## Cambridge IGCSE ${ }^{\text {TM }}$



## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 6 Investigation and Modelling (Extended)
October/November 2023
1 hour 40 minutes

You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer both part A (Questions 1 to 4 ) and part B (Questions 5 to 7 ).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 60 .
- The number of marks for each question or part question is shown in brackets [ ].


## Answer both parts A and B.

## A INVESTIGATION (QUESTIONS 1 TO 4)

## GEOMETRIC SEQUENCES (30 marks)

## You are advised to spend no more than 50 minutes on this part.

This investigation looks at sequences where the rule to get from one term to the next is a multiplication. These are called geometric sequences.

1 (a) These are the first four terms of a geometric sequence.

$$
\begin{array}{llll}
4 & 16 & 64 & 256
\end{array}
$$

(i) Work out the next term.
$\qquad$
(ii) Work out the 8th term.
$\qquad$
(iii) Find the $n$th term.
(b) The $n$th term of another geometric sequence is $3 \times 2^{n-1}$.
(i) Show that the first term of this sequence is 3 .
(ii) Complete the first four terms of this sequence.
(c) The $n$th term of a different geometric sequence is $36 \times 0.5^{n-1}$.

Work out the first four terms of this sequence.
(d) These are the first four terms of another sequence.
$\begin{array}{llll}5 & 15 & 45 & 135\end{array}$
Find the $n$th term.

2 The first two terms of a geometric sequence are 16 and 12.
Show that the 3 rd term is 9 .
[2]

3 (a) A geometric sequence has:
2 nd term $=12.5$
3 rd term $=62.5$.
(i) Work out the 1st term.
(ii) Find the $n$th term.
(b) Another geometric sequence has:

2 nd term $=24$
5 th term $=81$.
Work out the 6th term.

4 These are the first three terms of a geometric sequence.

$$
x \quad 2 x+3 \quad 11 x-6
$$

(a) (i) Give a reason why $\frac{2 x+3}{x}=\frac{11 x-6}{2 x+3}$.
$\qquad$
$\qquad$
(ii) Show that $7 x^{2}-18 x-9=0$.
(b) Solve $7 x^{2}-18 x-9=0$ by factorisation.

Give your answer as an integer and a fraction.

$$
x=.
$$

$$
x=
$$

(c) There are two different geometric sequences where the first three terms are $x, 2 x+3,11 x-6$. Use your answer to part (b) to find the fourth term of each of these sequences.

## B MODELLING (QUESTIONS 5 TO 7)

## FILLING CONTAINERS (30 marks)

You are advised to spend no more than 50 minutes on this part.
This task looks at models for filling containers with water.
In this task all containers are empty when the water starts to flow into them.
5


A container is a cuboid 20 cm long, 20 cm wide and 25 cm high.
(a) Calculate the volume of the container.
(b) Water flows into the container at a constant rate of $50 \mathrm{~cm}^{3}$ per second.
(i) Show that it takes 8 seconds to fill the container to a height of 1 cm .
(ii) How many seconds does it take to completely fill the container?
(c) (i) Explain why the model for the time to fill the container is

$$
t=8 h
$$

where $t$ seconds is the time taken and $h \mathrm{~cm}$ is the height of water.
$\qquad$
$\qquad$
(ii) Work out the height of the water in the container after 1 minute.
(iii) Sketch the model for the time to fill the container.


$$
\text { Volume, } V \text {, of a cone of radius } r \text {, height } h . \quad V=\frac{1}{3} \pi r^{2} h
$$



NOT TO
SCALE

A container is in the shape of a cone.
The circular top has a radius of 16 cm and the container has a height of 24 cm .
(a) Calculate the volume of the container.

Give your answer correct to 3 significant figures.
(b)


NOT TO
SCALE

Water is poured into the container to a height of 3 cm . The surface of the water is a circle with radius $x \mathrm{~cm}$.
(i) Show that $x=2$.
(ii) Work out the volume of water in the container.
(iii) Water flows into the empty container at a constant rate of $20 \mathrm{~cm}^{3}$ per second.

Work out the time it takes to fill the container to a height of 3 cm .
(c)


NOT TO
SCALE

The container is emptied.
Water is now poured into the container to a height of $h \mathrm{~cm}$.
The surface of the water is a circle with radius $r \mathrm{~cm}$.
(i) Water flows into the container at a constant rate of $20 \mathrm{~cm}^{3}$ per second. The time taken to fill the container to a height of $h \mathrm{~cm}$ is $t$ seconds.

Show that the model for the time to fill the container is $t=\frac{\pi h^{3}}{135}$.
(ii) How many seconds does it take to completely fill the container?
(iii) Sketch the model for the time to fill the container.

(d) The container is emptied.

Water is now poured into the container at a different rate.
The model for the time to fill the container is now $t=\frac{\pi h^{3}}{189}$.
Find the rate of flow for this model.

7 The containers in Question 5 and Question 6 are both empty
Water flows into the container in Question 5 at a rate of $50 \mathrm{~cm}^{3}$ per second. At the same time water flows into the container in Question 6 at a rate of $20 \mathrm{~cm}^{3}$ per second.
(a) Find the height of water when it is the same in both containers.
(b) Find the time it takes for the water to reach this height.

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