The width of the arch is 8 metres.

(a) The formula for the radius of the circle is

$$r = \frac{4h^2 + w^2}{8h}.$$

(i) Show that  $r = \frac{h}{2} + \frac{8}{h}$ .

[2]
-----

(ii) On the diagram, sketch the graph of r for  $0 \le h \le 8$ .



(iii) Find the height that gives the minimum radius.

(b) (i) The height of the arch is 1 metre.

Find the radius of the arch.



The angle at the centre of the circle that forms the arch is  $\theta$ .

Find the value of  $\theta$  correct to the nearest degree.

......[3]

(iii) Find the curved length of the arch.

.....[2]

## 7 Lancet arch model



This arch is made using two equal arcs.



The equal arcs are parts of the circumferences of two identical circles both of radius r metres. The base of the arch is on the line joining the centres of the two circles. The height is on the line of symmetry of the arch.

(a) Use Pythagoras' Theorem to show that the model for r in terms of h and w is

$$r = \frac{h^2}{w} + \frac{w}{4}.$$

(b) The width of the arch is 8 metres.

(i) Show that 
$$r = \frac{h^2}{8} + 2$$
.

[1]

(ii) On the diagram in Question 6(a)(ii), sketch the graph of r for  $0 \le h \le 8$ .

[2]

(iii) Find the value of *h* and the value of *r* when the graphs from Question 6(a)(ii) and Question 7(b)(ii) intersect.

Explain what your answers show about the shapes of the two arches at this point.

- (c) The width of the lancet arch is 8 metres. The radius is 10 metres.
  - (i) Find its height.



Use trigonometry to help you find the curved length of the arch.

......[4]

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