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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01**

Paper 1 **October/November 2008**

2 hours

Additional Materials: Answer Booklet/Paper
Mathematical tables

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

SPA (SLM/DT) T44100/5

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\text{cosec}^2 A = 1 + \cot^2 A.$$

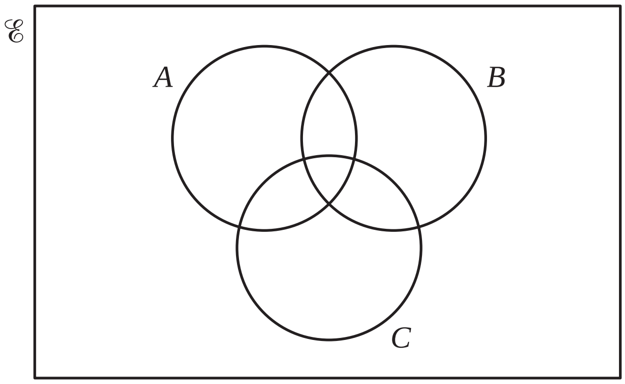
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\ .$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1



(i) Copy the Venn diagram above and shade the region that represents $A \cup (B \cap C)$. [1]

(ii) Copy the Venn diagram above and shade the region that represents $A \cap (B \cup C)$. [1]

(iii) Copy the Venn diagram above and shade the region that represents $(A \cup B \cup C)'$. [1]

2 Find the set of values of x for which $(2x + 1)^2 > 8x + 9$. [4]

3 Prove that $\dfrac{\sin A}{1 + \cos A} + \dfrac{1 + \cos A}{\sin A} \equiv 2\operatorname{cosec} A$. [4]

4 A function f is such that $f(x) = ax^3 + bx^2 + 3x + 4$. When $f(x)$ is divided by $x - 1$, the remainder is 3. When $f(x)$ is divided by $2x + 1$, the remainder is 6. Find the value of a and of b. [5]

5 Given that $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ and that $\mathbf{b} = p\mathbf{i} + \mathbf{j}$, find

(i) the unit vector in the direction of $\mathbf{a}$, [2]

(ii) the values of the constants p and q such that $q\mathbf{a} + \mathbf{b} = 19\mathbf{i} - 23\mathbf{j}$. [3]

6 **(i)** Solve the equation $2t = 9 + \dfrac{5}{t}$. [3]

(ii) Hence, or otherwise, solve the equation $2x^{\frac{1}{2}} = 9 + 5x^{-\frac{1}{2}}$. [3]

7 **(i)** Express $4x^2 - 12x + 3$ in the form $(ax + b)^2 + c$, where a, b and c are constants and $a > 0$. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve $y = 4x^2 - 12x + 3$. [2]

(iii) Given that $f(x) = 4x^2 - 12x + 3$, write down the range of f. [1]

8 A curve is such that $\frac{d^2y}{dx^2} = 4e^{-2x}$. Given that $\frac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-4})$, find the equation of the curve.

9 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(2 - 3x)^5$. [3]

The first 3 terms in the expansion of $(a + bx)(2 - 3x)^5$ in ascending powers of x are $64 - 192x + cx^2$.

(ii) Find the value of a, of b and of c. [5]

10 **(a)** Functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = 3 - x,$$
$$g(x) = \frac{x}{x+2}, \quad \text{where } x \neq -2.$$

(i) Find $fg(x)$. [2]

(ii) Hence find the value of x for which $fg(x) = 10$. [2]

(b) A function h is defined, for $x \in \mathbb{R}$, by $h(x) = 4 + \ln x$, where $x > 1$.

(i) Find the range of h. [1]

(ii) Find the value of $h^{-1}(9)$. [2]

(iii) On the same axes, sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [3]

11 Solve the equation

(i) $\tan 2x - 3\cot 2x = 0$, for $0° < x < 180°$, [4]

(ii) $\operatorname{cosec} y = 1 - 2\cot^2 y$, for $0° \leqslant y \leqslant 360°$, [5]

(iii) $\sec(z + \frac{\pi}{2}) = -2$, for $0 < z < \pi$ radians. [3]

12 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = \frac{x^2}{x+1}$.

(i) Find the coordinates of the stationary points of the curve. [5]

The normal to the curve at the point where $x = 1$ meets the x-axis at M. The tangent to the curve at the point where $x = -2$ meets the y-axis at N.

(ii) Find the area of the triangle MNO, where O is the origin. [6]

OR

A curve has equation $y = e^{x-2} - 2x + 6$.

(i) Find the coordinates of the stationary point of the curve and determine the nature of the stationary point. [6]

The area of the region enclosed by the curve, the positive x-axis, the positive y-axis and the line $x = 3$ is $k + e - e^{-2}$.

(ii) Find the value of k. [5]

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