

# **Cambridge International Examinations**

Cambridge Ordinary Level

| CANDIDATE<br>NAME |  |  |                     |  |  |
|-------------------|--|--|---------------------|--|--|
| CENTRE<br>NUMBER  |  |  | CANDIDATE<br>NUMBER |  |  |

#### **ADDITIONAL MATHEMATICS**

4037/11

Paper 1 May/June 2015

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

#### READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

#### Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.



This document consists of 15 printed pages and 1 blank page.

Turn over

# Mathematical Formulae

## 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ 

## 2. TRIGONOMETRY

*Identities* 

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (i) State the period of  $\sin 2x$ .

[1]

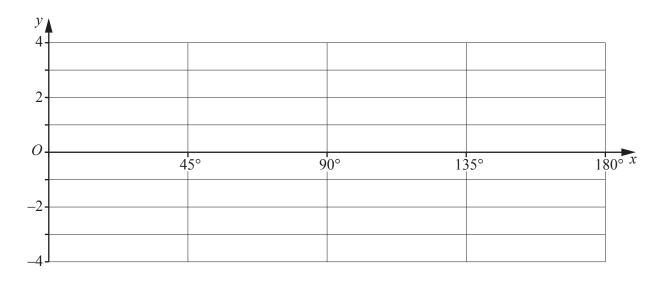
(ii) State the amplitude of  $1 + 2\cos 3x$ .

[1]

(iii) On the axes below, sketch the graph of

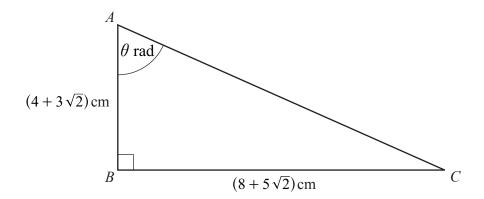
(a) 
$$y = \sin 2x$$
 for  $0^{\circ} \le x \le 180^{\circ}$ , [1]

**(b)** 
$$y = 1 + 2\cos 3x$$
 for  $0^{\circ} \le x \le 180^{\circ}$ . [2]



(iv) State the number of solutions of  $\sin 2x - 2\cos 3x = 1$  for  $0^{\circ} \le x \le 180^{\circ}$ . [1]

# 2 Do not use a calculator in this question.



The diagram shows the triangle *ABC* where angle *B* is a right angle,  $AB = (4 + 3\sqrt{2})$  cm,  $BC = (8 + 5\sqrt{2})$  cm and angle  $BAC = \theta$  radians. Showing all your working, find

(i)  $\tan \theta$  in the form  $a + b\sqrt{2}$ , where a and b are integers, [2]

(ii)  $\sec^2 \theta$  in the form  $c + d\sqrt{2}$ , where c and d are integers. [3]

| 3 | (i) | Find the first 4 terms in the expansion of | $(2+x^2)^6$ | in ascending powers of x. | [3] |
|---|-----|--|-------------|---------------------------|-----|
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(ii) Find the term independent of x in the expansion of 
$$(2+x^2)^6 \left(1-\frac{3}{x^2}\right)^2$$
. [3]

4 (a) Given that the matrix  $\mathbf{X} = \begin{pmatrix} 2 & -4 \\ k & 0 \end{pmatrix}$ , find  $\mathbf{X}^2$  in terms of the constant k. [2]

**(b)** Given that the matrix  $\mathbf{A} = \begin{bmatrix} a & 1 \\ b & 5 \end{bmatrix}$  and the matrix  $\mathbf{A}^{-1} = \begin{bmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$ , find the value of each of the integers a and b.

5 The curve  $y = xy + x^2 - 4$  intersects the line y = 3x - 1 at the points A and B. Find the equation of the perpendicular bisector of the line AB. [8]

| 6 | The polynomial   | $f(x) = ax^3 - 15x^2 + bx - 2$ | has a factor of | 2x - 1 | and a remainder of 5 when |
|---|------------------|--------------------------------|-----------------|--------|---------------------------|
|   | divided by $x -$ | 1.                             |                 |        |                           |

(i) Show that b = 8 and find the value of a. [4]

(ii) Using the values of a and b from part (i), express f(x) in the form (2x - 1) g(x), where g(x) is a quadratic factor to be found. [2]

(iii) Show that the equation f(x) = 0 has only one real root. [2]

| 7 | The point $A$ , where $x = 0$ , lies on the curve $x$ -axis at the point $B$ . | $y = \frac{\ln(4x^2 + 3)}{x - 1}$ | . The normal to | the curve at A meets th |
|---|--|-----------------------------------|-----------------|-------------------------|
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(i) Find the equation of this normal.

[7]

(ii) Find the area of the triangle AOB, where O is the origin.

[2]

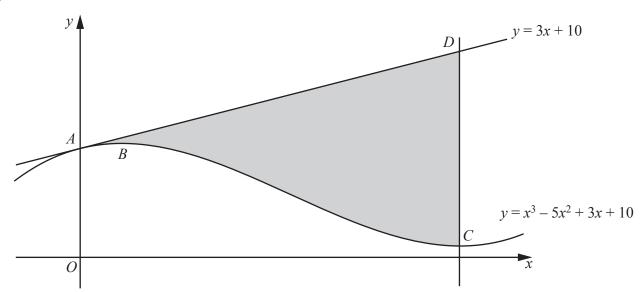
- 8 It is given that  $f(x) = 3e^{2x} \quad \text{for } x \ge 0,$  $g(x) = (x+2)^2 + 5 \quad \text{for } x \ge 0.$ 
  - (i) Write down the range of f and of g. [2]

(ii) Find  $g^{-1}$ , stating its domain. [3]

(iii) Find the exact solution of gf(x) = 41. [4]

(iv) Evaluate  $f'(\ln 4)$ . [2]

9



The diagram shows parts of the line y = 3x + 10 and the curve  $y = x^3 - 5x^2 + 3x + 10$ . The line and the curve both pass through the point A on the y-axis. The curve has a maximum at the point B and a minimum at the point C. The line through C, parallel to the y-axis, intersects the line y = 3x + 10 at the point D.

(i) Show that the line AD is a tangent to the curve at A. [2]

(ii) Find the x-coordinate of B and of C. [3]

| (iii) | Find the area of the shaded region ABCD, showing all your working. | [5] |
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**10** (a) Solve  $4\sin x = \csc x$  for  $0^{\circ} \le x \le 360^{\circ}$ . [3]

**(b)** Solve 
$$\tan^2 3y - 2 \sec 3y - 2 = 0$$
 for  $0^\circ \le y \le 180^\circ$ . [6]

(c) Solve  $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$  for  $0 \le z \le 2\pi$  radians. [3]

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