Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0245384637

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (LO) 214474

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

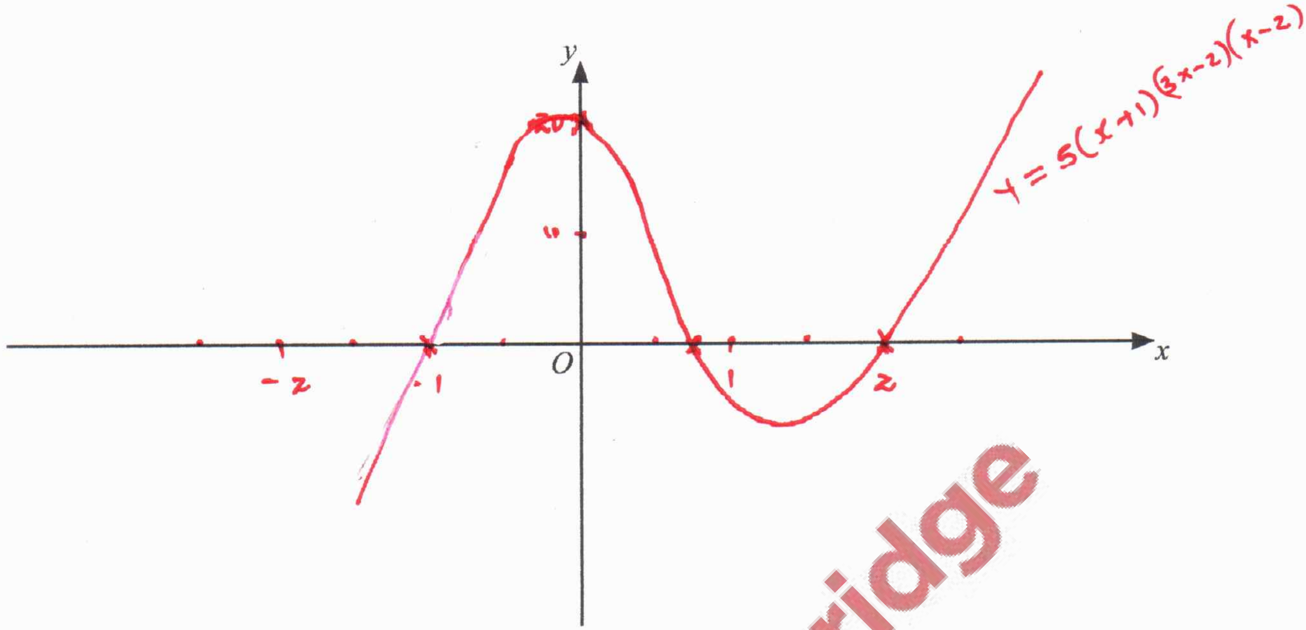
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

$x = -1$ $x = 2/3$ $x-2=0$ $x=2$ $= 20$

1 **(a)** On the axes, sketch the graph of $y = 5(x+1)(3x-2)(x-2)$, stating the intercepts with the coordinate axes. [3]



(b) Hence find the values of x for which $5(x+1)(3x-2)(x-2) > 0$. [2]

$-1 < x < 2/3$ and $x > 2$

2 Find $\int_3^5 \left(\frac{1}{x-1} - \frac{1}{(x-1)^2}\right) dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. [5]

Integrate; $\int_3^5 \frac{1}{x-1} - \int_3^5 \frac{1}{(x-1)^2}$

$\frac{1}{(x-1)^2} \rightarrow$ can be expressed as: $(x-1)^{-2}$

$[\ln|x-1|]_3^5 = \ln 4 - \ln 2$

$= \ln 4/2$

$= \ln 2$

$\int_3^5 (x-1)^{-2} dx = \left[\frac{(x-1)^{-1}}{-1}\right]_3^5$

$= -1\left[\frac{1}{4} - \frac{1}{2}\right]$

$= -1[-1/4] = 1/4$

$\ln 2 - 1/4 = -1/4 + \ln 2$

3 The polynomial $p(x) = ax^3 - 9x^2 + bx - 6$, where a and b are constants, has a factor of $x-2$. The polynomial has a remainder of 66 when divided by $x-3$.

(a) Find the value of a and of b. [4]

$x-2=0$ | $x-3=0$
$x=2$ | $x=3$

$P(3)=66$

$a(3^3)-9(3^2)+b(3)-6$

$27a-81+3b-6=66$

$27a+3b=153$ - (ii)

Solve the simultaneous equations (i) and (ii)

By substitution equation (i)

$b=21-4a$

When $x=2$ substitute $P(2)=a(2^3)-9(2^2)+2(b)-6$

$8a+2b-42=0$ Divide by 2

$\frac{8a}{2}+\frac{2b}{2}-\frac{42}{2}$

$4a+b=21$ - (i)

Using equation (ii) $b=21-4a$

$27a+3(21-4a)=153$

$27a+63-12a=153$

$\frac{15a}{15}=\frac{90}{15}$

$a=6$

$b=21-4a$

$21-4(6)$

$21-24=-3$

$a=6$

$b=-3$

(b) Using your values of a and b, show that $p(x) = (x-2)q(x)$, where $q(x)$ is a quadratic factor to be found. [2]

$P(x)=6x^3-9x^2-3x-6$

$$\begin{array}{r} 6x^2+3x+3 \\ x-2\,\big)\,6x^3-9x^2-3x-6 \\ 6x^3-12x^2 \\ \hline 3x^2-3x-6 \\ 3x^2-6x \\ \hline 3x-6 \\ 3x-6 \\ \hline 0 \end{array}$$

$P(x)=(x-2)(6x^2+3x+3)$

(c) Hence show that the equation $p(x) = 0$ has only one real solution. [2]

$P(x)=(x-2)(6x^2+3x+3)=0$

$x-2=0$ $\quad$ $6x^2+3x+3=0$

$x=2$ $\quad$ $b^2-4ac=9-4(6)(3)$

$=-63<0$

Since $b^2-4ac\le 0$, There are no real roots.

4 The first 3 terms in the expansion of $(a+x)^3\left(1-\frac{x}{3}\right)^5$, in ascending powers of x, can be written in the form $27+bx+cx^2$, where a, b and c are integers. Find the values of a, b and c. [8]

$3C_0(a)^3(x)^0 + 3C_1(a)^2(x)^1 + 3C_2(a)(x)^2 + 3C_3(a)^0(x)^3$

$1\cdot a^3\cdot 1 + 3\cdot a^2\cdot x + 3\cdot a\cdot x^2 + 1\cdot 1\cdot x^3$

$a^3 + 3a^2x + 3ax^2 + x^3.$

$5C_0(1)^5(-x/3)^0 + 5C_1(1)^4(-x/3)^1 + 5C_2(1)^3(-x/3)^2$

$1\cdot 1\cdot 1 + 5\cdot 1\cdot -x/3 + 10\cdot 1\cdot x^2/9$

$1 + -\frac{5}{3}x + \frac{10}{9}x^2$

$(a^3 + 3a^2x + 3ax^2 + x^3)\left(1 - \frac{5}{3}x + \frac{10}{9}x^2\right)$

$= a^3 - \frac{5}{3}a^3x + \frac{10}{9}a^3x^2 + 3a^2x - \frac{5}{3}(3a^2)x^2 + 3ax^2$

$= a^3 + \left(3a^2 - \frac{5}{3}a^3\right)x + \left(\frac{10}{9}a^3 - 5a^2 + 3a\right)x^2$

$27 + bx + cx^2$

(substitute $a=3$)

$a^3 = 27$

$a = \sqrt[3]{27}$

$a = \underline{\underline{3}}$

$b = 3a^2 - \frac{5}{3}a^3$

$= 3(3^2) - \frac{5}{3}(3^3)$

$= \underline{\underline{-18}}$

$c: \frac{10}{9}(3^3) - 5(3^2) + 3(3)$

$= \frac{270}{9} - 45 + 9$

$= \underline{\underline{-6}}$

Values: $a = 3$

$b = -18$

$c = \underline{\underline{-6}}$.

5 The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 1 + e^{2x} \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f. [2]

$f(x) = x^2 + 4x$

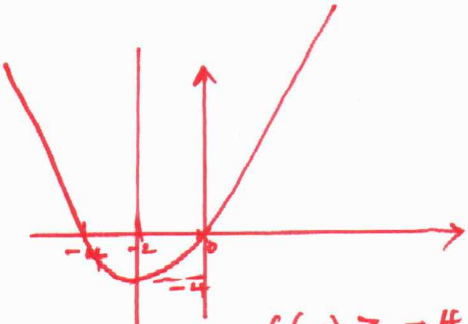
$x(x+4) = 0$

$x = 0 \mid x = -4$

$f(x) = (-2)^2 + 4(-2)$

$= 4 - 8$

$= -4$



range is $f(x) \geq -4$

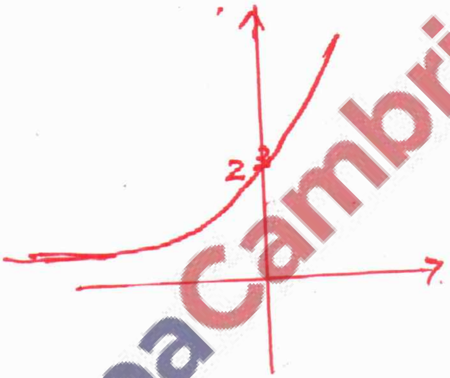
(b) Write down the range of g. [1]

$g(x) = 1 + e^{2x}$

$f(x) = 0$ $\quad g(x) = 1 + 1$

$= 2$

Any number raised to power of zero is usually 1.



$g(x) > 1$

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm. [4]

$f(g(x)) = [g(x)]^2 + 4[g(x)]$

$= [1 + e^{2x}]^2 + 4[1 + e^{2x}]$

$= 1 + 2e^{2x} + e^{4x} + 4 + 4e^{2x} = 21$

$e^{4x} + 6e^{2x} + 5 - 21 = 0$

$(e^{2x})^2 + 6e^{2x} - 16 = 0$

Let $e^{2x} = u$

$u^2 + 6u - 16 = 0$

$P = -16 \quad (8, -2)$

$S = 6$

$(u+8)(u-2) = 0$

$u + 8 = 0 \qquad u - 2 = 0$

$u = -8 \qquad u = 2$

Since $e^{2x} = -8$

$e^{2x} = 2$

$\frac{2x}{2} = \frac{\ln 2}{2}$

$x = \frac{1}{2} \ln 2$

$x = \ln\sqrt{2}$

6 (a) (i) Find how many different 5-digit numbers can be formed using the digits 1, 3, 5, 6, 8 and 9. No digit may be used more than once in any 5-digit number. [1]

$^6P_5 = 720$

1 3 5 6 8

$6 \times 5 \times 4 \times 3 \times 2 = 720$

(ii) How many of these 5-digit numbers are odd? [1]

1 3 5 6 8 9 →

3 5 6 8 1

$5 \times 4 \times 3 \times 2 \times 4$

$= 480$

(iii) How many of these 5-digit numbers are odd and greater than 60 000? [3]

1 3 5 6 8 9

1. start ~~6~~, 8

6 1 3 5 9

$2 \times 4 \times 3 \times 2 \times 4 = 192$

2. start 9 3 5 6 1

$1 \times 4 \times 3 \times 2 \times 3 = 72$

$192 + 72$

$= 264$

(b) Given that $45 \times {}^nC_4 = (n+1) \times {}^{n+1}C_5$, find the value of n. [4]

$^nC_r = \frac{n!}{(n-r)!\,r!}$

$^nC_4 = \frac{n!}{(n-4)!\,24}$

$^{n+1}C_5 = \frac{(n+1)!}{(n+1-5)!\,5!}$

$= \frac{(n+1)!}{(n-4)!\,120}$

$\frac{45 \cdot n!}{24(n-4)!} = \frac{(n+1)\cdot(n+1)!}{120(n-4)!}$

$= 225n! = (n+1)(n+1)!$

$= 225\cancel{n!} = (n+1)(n+1)\cancel{n!}$

$225 = (n+1)^2$

$2! = 2\times1$

$3! = 3\times2\times1$

$= 3\times2!$

$(n+1)! = (n+1)(n)!$

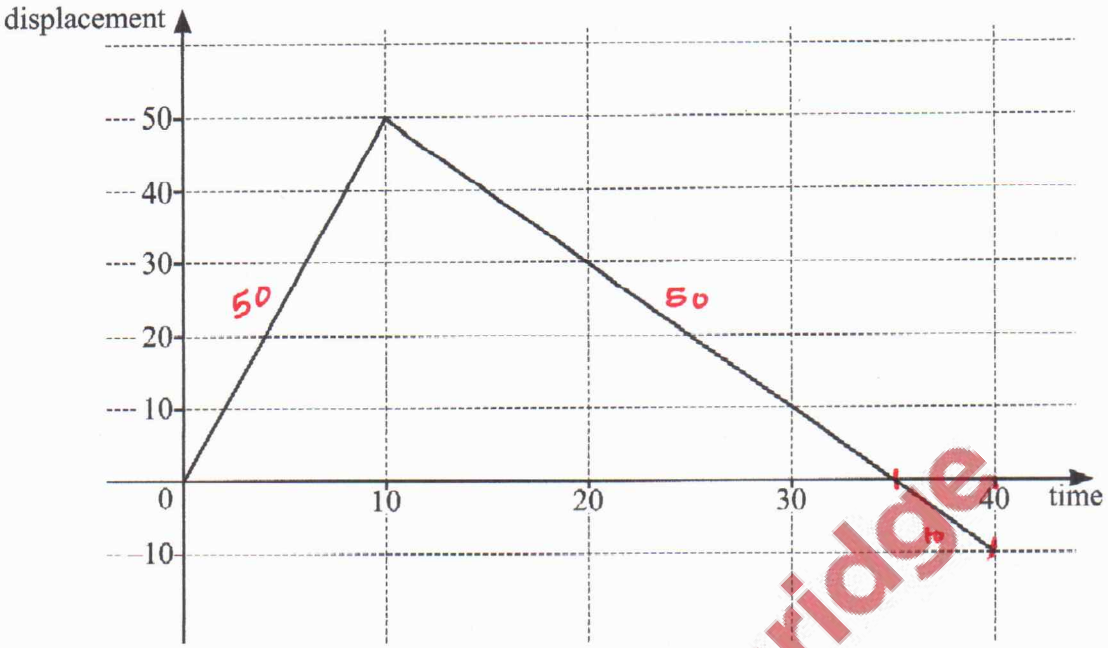
$n+1 = \sqrt{225}$

$n+1 = 15$

$n = 15 - 1$

$n = 14$

7 **(a)** In this question, all lengths are in metres and time, t, is in seconds.



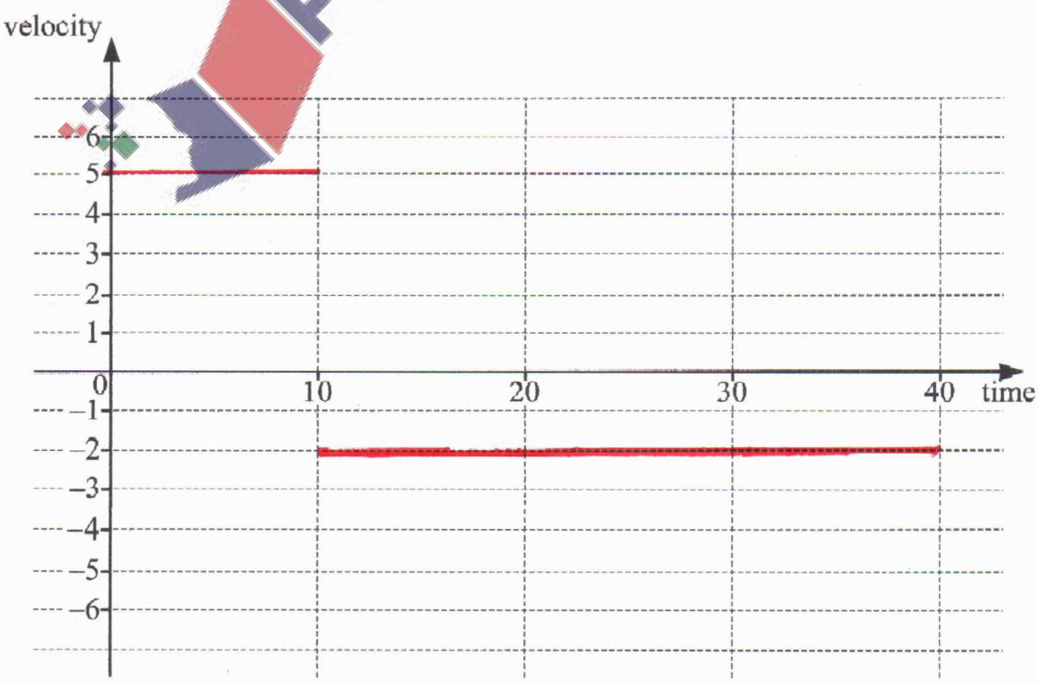
The diagram shows the displacement–time graph for a runner, for $0 \leqslant t \leqslant 40$.

(i) Find the distance the runner has travelled when $t = 40$. [1]

Distance $= 50 + 50 + 10$

$= 110\text{m}$

(ii) On the axes, draw the corresponding velocity–time graph for the runner, for $0 \leqslant t \leqslant 40$. [2]



(b) A particle, P, moves in a straight line such that its displacement from a fixed point at time t is s.

The acceleration of P is given by $(2t+4)^{-\frac{1}{2}}$, for $t > 0$.

(i) Given that P has a velocity of 9 when $t = 6$, find the velocity of P at time t. [3]

$a = (2t+4)^{-1/2}$

$V = \int a\,dt = \int (2t+4)^{-1/2}\,dt$

$= \dfrac{(2t+4)^{1/2}}{\cancel{\frac{1}{2}}(2)} + c$

$v = (2t+4)^{1/2} + c$

Since the velocity is 9, $t=6$ substitute the values.

$9 = (2(6)+4)^{1/2}$

$9 = (16)^{1/2} + c$

$9 = 4 + c$

$c = 9 - 4$

$c = \underline{5}$

$V = (2t+4)^{1/2} + 5$

(ii) Given that $s = \frac{1}{3}$ when $t = 6$, find the displacement of P at time t. [3]

Integrate the Velocity to obtain the displacement

$S = \int v\,dt$

$= (2t+4)^{1/2} + 5\,dt$

$S = \dfrac{(2t+4)^{3/2}}{\frac{3}{\cancel{2}}(2)} + 5t + d$

$S = \frac{1}{3}(2t+4)^{3/2} + 5t + d$

$\frac{1}{3} = \frac{1}{3}(16)^{3/2} + 30 + d$

$\frac{1}{3} = \frac{64}{3} + \frac{90}{3} + d$

$\frac{1}{3} = \frac{154}{3} + d$

$d = \frac{1}{3} - \frac{154}{3}$

$d = \underline{-51}$

The value of displacement $= \frac{1}{3}(2t+4)^{3/2} + 5t - 51$

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = (2-\sqrt{3})x^2 + x - 1$. The x-coordinate of a point A on the curve is $\frac{\sqrt{3}+1}{2-\sqrt{3}}$.

(a) Show that the coordinates of A can be written in the form $(p+q\sqrt{3}, r+s\sqrt{3})$, where p, q, r and s are integers. [5]

Rationalise the x-coordinate of a point A.

$\frac{\sqrt{3}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{\sqrt{3}(2+\sqrt{3})+1(2+\sqrt{3})}{2(2+\sqrt{3})-\sqrt{3}(2+\sqrt{3})}$

$= \frac{2\sqrt{3}+3+2+\sqrt{3}}{1}$

$x = 3\sqrt{3}+5$

$x^2 = (3\sqrt{3}+5)^2$

$= (3\sqrt{3}+5)(3\sqrt{3}+5)$

$= 3\sqrt{3}(3\sqrt{3}+5)+5(3\sqrt{3}+5)$

$= 27+15\sqrt{3}+15\sqrt{3}+25$

$= 52+30\sqrt{3}$

$y = (2-\sqrt{3})(52+30\sqrt{3})+3\sqrt{3}+5-1$

$y = 2(52+30\sqrt{3})-\sqrt{3}(52+30\sqrt{3})+3\sqrt{3}+5-1$

$y = 104+60\sqrt{3}-52\sqrt{3}-30(3)+3\sqrt{3}+4$

$y = 18+11\sqrt{3}$

$A = (3\sqrt{3}+5,\ 18+11\sqrt{3})$

(b) Find the x-coordinate of the stationary point on the curve, giving your answer in the form $a+b\sqrt{3}$, where a and b are rational numbers. [3]

For the stationary point $\frac{dy}{dx}=0$.

$y=(2-\sqrt{3})x^2+x-1$

$\frac{dy}{dx}=2(2-\sqrt{3})x+1+0=0$

$\frac{\cancel{2}(2-\sqrt{3})x}{\cancel{2}}=\frac{-1}{2}$

$(2-\sqrt{3})x=-\frac{1}{2}$

$x=\frac{-1}{2(2-\sqrt{3})}=\frac{-1}{4-2\sqrt{3}}$

Rationalise the value of x.

$\frac{-1}{4-2\sqrt{3}}\times\frac{4+2\sqrt{3}}{4+2\sqrt{3}}$

$=\frac{-4-2\sqrt{3}}{16-4(3)}$

$=\frac{-4-2\sqrt{3}}{16-12}$

$=\frac{-4-2\sqrt{3}}{4}$

x-coordinate $=-1-\frac{1}{2}\sqrt{3}$

9 (a) (i) Write $6xy+3y+4x+2$ in the form $(ax+b)(cy+d)$, where a, b, c and d are positive integers. [1]

Consider the $6xy+3y+4x+2$

Common factors; $3y(2x+1)+2(2x+1)$

then Factorise; $(3y+2)(2x+1)$

(ii) Hence solve the equation $6\sin\theta\cos\theta+3\cos\theta+4\sin\theta+2=0$ for $0° < \theta < 360°$. [4]

$6\sin\theta\cos\theta+3\cos\theta+4\sin\theta+2=0$

This equation is represented by: $6xy+3y+4x+2=0$

$(2\sin\theta+1)(3\cos\theta+2)=0$

$\frac{2\sin\theta}{2}+\frac{1}{2}=0$ $\qquad$ $\frac{3\cos\theta}{3}+\frac{2}{3}=0$

$\sin\theta=-1/2$ $\qquad$ $\cos\theta=-2/3$

$\theta=\sin^{-1}(1/2)$ $\qquad$ $\theta=\cos^{-1}(2/3)$

$\text{"} = 30°$ $\qquad$ $\theta = 48.2°$

$\theta° = 180°+30°,\ 360°-30°$ $\qquad$ $\theta = 180-48.2=131.8°$

$= 210°,\ 330°$ $\qquad$ $\theta = 180+48.2=228.2°$

$\theta = 131.8,\ 210°,\ 228.2°,\ 330°$

(b) Solve the equation $\frac{1}{2}\sec\left(2\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ for $-\pi < \phi < \pi$, where ϕ is in radians. Give your answers in terms of π. [5]

$\sec = \frac{1}{\cos}$

$\sec y = \frac{2}{\sqrt{3}}$

$\cos y = \frac{\sqrt{3}}{2}$

$y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$y = 30^\circ$

$y = \frac{\pi}{6},\ 2\pi - \frac{1}{6} = 1\frac{5}{6}\pi = \frac{11}{6}\pi,\ \frac{\pi}{6} + 2 = \frac{13}{6}\pi,\ \frac{13}{6}\pi + 2 = \frac{23}{6}\pi$

$y = \frac{\pi}{6},\ \frac{11}{6}\pi,\ \frac{13\pi}{6},\ \frac{23}{6}\pi,\ \boxed{-\frac{11}{6}\pi,\ -\frac{1}{6}\pi}$

Due to the $-\pi$ we can as well subtract 2π since the domain is in negative side

$y = 2\phi + \frac{\pi}{4}$

$2\phi = \frac{\pi}{6} - \frac{\pi}{4},\ \frac{11}{6}\pi - \frac{\pi}{4},\ \frac{13}{6}\pi - \frac{\pi}{4},\ \frac{23}{6}\pi - \frac{\pi}{4},\ -\frac{11}{6}\pi - \frac{1}{4}\pi$

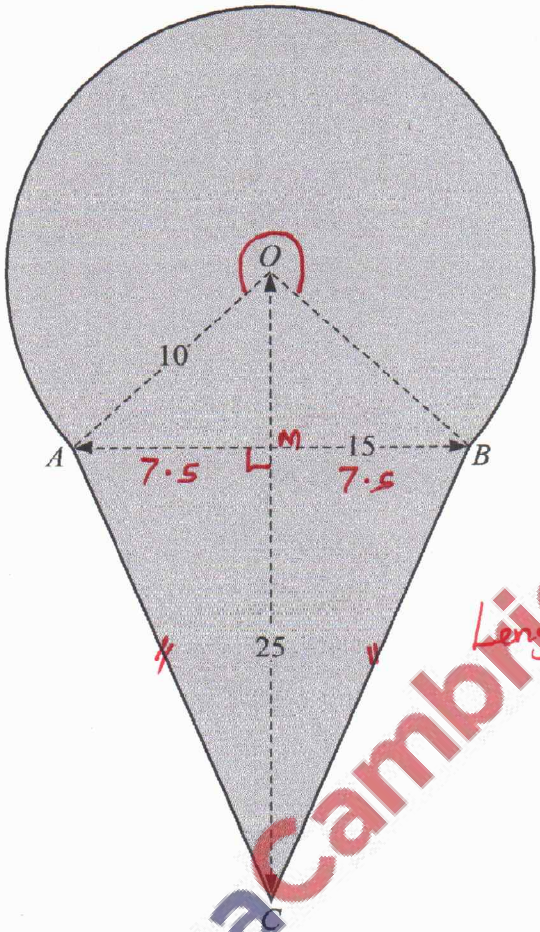
$= \frac{-\frac{1}{12}\pi}{2},\ \frac{\frac{19}{12}\pi}{2},\ \frac{\frac{23}{12}\pi}{2},\ \frac{\frac{43}{12}\pi}{2},\ \frac{\frac{-25}{12}}{2},\ \frac{-\frac{5}{12}\pi}{2}$

Make ϕ subject

divide by 2. $\phi = \frac{1}{24}\pi,\ \frac{19}{24}\pi,\ \frac{23}{24}\pi,\ \frac{43}{24}\pi,\ -\frac{25}{24}\pi,\ -\frac{5}{24}\pi$

$\phi = \frac{-5}{24}\pi,\ -\frac{1}{24}\pi,\ \frac{1}{24}\pi,\ \frac{43}{24}\pi$

10 In this question all lengths are in centimetres.



Angle $= 2\pi - 1.70$
$= \underline{4.583}$

$OM^2 = 10^2 - 7.5^2$
$OM^2 = 100 - 56.25$
$OM^2 = \sqrt{43.75}$
$OM = \underline{6.614}$

Length $MC = 25 - 6.614$
$= \underline{18.386}$

The diagram shows a shaded shape. The arc AB is the major arc of a circle, centre O, radius 10. The line AB is of length 15, the line OC is of length 25 and the lengths of AC and BC are equal.

(a) Show that the angle AOB is 1.70 radians correct to 2 decimal places. [2]

Using cosine rule $\angle AOB = \dfrac{10^2+10^2-15^2}{2\times10\times10}$

$= \dfrac{100+100-225}{200}$

$= \dfrac{-25}{200}$ $\quad \cos^{-1}(25/200) = 1.696$ radians $\approx \underline{1.70 \text{ radians}}$

(b) Find the perimeter of the shaded shape. [4]

Length of Major arc $= r\theta$
$= 10(4.583)$
$= \underline{45.83}$

Length of $AC = \sqrt{7.5^2 + 18.38^2}$
$= 19.856$
$= \underline{19.9\text{cm}}$

Perimeter $= 45.8 + 2(19.856)$

$= \underline{85.5\ \text{cm}}$

(c) Find the area of the shaded shape. [5]

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 10 \times 10 \times 4.583$

$= \underline{229.15\ \text{cm}^2}$

Area of the Triangle $= \frac{1}{2} \times b \times h$

$= \left(\frac{1}{2} \times 25 \times 7.5\right) \times \cancel{2}$

$= \underline{187.5\ \text{cm}^2}$

Total area of shaded shape

$= 229.15 + 187.5$

$= 416.65\ \text{cm}^2$

$\approx \underline{417.0\ \text{cm}^2}$