

MARK SCHEME for the October/November 2006 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \checkmark implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts.

- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
- OW -1,2 This is deducted from A or B marks when essential working is omitted.
- PA -1 This is deducted from A or B marks in the case of premature approximation.
- S -1 Occasionally used for persistent slackness – usually discussed at a meeting.
- EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

<p>1 (i) $x \notin A$, (ii) $n(B') = 16$ (iii) $C \cap D = \phi$ or $n(C \cap D) = 0$ (any other correct notations accepted) Nb $C \cap D = 0$ in (iii) gets B0 etc</p>	<p>B1 B1 B1 [3]</p>	<p>co co co</p>
<p>2 (i) $a = 2$ (ii) $b = 3$ (iii) $c = -1$</p>	<p>B1 B1 B1 [3]</p>	<p>co co co</p>
<p>3 $y = \frac{8}{(3x-4)^2}$ (i) $dy/dx = -16(3x-4)^{-3} \times 3$ (or by quotient rule.) → -6 (ii) $\delta y = dy/dx \times \delta x$ → -6p</p>	<p>B1 M1 A1 [3] M1 A1√ [2]</p>	<p>B1 for expression without the "x3" M1 Must appreciate "fn of a fn". co . For multiplying his ans to "i" by "p" $\Delta x = 2+p$ gets M0</p>
<p>4 (i) Modulus of $(3i - 4j)$ or $(4i + 3j) = 5$ $\overline{OP} = (3i - 4j) \times (10 \div 5) = 6i - 8j$ $\overline{OQ} = (4i + 3j) \times (15 \div 5) = 12i + 9j$ (ii) $\overline{PQ} = 12i + 9j - (6i - 8j) = 6i + 17j$ Magnitude = $\sqrt{6^2 + 17^2} = \sqrt{325} = 5\sqrt{13}$ $\lambda = 5$</p>	<p>B1 M1 A1 [3] M1 M1 A1 [3]</p>	<p>Anywhere. Mult. by 10 (or 15) ÷ modulus – once. Both correct. q-p or p-q Allow if p+q used. Allow if p-q used.</p>

<p>5(i)</p> $(5 \ 8 \ 4 \ 10) \begin{pmatrix} 300 & 60 & 40 \\ 150 & 50 & 20 \\ 120 & 40 & 0 \\ 100 & 0 & 0 \end{pmatrix}$ <p>(ii) (4180 860 360)</p> <p>(iii) $\begin{pmatrix} .05 \\ .10 \\ .20 \end{pmatrix}$</p> <p>(iv) 367</p>	<p>B1 B1</p> <p>M1 A1</p> <p>B1</p> <p>B1</p> <p>[6]</p>	<p>These two B marks are for a correct 3x4 or 4x3, and for 1x4 or 4x1, even if the two given are not compatible.</p> <p>The two must be compatible and written in the correct order. The resulting matrix must be correct to his two matrices. Allow if in part (i).</p> <p>Must be a row matrix if (ii) is column matrix and vice versa.</p> <p>co – even if arithmetic has been used.</p>
<p>6 $\left(2 - \frac{x}{2}\right)^6$</p> <p>Coefficient of x is $2^5 \left(\frac{-x}{2}\right) 6C1 = -96$</p> <p>Coefficient of x^2 is $2^4 \left(\frac{-x}{2}\right)^2 6C2 = 60$</p> <p>$(k+x)(60x^2 - 96x) \rightarrow 60k - 96 = 84$</p> <p>$\rightarrow k = 3$</p>	<p>M1 A1</p> <p>M1A1</p> <p>M1</p> <p>A1√</p> <p>[6]</p>	<p>Unsimplified with 6C1. co.</p> <p>Unsimplified with 6C2. co.</p> <p>Must be considering 2 terms.</p> <p>For his incorrect coefficients.</p>
<p>7 $f(x) = 9\left(x - \frac{1}{3}\right)^2 - 11$.</p> <p>Minimum at $x = \frac{1}{3}$</p> <p>(i) Range is -11 to 89.</p> <p>(ii) (a) $\left(\frac{1}{3}, -11\right)$ Minimum.</p> <p>(b) $\left(\frac{1}{3}, 11\right)$ Maximum</p>	<p>M1 A1</p> <p>B1 B1</p> <p>B1</p> <p>B1√B1√</p> <p>[7]</p>	<p>Correct method for x co-ord of min.pt.</p> <p>B1 for each value. ≥ 89 gets B0.</p> <p>For "Minimum" – ignore any working.</p> <p>Correct follow through from his coordinates and nature of stationary point.</p>
<p>8 (a) $\lg(x+12) = 1 + \lg(2-x)$</p> <p>$1 = \lg 10$</p> <p>$(x+12) = 10(2-x)$</p> <p>$\rightarrow x = \frac{8}{11}$</p> <p>(b) $\log_2 p = a \log_8 q = b$</p> <p>$p = 2^a$ and $q = 8^b$</p> <p>$2^c = \frac{2^a}{8^b} \rightarrow c = a - 3b$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Anywhere.</p> <p>Must be a product ie 1 expressed as log.</p> <p>co – or decimal equivalent.</p> <p>M1 for one correct power equation.</p> <p>A mark for both correct.</p> <p>Attempt at powers of 2 (or 8).</p> <p>co.</p>

<p>9 $y = \frac{2x-4}{x+3}$</p> <p>(i) $\frac{dy}{dx} = \frac{(x+3)2 - (2x-4)}{(x+3)^2} = \frac{10}{(x+3)^2}$</p> <p>Numerator $\neq 0$ for any value of x \rightarrow No turning points.</p> <p>(ii) P(2,0) At $x=2$, $m = \frac{2}{5}$ Eqn of tangent $y-0 = \frac{2}{5}(x-2)$ At $x=0$, $y = -\frac{4}{5}$ Q $(0, -\frac{4}{5})$ \rightarrow Area = $\frac{1}{2} \times 2 \times \frac{4}{5} = \frac{4}{5}$</p>	<p>M1 A1</p> <p>B1√ [3]</p> <p>B1</p> <p>M1 M1</p> <p>M1 A1 [5]</p>	<p>Use of correct formula. Numerical value for numerator. Product rule ok.</p> <p>Allow if constant numerator has been obtained for dy/dx.</p> <p>co.</p> <p>Must be numerical tangent, not normal. Correct form of line, even if normal.</p> <p>Use of $\frac{1}{2}bh$ or equivalent. co.</p>
<p>10 (i) $f(x) = (x-1)(x-k)(x-k^2)$ $f(2) = (2-k)(2-k^2)$</p> <p>$\rightarrow k^3 - 2k^2 - 2k - 3 = 0$</p> <p>(ii) Try numbers $\rightarrow k=3$ fits Divide by $(k-3) \rightarrow k^2 + k + 1$ Use of b^2-4ac or full formula Arrives at $\sqrt{\text{negative number } (-3)}$ \rightarrow No real solutions.</p>	<p>M1 M1</p> <p>A1 ag [3]</p> <p>B1 M1A1</p> <p>M1 A1 [5]</p>	<p>Forming cubic correctly Subbing in $x=2$</p> <p>co (answer given)</p> <p>First solution. Divides by x-“his value”. co.</p> <p>Full formula ok. Correct deduction – needs -3.</p>
<p>11 (a) $\cot x = \frac{1}{\tan x}$ $\rightarrow \tan^2 x + \tan x - 2 = 0$</p> <p>$\tan x = -2 \rightarrow x = 116.6^\circ$ or 296.6°</p> <p>$\tan x = 1 \rightarrow x = 45^\circ$ or 225°</p> <p>(b) $\sin(2y+1) = -\frac{5}{6}$ Base angle in radians = 0.985 $2y+1 = \pi + 0.985$ $y = 1.56$ or $2y+1 = 2\pi - 0.985$ $y = 2.15$</p> <p>Extra values in range, loses last A1 Extra values outside range – no penalty.</p>	<p>B1 M1</p> <p>A1 B1√ A1 [5]</p> <p>M1</p> <p>M1 A1 M1 A1 [5]</p>	<p>Used somewhere. Forming and solving quadratic.</p> <p>One value correct. For the two second values. One value correct.</p> <p>Making $\sin(2y+1)$ subject</p> <p>Realising $2y+1 = \pi +$ Realising that $2y+1 = 2\pi -$</p>

<p>12 EITHER</p> <p>(i) At A $y = 0$ $x = -\ln 2$ or -0.693 At B $x = 0$ $y = 3$</p> <p>(ii) $dy/dx = 2e^{-2x}$ At $x = 0$, $m = 2$ Gradient of normal = $-\frac{1}{2}$ Eqn of normal $y - 3 = -\frac{1}{2}x$ At C, $y = 0$ $x = 6$.</p> <p>(iii) $\int 4 - e^{-2x} dx = 4x + \frac{1}{2}e^{-2x}$ Area to left of y-axis = [] from $-\ln 2$ to 0 $= \frac{1}{2} - (-4\ln 2 + \frac{1}{2} \cdot 4) = 4\ln 2 - 1\frac{1}{2} = (1.27)$</p> <p>Area of triangle BOC = $\frac{1}{2} \times 3 \times 6 = 9$</p> <p>Shaded area = $4\ln 2 + 7\frac{1}{2} = 10.3$</p>	<p>B1 B1 [2]</p> <p>B1</p> <p>M1 M1 A1 [4]</p> <p>B1 B1</p> <p>M1</p> <p>M1</p> <p>A1 ag [5]</p>	<p>co. co.</p> <p>Anywhere.</p> <p>Use of m_1, m_2 with dy/dx. m numeric. For equation of line (even if tangent) co.</p> <p>For each term.</p> <p>Limits used correctly in an integral.</p> <p>Use of $\frac{1}{2}bh$ or integration under line.</p> <p>co – answer was given.</p>														
<p>12 OR</p> <p>(i)</p> <table border="1" data-bbox="327 963 726 1030"> <tr> <td>x</td> <td>15</td> <td>20</td> <td>25</td> <td>30</td> </tr> <tr> <td>lgy</td> <td>-0.82</td> <td>-0.42</td> <td>-0.02</td> <td>0.37</td> </tr> </table> <table border="1" data-bbox="327 1052 502 1120"> <tr> <td>35</td> <td>40</td> </tr> <tr> <td>0.77</td> <td>1.17</td> </tr> </table> <p>Knows what to do. Straight line.</p> <p>(ii) $A = 2$ (± 0.05) $m = lgb = 0.079 \rightarrow b = 1.18$ to 1.22</p> <p>(iii) $y = 10 \rightarrow lgy = 1$ "1" on lgy axis. $x = 37.5$ to 38.5</p> <p>(iv) $y^5 = 10^{-x} \rightarrow lgy = -\frac{x}{5}$ Line drawn. $\rightarrow x = 6.5$ to 7.5</p>	x	15	20	25	30	lgy	-0.82	-0.42	-0.02	0.37	35	40	0.77	1.17	<p>M1 A1 [2]</p> <p>M1 A1</p> <p>M1 A1 [4]</p> <p>M1 A1 [2]</p> <p>B1 M1</p> <p>A1 [3]</p>	<p>Must use values of lgy on one axis, values of x on other axis. Mark by "eye" – points are in line.</p> <p>Knows "c" = -A. co. (may need to interpolate) Knows that $m = lgb$ (statement only)</p> <p>Must realise that $lgy = 1$, not $y = 1$. co.</p> <p>For correctly converting to logs. Must make "lgy" the subject.</p> <p>co.</p>
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<p>DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.</p> <p><u>Formula.</u> Must be correct – ignore arithmetic and algebraic slips.</p> <p><u>Factors</u> Must attempt to put quadratic into 2 factors. Each factor then equated to 0.</p>																