Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the inequality $(x-8)(x-10) > 35$. [4]

$x^2-18x+80>35$

$x^2-18x+80-35>0$

$x^2-18x+45>0$

$x^2-18x+45=0$

$P=45$ $(-15,-3)$

$S=-18$

$x^2-15x-3x+45=0$

$x(x-15)-3(x-15)=0$

$(x-15)(x-3)=0$

$x-3=0$, $x=\underline{3}$ | $x-15=0$, $x=\underline{15}$

$\underline{x<3}$ and $\underline{x>15}$

$(x-8)(x-10)$

$x(x-10)-8(x-10)$

$=x^2-10x-8x+80$

$=\underline{x^2-18x+80}$

2 Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$. [4]

$\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$

$\frac{2^{2(x+1)}}{2^{x-1}} = 2^{5\left(\frac{x}{3}\right)} \times 2^{3\left(\frac{1}{3}\right)}$

$2^{(2x+2)} = 2^{\frac{5x}{3}} \times 2^{1}$

$2x+2-(x-1) = \frac{5x}{3}+1$

$2x+2-x+1 = \frac{5x}{3}+1$

$x+3 = \frac{5x}{3}+1$

$2 = \frac{5x}{3}-\frac{3x}{1}$

$3\times 2 = \frac{2x}{3}\times 3$

$\frac{6}{2} = \frac{2x}{2}$ $x = \underline{3}$

$\frac{2^a}{2^b} = 2^{a-b}$

$2^a \times 2^b = 2^{a+b}$

3 (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$. [5]

$(12, 1) \quad (4, 3)$

Gradient $= \frac{3-1}{4-12}$

$= \frac{2}{-8}$

$m_1 = -\frac{1}{4}$

$m_1 \times m_2 = -1$

$-\frac{1}{4} \times m_2 = -1$

$m_2 = 4$

$m_2, 4 \quad , (8, 2)$

$\frac{y-2}{x-8} = \frac{4}{1}$

$y-2 = 4(x-8)$

$y-2 = 4x-32$

$y = 4x - 32 + 2$

$y = 4x - 30$

Two lines are Perpendicular When the products of their gradients is -1.

(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB. [3]

At x-axis, $y = 0$

$0 = 4x - 30$

$\frac{30}{4} = \frac{4x}{4}$

$x = \frac{30}{4}$

$x = 7.5 \quad A(7.5, 0)$

At y-axis; $x = 0$.

$y = 4x - 30$

$y = 4(0) - 30$

$y = -30$

$B(0, -30)$

Length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-7.5)^2 + (-30)^2}$

$= \sqrt{56.25 + 900}$

$= \sqrt{956.25}$

$= 30.923$

$= 30.9$

4 Solve the simultaneous equations.

$$\log_3(x+y) = 2$$

$$2\log_3(x+1) = \log_3(y+2)$$

[6]

$\log_3(x+y) = 2$

$x+y = 3^2$

$x+y = 9$ --(i)

$x+y=9$

$\underline{y=9-x}$

$2\log 3\,(x+1) = \log_3(y+2)$

$\cancel{\log_3}(x+1)^2 = \cancel{\log_3}(y+2)$

$(x+1)^2 = y+2$

$x^2+2x+1 = y+2$

$x^2+2x+1-2 = y$

$x^2+2x-1 = y$ --(ii)

Substitute equation (i) to (ii) as $y = 9-x$

$x^2+2x-1 = 9-x$

$x^2+2x+x-1 = 9$

$x^2+3x = 10$

$x^2+3x-10 = 0$

$P = -10$ $(5, -2)$

$S = 3$

$(x+5)(x-2) = 0$

$x+5=0$ | $x-2=0$

$x=\underline{-5}$ | $x=\underline{2}$

$y=9-x$ | $y=9-x$

$y=9-(-5)$ | $y=9-2$

$y=9+5$ | $y=\underline{7}$

$y=\underline{14}$

$(x+1)^2$

$=(x+1)(x+1)$

$=x(x+1)1(x+1)$

$=x^2+x+x+1$

$=x^2+2x+1$

$x=2,\ y=7$

$(x=-5\ \ y=\underline{14}) \rightarrow$ reject

Since no logarithms numbers that has negative value

$x=\underline{2}\quad y=\underline{7}$

5 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

(a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$. [4]

$y = x^3 - 6x^2 + 3x + 10$

When $x = 1$ $y = 1^3 - 6(1^2) + 3(1) + 10$

$y = 1 - 6 + 3 + 10$

$y = \underline{8}$ $(1, 8)$

$\frac{dy}{dx} = 3x^2 - 12x + 3$

$= 3(1) - 12(1) + 3$

gradient $= -\underline{6}$

$\frac{y-8}{x-1} = \frac{-6}{1}$

$y - 8 = -6(x-1)$

$y - 8 = -6x + 6$

$y = -6x + 6 + 8$

$y = \underline{-6x + 14}$

(b) Find the coordinates of the point where this tangent meets the curve again. [5]

$y = x^3 - 6x^2 + 3x + 10$

$-6x + 14 = x^3 - 6x^2 + 3x + 10$

$x^3 - 6x^2 + 9x - 4 = 0$

$$\begin{array}{r|l} & x^2 - 5x + 4 \\ x-1 & x^3 - 6x^2 + 9x - 4 \\ & x^3 - x^2 \\ \hline & -5x^2 + 9x \\ & -5x^2 + 5x \\ \hline & 4x - 4 \\ & 4x - 4 \\ \hline & 0 \end{array}$$

$(x-1)(x^2 - 5x + 4) = 0$

$(x-1)(x-1)(x-4) = 0$

$x = \underline{1}$ $x = \underline{1}$ $x = \underline{4}$

$x = \underline{4}$ $(y = -6x + 14$

$= -6(4) + 14$

$= -24 + 14$

$= -10$

$\underline{(4, -10)}$

6 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$. [6]

$$\int_2^4 1 + \frac{2}{x} + x^{-2}\,dx$$

$$= \left[x + 2\ln x + \frac{x^{-1}}{-1}\right]_2^4$$

$$= \left[4 + 2\ln 4 + \frac{4^{-1}}{-1}\right] - \left[2 + 2\ln 2 + \frac{2^{-1}}{-1}\right]$$

$$= \left[4 + \ln 2^4 + \frac{1}{4}\right] - \left[2\ln 2^2 - \frac{1}{2}\right]$$

$$= \left(4 + \ln 16 - \frac{1}{4}\right) - \left[2\ \ln 4 + \frac{1}{2}\right]$$

$$= 4 + \ln 16 - \frac{1}{4} - 2 - \ln 4 + \frac{1}{2}$$

$$= 2\frac{1}{4} + \ln 16 - \ln 4$$

$$= 2\frac{1}{4} + \ln\frac{16}{4}$$

$$= 2\frac{1}{4} + \ln 4$$

$$= 2\frac{1}{4} + \ln 4$$

$$\frac{(x+1)^2}{x^2} = \frac{x^2+2x+1}{x^2}$$

$$= 1 + \frac{2}{x} + \frac{1}{x^2}$$

$\lg a - \log b$

$= \lg \frac{a}{b}$

7 A geometric progression has a first term of 3 and a second term of 2.4. For this progression, find

(a) the sum of the first 8 terms, [3]

$3r = 2.4$

$r = 0.8$

$a = \underline{3}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{3(1-0.8^8)}{1-0.8}$

$= \underline{\underline{12.5}}$

(b) the sum to infinity, [1]

$S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-0.8}$

$= \underline{15}$

(c) the least number of terms for which the sum is greater than 95% of the sum to infinity. [4]

$S_n > 95\% \; S_\infty$

$\frac{a(1-r^n)}{1-r} > 0.95 S_\infty$

$\frac{3(1-0.8^n)}{1-0.8} > 0.95(15)$

$1-0.8^n > 0.95$

$0.8^n > 0.95 - 1$

$0.8^n < 0.05$

$\log 0.8^n < \log 0.05$

$n \log 0.8 < \log 0.05$

$\frac{n(-0.0969)}{-0.0969} < \frac{+1.301}{+0.0969}$

$n = 13.4265$

$n = 14$

$(2\sqrt{3}+1)(2\sqrt{3}-1)$

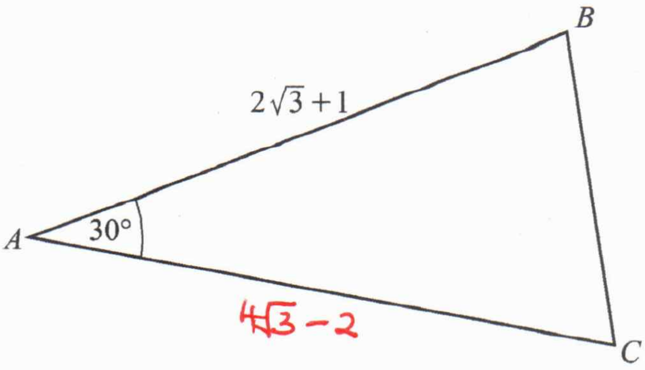
$2\sqrt{3}(2\sqrt{3}-1)+1(2\sqrt{3}-1)$

$= 12 - \cancel{2\sqrt{3}} + \cancel{2\sqrt{3}} - 1$

$= \underline{11}$

8 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

In this question lengths are in centimetres.



You may use the following trigonometric ratios.

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

(a) Given that the area of the triangle ABC is $5.5\,\text{cm}^2$, find the exact length of AC. Write your answer in the form $a+b\sqrt{3}$, where a and b are integers. [4]

Area of Triangle $= \frac{1}{2}ab\sin C$

$5.5\,\text{cm}^2 = \frac{1}{2} \times 2\sqrt{3}+1 \times b \sin 30°$

$5.5 = \frac{1}{2} \times 2\sqrt{3}+1 \times b\left(\frac{1}{2}\right)$

$4 \times 5.5 = \frac{1}{4} \times 2\sqrt{3}+1 \times b \times 4$

$\frac{22}{2\sqrt{3}+1} = \frac{\cancel{(2\sqrt{3}+1)} \times b}{\cancel{2\sqrt{3}+1}}$

$b = \frac{22}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$

$\frac{22(2\sqrt{3}-1)}{(2\sqrt{3}+1)(2\sqrt{3}-1)}$

$= \frac{44\sqrt{3}-22}{11}$

$= \underline{4\sqrt{3}-2}$

$AC = \underline{-2+4\sqrt{3}}$

(b) Show that $BC^2 = c+d\sqrt{3}$, where c and d are integers to be found. [4]

$\cos 30° = \frac{(2\sqrt{3}+1)^2 + (-2+4\sqrt{3})^2 - BC^2}{(2\sqrt{3}+1)^2 + (4\sqrt{3}-2)^2 - BC^2}$

$\frac{\sqrt{3}}{2}$

$= 2(2\sqrt{3})+1)(4\sqrt{3}-2)$

$= 8(3) - 4\sqrt{3} + 4\sqrt{3} - 2$

$= 24 - 2$

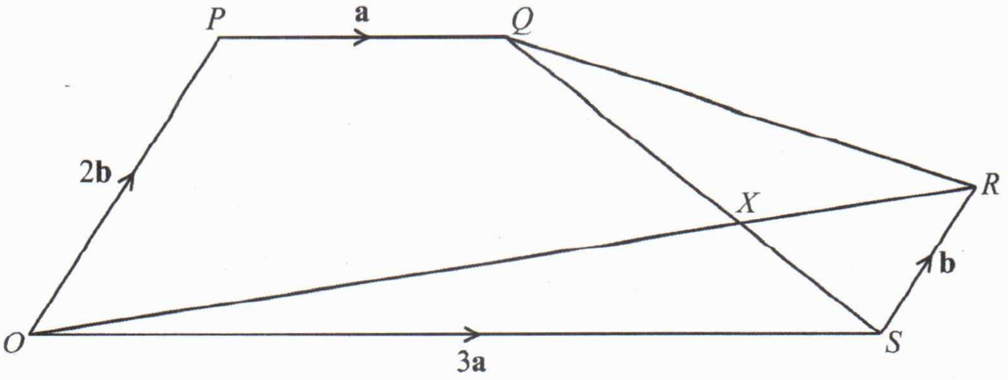
$= \underline{22}\sqrt{3} = 12 + 4\sqrt{3} + 1 + 48 - 8\sqrt{3} + 4 - BC^2$

$22\sqrt{3} = 65 - 4\sqrt{3} - BC^2$

$BC^2 = 65 - 4\sqrt{3} - 22\sqrt{3}$

$BC^2 = \underline{65 - 26\sqrt{3}}$

9



In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

(a) Find $\overrightarrow{OQ}$ in terms of **a** and **b**. [1]

$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= 2b + a$

(b) Find $\overrightarrow{QS}$ in terms of **a** and **b**. [1]

$\overrightarrow{QS} = \overrightarrow{QO} + OS$

$= -a - 2b + 3a$

$= 2a - 2b$

$= 2(a-b)$

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and μ. [1]

$\overrightarrow{QX} = \mu \overrightarrow{QS}$

$= \mu(2a-2b)$

$OX = \mu(2a-2b) + 2b + a$

$= 2\mu a - 2\mu b + 2b + a$

$= (2\mu+1)a + (2-2\mu)b$

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and λ. [1]

$\overrightarrow{OR} = \overrightarrow{OS} + \overrightarrow{SR}$

$= 3a + b$

$OX = \lambda(\overrightarrow{OR})$

$= \lambda(3a+b)$

$= 3\lambda a + \lambda b$

(e) Find the value of λ and of μ. [3]

$\vec{OX} = (2\mu+1)a + (2-2\mu)b$

$\vec{OX} = 3\lambda a + \lambda b$

$(2\mu+1)a + (2-2\mu)b = 3\lambda a + \lambda b$

$(2\mu+1)\cancel{a} = 3\lambda \cancel{a}$

$2\mu+1 = 3\lambda$

$2\mu+1 = 6-6\mu$

$2\mu+6\mu = 5$

$\frac{\cancel{8}\mu}{\cancel{8}} = \frac{5}{8}$

$\mu = 5/8$

$2-2\mu = \lambda$

$2-2(5/8) = \lambda$

$2 - \frac{10}{8} = \lambda$

$3/4 = \lambda$

$\mu = 5/8$

$\lambda = 3/4$

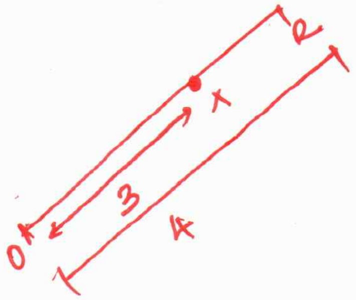
(f) Find the value of $\frac{QX}{XS}$. [1]

$SX = 3/8\ QS$

$QX = \frac{5/8\ QS}{3/8\ QS}$

$= 5/3$

(g) Find the value of $\frac{OR}{OX}$. [1]



$OX = \lambda OR$

$OX = 3/4\ OR$

$= 4/3$

10 The number, b, of bacteria in a sample is given by $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week.

(a) Find the value of P and of Q. $t=0, \quad b=500, \quad t=1 \quad b=600$ [4]

$b = P + Qe^{2t}$

$500 = P + Qe^{2(0)}$

$500 = P + Q$ —(i)

$600 = P + Qe^{2t}$

$600 = P + Qe^{2(1)}$

$600 = P + Qe^{2}$ --(ii)

$600 - Qe^{2} + Q = 500$

$-Qe^{2} + Q = 500 - 600$

$-Qe^{2} + Q = -100$

$\dfrac{Q(\cancel{-e^{2}+1})}{\cancel{-e^{2}+1}} = \dfrac{-100}{-e^{2}+1}$

$Q = 15.6517$

≈ 15.7

$P = 600 - (15.6517)e^{2}$

$= 484.3$

$= 484$

$P = 484 \quad Q = 15.7$

(b) Find the number of bacteria present after 2 weeks. [1]

$t = 2$

$b = 484 + 15.7e^{2t}$

$b = 484 + 15.7e^{2(2)}$

$b = 484 + 15.7e^{4}$

$b = 1341.19$

$b = \underline{\underline{1340}}$

(c) Find the first week in which the number of bacteria is greater than 1 000 000. [3]

$b > 1000000$

$484 + 15.7e^{2t} > 1000000 - 484$

$\frac{\cancel{15.7}e^{2t}}{\cancel{15.7}} > \frac{999516}{15.7}$

$e^{2t} > 63663.43949$

$2t > \ln 63663.43949$

$\frac{\cancel{2}t}{\cancel{2}} > \frac{11.06}{2}$

$t > 5.53$

$\simeq \underline{\underline{6}}$

$\boxed{\begin{array}{l} e^{a} = b \\ a = \ln b \end{array}}$

11 **(a)** Show that $\dfrac{\sin x \tan x}{1-\cos x} = 1 + \sec x$. [4]

$\tan x = \dfrac{\sin x}{\cos x}$

$$\frac{\sin x \cdot \frac{\sin x}{\cos x}}{1-\cos x} = \frac{\frac{\sin^2 x}{\cos x}}{1-\cos x} = \frac{\sin^2 x}{\cos x(1-\cos x)}$$

$$= \frac{\sin^2 x}{\cos x - \cos^2 x}$$

$$= \frac{1-\cos^2 x}{\cos x - (1-\cos x)}$$

$$= \frac{(1)^2 - \cos^2 x}{\cos x(1-\cos x)}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cos x\cancel{(1-\cos x)}}$$

$$= \frac{1+\cos x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$= \underline{\underline{1 + \sec x}}$$

(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0° \leqslant x \leqslant 360°$. [6]

$\tan x = \frac{s}{c}$

$\cot x = \frac{c}{s}$

$\sec x = \frac{1}{\cos}$.

$$\frac{5\sin x}{\cos x} - \frac{3\cos x}{\sin x} = \frac{2}{\cos x}$$

$$\frac{5\sin^2 x - 3\cos^2 x}{\cancel{\cos x}\sin x} = \frac{2}{\cancel{\cos x}}$$

$\cos^2 x = 1 - \sin^2 x$

$$5\sin^2 x - 3\cos^2 x = 2\sin x$$

$$5\sin^2 x - 3(1-\sin^2 x) = 2\sin x$$

$$5\sin^2 x - 3 + 3\sin^2 x = 2\sin x$$

$$8\sin^2 x - 2\sin x - 3 = 0$$

$p = -24$

$s = -2$ $(-6, 4)$

$$8\sin^2 x - 6\sin x + 4\sin x - 3 = 0$$

$$(4\sin x - 3)(2\sin x + 1) = 0$$

$4\sin x - 3 = 0$

$\frac{\cancel{4}\sin x}{\cancel{4}} = 3/4$

$\sin x = 0.75$

$= 48.6°$

$180° - 48.6°$

$= 131.4°$

$48.6°, 131.4°$

$\frac{\cancel{2}\sin x}{\cancel{2}\sin x} = \frac{-1}{2}$

$\sin x = -0.5$

$\sin^{-1}(x) = 30°$

$x = (180+30)$

$= 210°, 330°$