Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME							
CENTRE NUMBER						CANDIDATE NUMBER	

4936059356

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2024**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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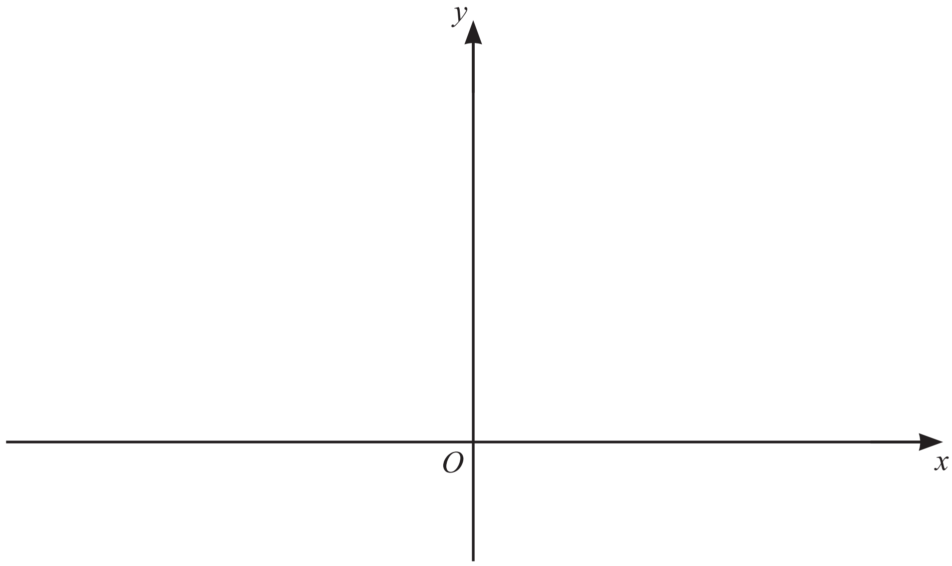
1 **(a)** On the axes, sketch the graph of $y = |4x - 6|$, showing the points where the graph meets the axes. [2]



(b) Solve the equation $|4x - 6| = |2x|$. [3]

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[Turn over

2 **(a)** Write $3+4x-2x^2$ in the form $a+b(x+c)^2$, where a, b and c are integers. [3]

(b) Hence write down the range of the function $f(x) = 3+4x-2x^2$, where $x \in \mathbb{R}$. [1]

3 Use algebra to show that the equation $5x(x-3) = 5x-26$ has no real solutions. [3]

4 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the exact distance between the two points where the curve $9(x-1)^2+4(y-3)^2=36$ cuts the y-axis. [4]

(b) Find the coordinates of the points where the curve with equation $2x^2+83xy=x^3y-20x$ intersects the curve with equation $y=\frac{1}{x}$. Give each of your answers in the form $a+b\sqrt{c}$, where a and b are rational and c is the smallest integer possible. [6]

5 There are 3 women, 2 men and 4 children in a choir.

(a) The choir stands in a single straight line.

(i) Find the number of possible arrangements if the first person and last person are both women. [2]

(ii) Find the number of possible arrangements if all the children stand next to each other. [2]

(b) Four of the choir are selected to sing in a group.

(i) Find the number of different selections if no man is chosen. [2]

(ii) Find the number of different selections if at least 2 women are chosen. [2]

6 Variables x and y are such that $y = \cos x \sin^2 x$. Use differentiation to find the approximate change in y as x increases from 3 to $3 + h$, where h is small. [5]

7 It is given that $y = mx^2 + \frac{x}{2} + n$, where m and n are non-zero constants. It is also given that $3\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - y$ for all values of x. Find the values of m and n. [4]

8 **(a)** In an arithmetic progression, the sum of the first 30 terms is -1065.
The sum of the **next** 20 terms is -2210.
Find the first term and the common difference. [5]

(b) A geometric progression is such that the first term is 4 and the sum of the first three terms is 7.

Find the two possible values of the common ratio and find the sum to infinity for the convergent progression. [5]

9 The functions f and g are defined by

$$\mathrm{f}(x) = \frac{3x^2}{4x-1} \quad \text{for } x < 0$$
$$\mathrm{g}(x) = \frac{1}{x^2} \quad \text{for } x < 0.$$

(a) Explain why the function fg does **not** exist. [1]

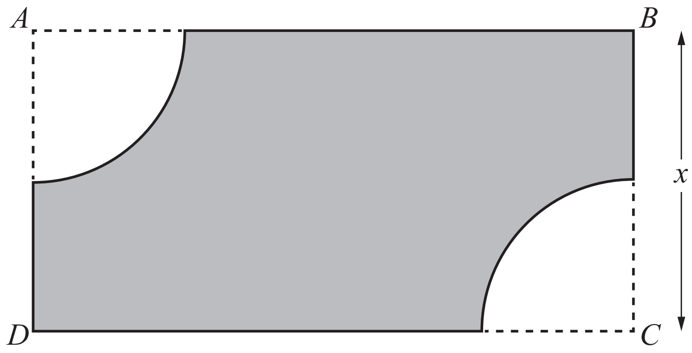
(b) Given that the function gf does exist, find and simplify an expression for $\mathrm{gf}(x)$. [2]

(c) Show that $\mathrm{f}^{-1}(x)$ can be written as $\dfrac{px - \sqrt{x(qx+r)}}{3}$ where p, q and r are integers. [4]

10 **(a)** Show that $(\tan x + \sec x)^2$ can be written as $\dfrac{1+\sin x}{1-\sin x}$. [4]

(b) Hence solve the equation $(\tan 3\theta + \sec 3\theta)^2 = 6$ for $0° \leqslant \theta \leqslant 180°$. [4]

11 In this question all lengths are in centimetres.



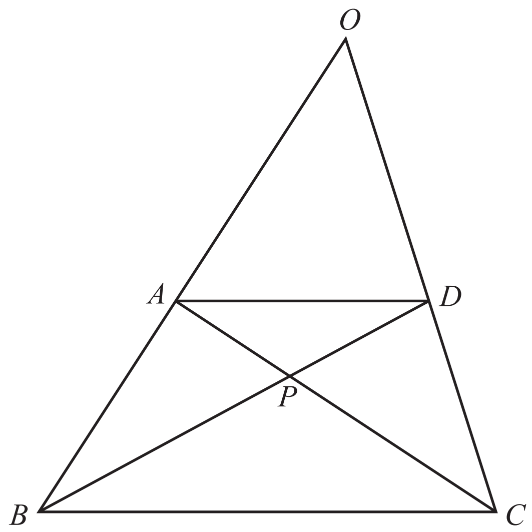
The diagram shows a rectangle $ABCD$ with $BC = x$.
The area of the rectangle is $400\,\text{cm}^2$.

Two identical quarter-circles of radius $\frac{x}{2}$, with centres A and C, are removed from the rectangle to make the shaded shape.

Given that x can vary, find the value of x that gives the minimum value of the perimeter of the shaded shape and hence find this minimum value. [7]

Continuation of working space for Question 11.

12



The diagram shows a triangle OBC.
$OA : OB = 4 : 7$ and $OD : OC = 4 : 7$.

$$\overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

The point P is the point of intersection of AC and BD such that $\overrightarrow{AP} = \lambda\overrightarrow{AC}$ and $\overrightarrow{BP} = \mu\overrightarrow{BD}$ where λ and μ are scalars.

(a) Find two expressions for $\overrightarrow{OP}$, each in terms of **b**, **c** and a scalar, and hence show that P divides both AC and DB in the ratio $4 : 7$. [7]

(b) The point Q is such that $\overrightarrow{OQ} = \frac{2}{7}\mathbf{b} + \frac{2}{7}\mathbf{c}$.

Use a vector method to show that O, Q and P are collinear. Justify your answer. [2]

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