Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 0 4 0 9 0 7 2 9 4 *

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2024**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

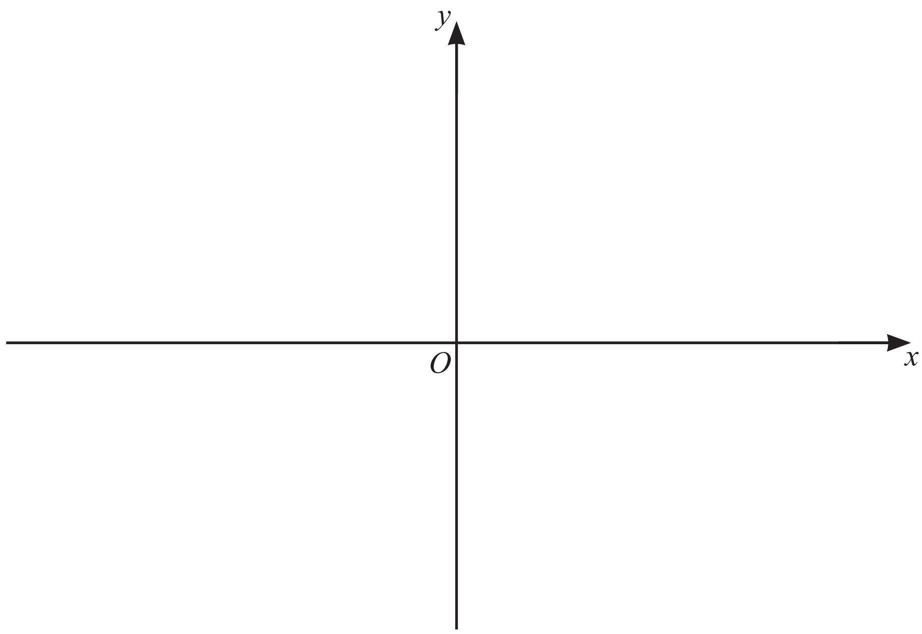
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

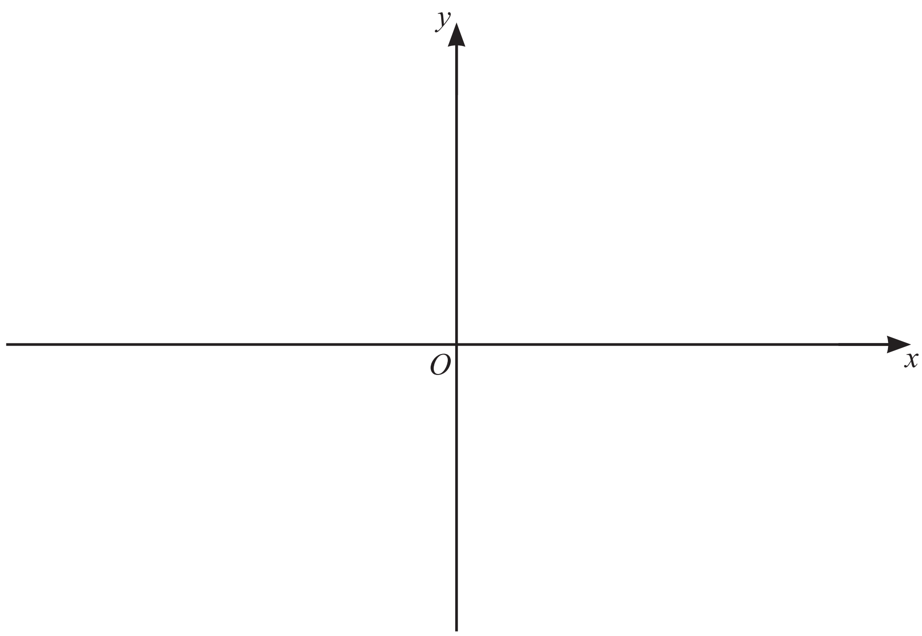
1 **(a)** On the axes, sketch the graph of $y = (2x - 5)(x + 3)(1 - x)$, stating the intercepts with the coordinate axes. [3]



(b) Hence

(i) solve the inequality $(2x - 5)(x + 3)(1 - x) \leqslant 0$ [2]

(ii) on the axes below, sketch the graph of $y = |(2x - 5)(x + 3)(1 - x)|$. [1]



2 **(a)** Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos\frac{x}{4}\,dx$. You must show all your working. [4]

(b) Find $\int\left(\frac{1}{4x-3}+\frac{1}{x^3}\right)dx$. [3]

3 **(a)** Determine whether the equation $\frac{(4x+1)(3x+2)}{5x-3} = x+1$ has two distinct real roots, two equal roots or no real roots. [4]

(b) Solve the equation $\frac{12}{\sqrt[3]{x}} - \sqrt[3]{x} = 4$. [4]

4 The polynomial p is such that $\mathrm{p}(x) = 6x^3 + x^2 - 12x + 5$.

(a) Find the remainder when $\mathrm{p}(x)$ is divided by $x - 2$. [1]

(b) **(i)** Show that $2x - 1$ is a factor of $\mathrm{p}(x)$. [1]

(ii) Hence write $\mathrm{p}(x)$ as a product of linear factors. [3]

(iii) Hence solve the equation $6\sin^3\theta + \sin^2\theta - 12\sin\theta + 5 = 0$ for $0° \leqslant \theta \leqslant 90°$. [2]

5 A curve has equation $y = 5e^{2x-1} + e$. The tangent to the curve at the point where $x = 1$ cuts the x-axis at the point P.

Find the equation of the tangent in the form $y = mx + c$, where m and c are exact values, and hence find the x-coordinate of P. [6]

6 **(a)** Show that $\sin^3 x\left(\dfrac{\operatorname{cosec} x}{\cot x}\right)$ can be written as $\sin^2 x \tan x$. [3]

(b) Solve the equation $\cos^2 x \tan x - \dfrac{1}{2}\tan x = 0$ for $-\pi < x < \pi$. [5]

7 Find the number of different ways the 9 letters of the word POLYMATHS can be arranged when

(a) the O and A are **not** next to each other [2]

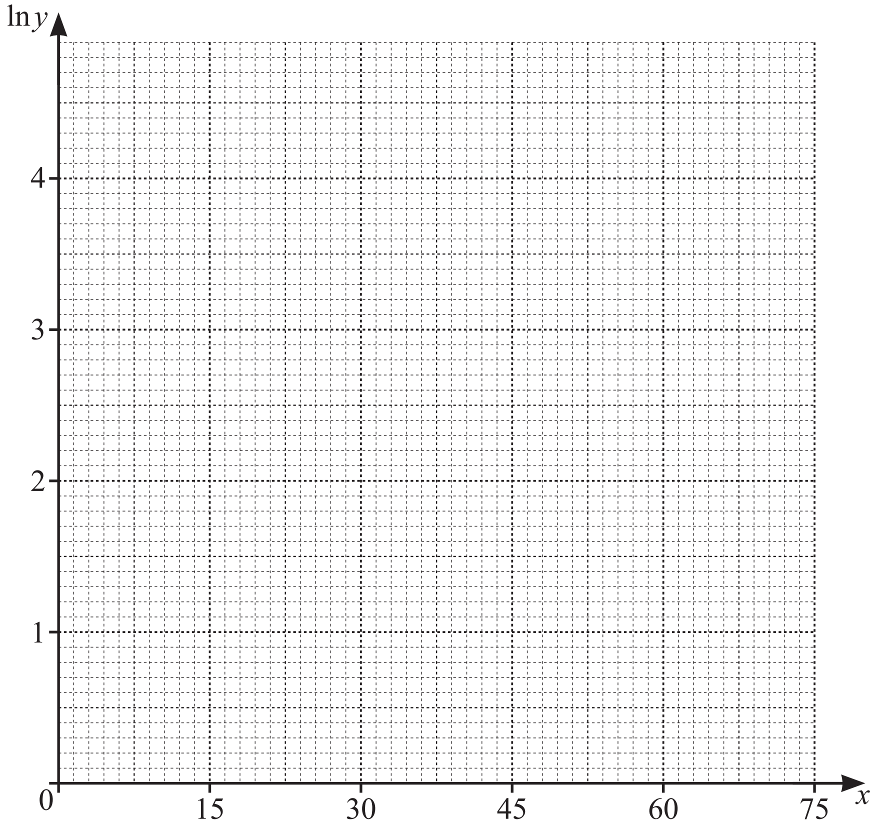
(b) the letters MATHS are together in this order. [2]

8 An experiment was carried out and values of y for certain values of x were recorded. The table shows the values recorded.

x	15	30	45	60	75
y	10	13	22	35	50

The relationship between y and x is modelled by $y = Ae^{kx}$, where A and k are constants.

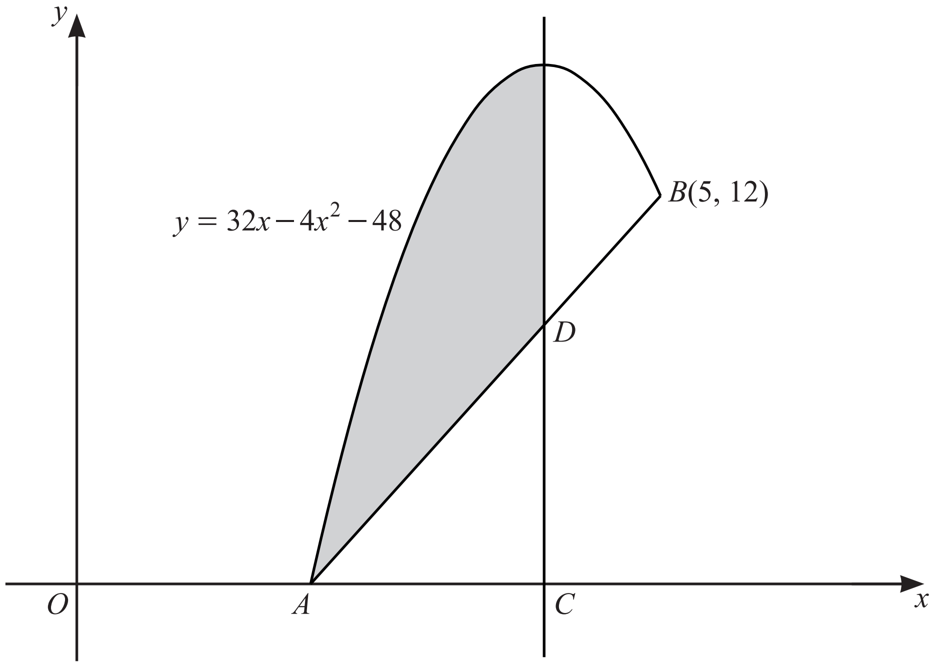
(a) Draw a straight line graph for $\ln y$ against x. [2]



(b) Find the equation of the line in **part (a)** and hence find the values of A and k. Give each value correct to 1 significant figure. [5]

(c) Find the value of x for which $y = 17$. [2]

9



The diagram shows part of the curve $y = 32x - 4x^2 - 48$ and the line AB.
The curve and the line AB meet the x-axis at A and meet again at the point $B(5, 12)$.
The line CD extended is parallel to the y-axis and passes through the maximum point of the curve.
Find the area of the shaded region. [9]

Continuation of working space for Question 9.

10 The functions f and fg are defined by

$$f(x) = e^{x^2+3} \quad \text{for } x < 0$$

$$fg(x) = e^{2x} \quad \text{for } x > \frac{3}{2}.$$

(a) Explain why f^{-1} exists. [1]

(b) Find an expression for $f^{-1}(x)$ and state the domain and range of f^{-1}. [5]

(c) Hence find and simplify an expression for $g(x)$. [2]

11 In the binomial expansion of $\left(2+\frac{x}{2}\right)^n$, the first three terms in increasing powers of x are $b+abx+\frac{9}{8}abx^2$. Find the values of the constants n, a and b. [8]

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