

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Ordinary Level

**ADDITIONAL MATHEMATICS**

**4037/02**

Paper 2

October/November 2004

**2 hours**

Additional Materials: Answer Booklet/Paper  
Graph paper  
Mathematical tables

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.  
Write your answers on the separate Answer Booklet/Paper provided.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 80.  
The use of an electronic calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

*Mathematical Formulae***1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

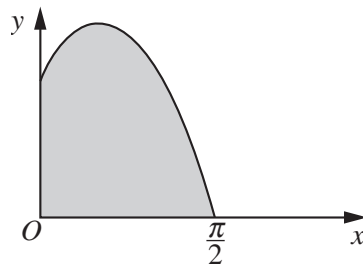
$$\Delta = \frac{1}{2} bc \sin A.$$

- 1 Given that  $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}$ , find  $\mathbf{A}^{-1}$  and **hence** solve the simultaneous equations

$$\begin{aligned} 2x + 3y + 4 &= 0 \\ -5x + 4y + 13 &= 0. \end{aligned}$$

- 2 Given that  $\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$ , where  $a$  and  $b$  are integers, find, without using a calculator, the value of  $a$  and of  $b$ . [4]

- 3 The diagram shows part of the curve  $y = 3\sin 2x + 4\cos x$ .



Find the area of the shaded region, bounded by the curve and the coordinate axes. [5]

- 4 Find the values of  $k$  for which the line  $y = x + 2$  meets the curve  $y^2 + (x + k)^2 = 2$ . [5]

- 5 Solve the equation  $\log_{16}(3x - 1) = \log_4(3x) + \log_4(0.5)$ . [6]

- 6 Given that  $x = 3\sin\theta - 2\cos\theta$  and  $y = 3\cos\theta + 2\sin\theta$ ,

(i) find the value of the acute angle  $\theta$  for which  $x = y$ , [3]

(ii) show that  $x^2 + y^2$  is constant for all values of  $\theta$ . [3]

- 7 Given that  $6x^3 + 5ax - 12a$  leaves a remainder of  $-4$  when divided by  $x - a$ , find the possible values of  $a$ . [7]

- 8 A motor boat travels in a straight line across a river which flows at  $3 \text{ ms}^{-1}$  between straight parallel banks 200 m apart. The motor boat, which has a top speed of  $6 \text{ ms}^{-1}$  in still water, travels directly from a point  $A$  on one bank to a point  $B$ , 150 m downstream of  $A$ , on the opposite bank. Assuming that the motor boat is travelling at top speed, find, to the nearest second, the time it takes to travel from  $A$  to  $B$ . [7]

9 In order that each of the equations

(i)  $y = ab^x$ ,

(ii)  $y = Ax^k$ ,

(iii)  $px + qy = xy$ ,

where  $a, b, A, k, p$  and  $q$  are unknown constants, may be represented by a straight line, they each need to be expressed in the form  $Y = mX + c$ , where  $X$  and  $Y$  are each functions of  $x$  and/or  $y$ , and  $m$  and  $c$  are constants. Copy the following table and insert in it an expression for  $Y, X, m$  and  $c$  for each case.

	$Y$	$X$	$m$	$c$
$y = ab^x$				
$y = Ax^k$				
$px + qy = xy$				

[7]

10 The function  $f$  is defined by  $f: x \mapsto |x^2 - 8x + 7|$  for the domain  $3 \leq x \leq 8$ .

(i) By first considering the stationary value of the function  $x \mapsto x^2 - 8x + 7$ , show that the graph of  $y = f(x)$  has a stationary point at  $x = 4$  and determine the nature of this stationary point. [4]

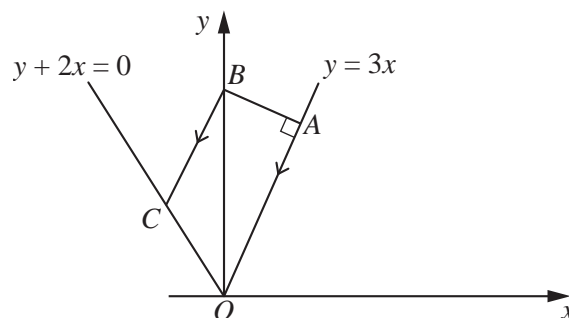
(ii) Sketch the graph of  $y = f(x)$ . [2]

(iii) Find the range of  $f$ . [2]

The function  $g$  is defined by  $g: x \mapsto |x^2 - 8x + 7|$  for the domain  $3 \leq x \leq k$ .

(iv) Determine the largest value of  $k$  for which  $g^{-1}$  exists. [1]

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The diagram shows a trapezium  $OABC$ , where  $O$  is the origin. The equation of  $OA$  is  $y = 3x$  and the equation of  $OC$  is  $y + 2x = 0$ . The line through  $A$  perpendicular to  $OA$  meets the  $y$ -axis at  $B$  and  $BC$  is parallel to  $AO$ . Given that the length of  $OA$  is  $\sqrt{250}$  units, calculate the coordinates of  $A$ , of  $B$  and of  $C$ .

[10]

12 Answer only **one** of the following two alternatives.

**EITHER**

A particle, travelling in a straight line, passes a fixed point  $O$  on the line with a speed of  $0.5 \text{ ms}^{-1}$ . The acceleration,  $a \text{ ms}^{-2}$ , of the particle,  $t$  s after passing  $O$ , is given by  $a = 1.4 - 0.6t$ .

- (i) Show that the particle comes instantaneously to rest when  $t = 5$ . [4]
- (ii) Find the total distance travelled by the particle between  $t = 0$  and  $t = 10$ . [6]

**OR**

Each member of a set of curves has an equation of the form  $y = ax + \frac{b}{x^2}$ , where  $a$  and  $b$  are integers.

- (i) For the curve where  $a = 3$  and  $b = 2$ , find the area bounded by the curve, the  $x$ -axis and the lines  $x = 2$  and  $x = 4$ . [4]

Another curve of this set has a stationary point at  $(2, 3)$ .

- (ii) Find the value of  $a$  and of  $b$  in this case and determine the nature of the stationary point. [6]





