

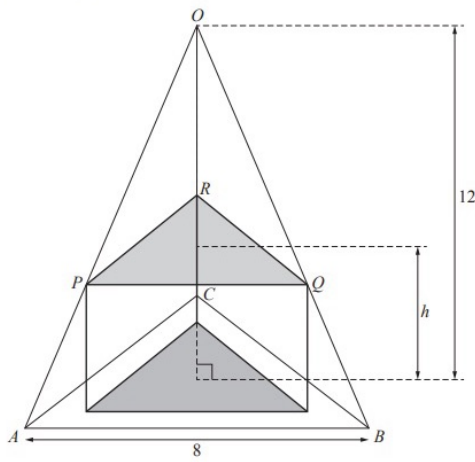


**Difficult Questions Explained for  
Cambridge O Level**

# **Additional Mathematics (4037)**

1st edition, for examination until 2027

12 In this question all lengths are in centimetres.



The diagram shows a right triangular prism of height  $h$  inside a right pyramid. The pyramid has a height of 12 and a base that is an equilateral triangle,  $ABC$ , of side 8. The base of the prism sits on the base of the pyramid. Points  $P$ ,  $Q$  and  $R$  lie on the edges  $OA$ ,  $OB$  and  $OC$ , respectively, of the pyramid  $OABC$ . Pyramids  $OABC$  and  $OPQR$  are similar.

(a) Show that the volume,  $V$ , of the triangular prism is given by  $V = \frac{\sqrt{3}}{9}(ah^3 + bh^2 + ch)$  where  $a$ ,  $b$  and  $c$  are integers to be found. [4]

$$\frac{144 - 24h + h^2}{144} = \frac{x}{16\sqrt{3}}$$

$$\Delta PQR = \frac{\sqrt{3}}{9}(h^2 - 24h + 144)$$

cross-sectional area x height ( $h$ )

$$\frac{\sqrt{3}}{9}(h^2 - 24h + 144)h$$

$$\frac{\sqrt{3}}{9}(h^3 - 24h^2 + 144h) \#$$

Final Answer

$$\frac{1}{2}(8^2) \times \sin 60^\circ = \text{Area of } ABC$$

$$\text{area of } ABC = 16\sqrt{3}$$

$$\frac{12-h}{12} - \text{ratio of } PQR \text{ \& } PQO \text{ length}$$

$$\left(\frac{12-h}{12}\right)^2 - \text{ratio of areas}$$

$$\text{Ratio of Areas} = \frac{144 - 24h + h^2}{144}$$

$$\frac{144 - 24h + h^2}{144} = \frac{\text{Area of } PQR}{\text{Area of } ABC (16\sqrt{3})}$$

Solve for PQR

(b) It is given that, as  $h$  varies,  $V$  has a maximum value. Find the value of  $h$  that gives this maximum value of  $V$ . [3]

#1: differentiate the volume equation with respect to  $h$

$$\frac{dV}{dh} = \frac{\sqrt{3}}{9}(3h^2 - 48h + 144) \quad \frac{\sqrt{3}}{9}(6h - 48)$$

second derivative

Maximum!

$$\frac{\sqrt{3}}{9}(6(\underline{4}) - 48) = \frac{-8\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{9}(6(12) - 48) = \frac{8\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{9}(3h^2 - 48h + 144) = 0$$

use 2nd derivative to find max

solve this

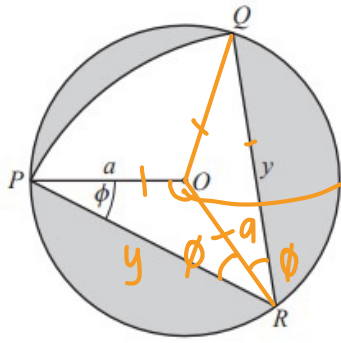
$$h = 4 \text{ final answer}$$

# 4037\_m23\_qp\_22

9 In this question, all lengths are in centimetres and all angles are in radians.

(b)

Triangle  $POR$  is  
congruent to  
triangle  $QOR$   
 $PRQ = 2\phi$



$$180 = \pi$$

$$\angle POR = \pi - 2\phi$$

The diagram shows a circle with centre  $O$  and radius  $a$ . Sector  $PQR$  is a sector of a different circle with centre  $R$  and radius  $y$ . Angle  $OPR$  is  $\phi$ . Find, in terms of  $a$  and  $\phi$  only, the total area of the three shaded regions. Simplify your answer. [4]

$$\frac{1}{2} r^2 \theta \rightarrow \text{Area of Sector}$$

$$\pi r^2 \rightarrow \text{Circle}$$

$$\pi (a)^2 - \frac{1}{2} (y)^2 2\phi$$

Find  $y$  in terms of  $\phi$

- use cosine / sine rule

COSINE

$$y^2 = a^2 + a^2 - 2a^2 \cos(\pi - 2\phi)$$

Substitute  $y^2$  into  $\pi (a)^2 - \frac{1}{2} (y)^2 2\phi$

$$\pi (a)^2 - \frac{1}{2} (a^2 + a^2 - 2a^2 \cos(\pi - 2\phi)) 2\phi$$

$$\pi a^2 - (2a^2 - 2a^2 \cos(\pi - 2\phi))$$

↑  
Final answer

SINE

$$\frac{a}{\sin \phi} = \frac{y}{\sin(\pi - 2\phi)}$$

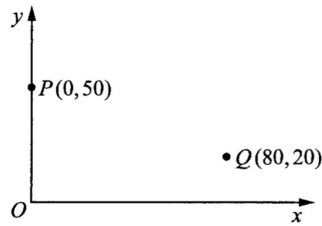
$$\frac{a}{\sin \phi} (\sin(\pi - 2\phi)) = y$$

$$\frac{a \sin(\pi - 2\phi)}{\sin \phi} = y$$

Substitute  $y$  into  $\pi a^2 - \frac{1}{2} (y)^2 2\phi$

$$\pi a^2 - \frac{1}{2} \left( \frac{a \sin(\pi - 2\phi)}{\sin \phi} \right)^2 2\phi$$

$$\pi a^2 - \frac{a^2 \phi \sin^2(\pi - 2\phi)}{\sin^2 \phi}$$



At 1200 hours, ship  $P$  is at the point with position vector  $50\mathbf{j}$  km and ship  $Q$  is at the point with position vector  $(80\mathbf{i} + 20\mathbf{j})$  km, as shown in the diagram. Ship  $P$  is travelling with velocity  $(20\mathbf{i} + 10\mathbf{j})$  km  $\text{h}^{-1}$  and ship  $Q$  is travelling with velocity  $(-10\mathbf{i} + 30\mathbf{j})$  km  $\text{h}^{-1}$ .

- (i) Find an expression for the position vector of  $P$  and of  $Q$  at time  $t$  hours after 1200 hours. [3]  
 (ii) Use your answers to part (i) to determine the distance apart of  $P$  and  $Q$  at 1400 hours. [3]  
 (iii) Determine, with full working, whether or not  $P$  and  $Q$  will meet. [2]

$$i. \quad r_p = 50\hat{j} + t(20\hat{i} + 10\hat{j})$$

$$r_q = 80\hat{i} + 20\hat{j} + t(-10\hat{i} + 30\hat{j})$$

$r = a + tv$   
 where  $r$  is the final position vector  
 $a$  is Original position vector  
 $t$  is time  
 $v$  is Velocity Vector

ii. Substitute 2 into  $t$ , 1200  $\rightarrow$  1400 (2 hours)

$$r_p = 50\hat{j} + 2(20\hat{i} + 10\hat{j})$$

$$= 40\hat{i} + 70\hat{j}$$

$$r_q = 80\hat{i} + 20\hat{j} + 2(-10\hat{i} + 30\hat{j})$$

$$= 60\hat{i} + 80\hat{j}$$

Distance between  $Q$  &  $P$  so  $r_q - r_p$

$$60\hat{i} + 80\hat{j} - (40\hat{i} + 70\hat{j})$$

$$= 20\hat{i} + 10\hat{j}$$

$\hookrightarrow$  Find magnitude of vector by using Pythagoras

$$\sqrt{20^2 + 10^2}$$

$$= 10\sqrt{5} \rightarrow \text{Final Answer!}$$

iii. When/if  $P$  &  $Q$  meet, they will be equal to each other

$$50\hat{j} + t(20\hat{i} + 10\hat{j}) = 80\hat{i} + 20\hat{j} + t(-10\hat{i} + 30\hat{j})$$

$$50\hat{j} + 20t\hat{i} + 10t\hat{j} = 80\hat{i} + 20\hat{j} - 10t\hat{i} + 30t\hat{j}$$

$$20t\hat{i} + \hat{j}(50 + 10t) = \hat{i}(80 - 10t) + \hat{j}(20 + 30t)$$

$$\cancel{\hat{j}}(50 + 10t) = \cancel{\hat{j}}(20 + 30t) \quad 20t\hat{i} = \hat{i}(80 - 10t)$$

$$50 + 10t = 20 + 30t$$

$$20t + 10t = 80$$

$$50 - 20 = 30t - 10t$$

$$30t = 80$$

$$30 = 20t$$

$$t = \frac{8}{3}$$

$$\frac{3}{2} = t$$

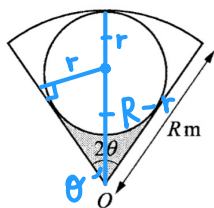
$\frac{3}{2} \neq \frac{8}{3}$  hence,  $P$  and  $Q$  don't meet.



# 4037\_w02\_qp\_01.

12 Answer only **one** of the following two alternatives.

**EITHER**



The diagram shows a garden in the form of a sector of a circle, centre  $O$ , radius  $R$  m and angle  $2\theta$ . Within this garden a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is  $r$  m,

(i) show that  $R = r \left( 1 + \frac{1}{\sin\theta} \right)$ . [4]

Given also that  $\theta = 30^\circ$ ,

(ii) calculate the fraction of the garden that is to be planted with roses. [4]

When the circular plot has been constructed, the remainder of the garden consists of three regions. Given further that  $R = 15$ ,

(iii) calculate, to 1 decimal place, the length of fencing required to fence along the perimeter of the shaded region. [3]

$$i. \sin \theta = \frac{r}{R-r}$$

$$R-r = \frac{r}{\sin \theta}$$

$$R = \frac{r}{\sin \theta} + r$$

$$R = r \left( \frac{1}{\sin \theta} + 1 \right)$$

ii. Convert  $30^\circ \rightarrow$  radians

$$30^\circ \times \frac{\pi}{180} = \frac{\pi}{6}$$

$$R = r \left( \sin \frac{\pi}{6} + 1 \right)$$

$$R = 3r$$

Area of circle  $\cdot \pi r^2$

Area of Sector:  $\frac{1}{2} R^2 2\theta$

Substitute  
 $R = 3r$

$$: R^2 \theta$$

$$: (3r)^2 \theta$$

$$: 9r^2 \theta$$

Substitute  
 $\frac{\pi}{6}$

Fraction:  $\frac{\text{roses}}{\text{garden}}$

$$= \frac{\pi r^2}{9r^2 \left( \frac{\pi}{6} \right)}$$

$$= \frac{\pi^2}{9\pi} = \frac{2}{3} \#$$

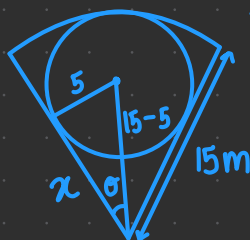
Final Answer.

iii.  $R = 3r$

$$15 = 3r$$

$$\frac{15}{3} = r$$

$$5 = r$$



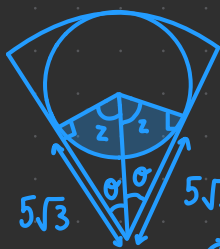
$$\tan \theta = \frac{r}{x}$$

$$\tan \left( \frac{\pi}{6} \right) = \frac{5}{x}$$

$$\frac{5}{\tan \left( \frac{\pi}{6} \right)} = x$$

$$x = 5\sqrt{3}$$

Finding  $y$ :



$$z = \pi - \theta - \frac{\pi}{2}$$

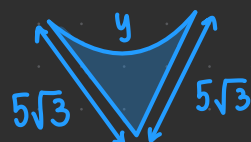
$$= \pi - \frac{\pi}{6} - \frac{\pi}{2}$$

$$= \frac{\pi}{3}$$

$$2z = \frac{2}{3} \pi$$

$$y = 5 \times \frac{2\pi}{3}$$

$$y = \frac{10\pi}{3}$$



Perimeter: Add all the sides

$$5\sqrt{3} + 5\sqrt{3} + \frac{10\pi}{3}$$

$$= 27.79$$

1 decimal place!

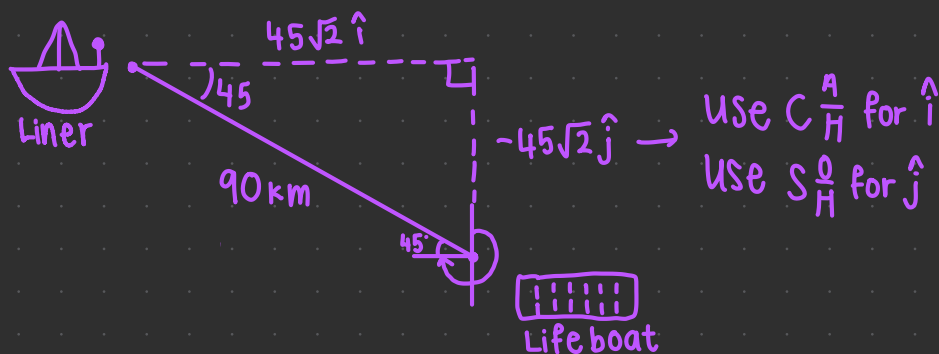
$$27.8 \text{ m}$$

↑ don't forget the unit :)

# 4037\_s03\_qp\_01

- 4 An ocean liner is travelling at  $36 \text{ km h}^{-1}$  on a bearing of  $090^\circ$ . At 0600 hours the liner, which is 90 km from a lifeboat and on a bearing of  $315^\circ$  from the lifeboat, sends a message for assistance. The lifeboat sets off immediately and travels in a straight line at constant speed, intercepting the liner at 0730 hours. Find the speed at which the lifeboat travels. [5]

$t = 1.5$   
 6:00 → 7:30  
 1.5 hours!



Final position vector

$r = a + tv$   
 where  $r$  is the final position vector  
 $a$  is Original position vector  
 $t$  is time  
 $v$  is Velocity Vector

Let original Liner position =  $0, 0$  Life boat original position :  $45\sqrt{2} \hat{i} - 45\sqrt{2} \hat{j}$

$$R_{\text{Liner}} = 36t \hat{i} \quad t = 1.5$$

$$= 36(1.5) \hat{i}$$

$$= 54 \hat{i} + 0 \hat{j}$$

$$R_{\text{Lifeboat}} = (45\sqrt{2} \hat{i} - 45\sqrt{2} \hat{j}) + 1.5(x \hat{i} + y \hat{j})$$

$$= 45\sqrt{2} \hat{i} - 45\sqrt{2} \hat{j} + 1.5x \hat{i} + 1.5y \hat{j}$$

$$= \hat{i} (45\sqrt{2} + 1.5x) + \hat{j} (1.5y - 45\sqrt{2})$$

For Liner & Lifeboat to meet  
 the  $R_{\text{Liner}} = R_{\text{Lifeboat}}$

$$54 \hat{i} + 0 \hat{j} = \hat{i} (45\sqrt{2} + 1.5x) + \hat{j} (1.5y - 45\sqrt{2})$$

$$54 = 45\sqrt{2} + 1.5x \quad 0 = 1.5y - 45\sqrt{2}$$

$$54 - 45\sqrt{2} = 1.5x \quad 45\sqrt{2} = 1.5y$$

$$\frac{54 - 45\sqrt{2}}{1.5} = x \quad \frac{45\sqrt{2}}{1.5} = y$$

$$x = 36 - 30\sqrt{2} \quad y = 30\sqrt{2}$$

This is velocity vector,  
 to convert to speed, use  
 Pythagoras theorem.

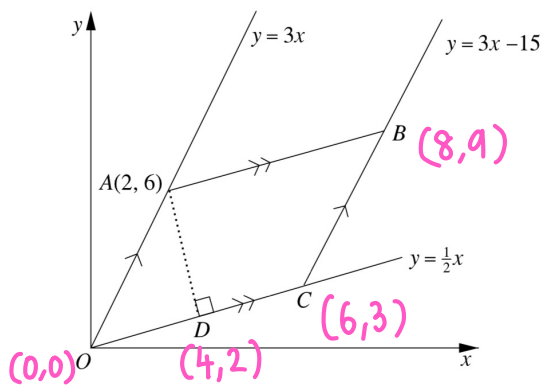
$$\sqrt{(36 - 30\sqrt{2})^2 + (30\sqrt{2})^2}$$

$$= 42.9 \text{ km/h}$$

↑ this is the final answer!

# 4037\_w03\_qp\_01

11 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a parallelogram  $OABC$  where  $O$  is the origin and  $A$  is the point  $(2, 6)$ . The equations of  $OA$ ,  $OC$  and  $CB$  are  $y = 3x$ ,  $y = \frac{1}{2}x$  and  $y = 3x - 15$  respectively. The perpendicular from  $A$  to  $OC$  meets  $OC$  at the point  $D$ . Find

- (i) the coordinates of  $C$ ,  $B$  and  $D$ , [8]  
 (ii) the perimeter of the parallelogram  $OABC$ , correct to 1 decimal place. [3]

ii.  $AO + AB + BC + CO$

$$AO = BC$$

$$AB = CO$$

$$2AO + 2CO$$

$$AO = \sqrt{6^2 + 2^2} \quad (\text{Pythagoras})$$

$$AO = 2\sqrt{10}$$

$$CO = \sqrt{6^2 + 3^2}$$

$$CO = 3\sqrt{5}$$

$$\begin{aligned} \text{Perimeter} &= 2(2\sqrt{10}) + 2(3\sqrt{5}) \\ &= 4\sqrt{10} + 6\sqrt{5} \\ &= 26.1 \text{ units} \end{aligned}$$

i.  $C$ : the point where line  $y = 3x - 15$  &  $y = \frac{1}{2}x$  intersect

$$C: 3x - 15 = \frac{1}{2}x$$

$$3x - \frac{1}{2}x = 15$$

$$\frac{5}{2}x = 15$$

$$x = 15 \div \frac{5}{2}$$

$$x = 6$$

\* After  $x$  is found substitute the value into either equations ( $3x - 15 = y$  or  $\frac{1}{2}x = y$ ) to find  $y$  coordinate.

$$y = \frac{1}{2}x$$

$$= \frac{1}{2}(6)$$

$$y = 3$$

$$C: (6, 3)$$

B. the line  $B$  is on is parallel to  $y = \frac{1}{2}x$ , hence they have same gradient ( $\frac{1}{2}$ )

Use  $A$  coords to find line equation.

$$y = \frac{1}{2}x + c \quad \text{Line equation}$$

$$A: (2, 6) \quad \text{for } AB =$$

$$6 = \frac{1}{2}(2) + c \quad y = \frac{1}{2}x + 5$$

$$6 - \frac{1}{2}(2) = c \quad \text{Let } AB \text{ line equation equal to } y = 3x - 15$$

$$c = 5$$

Because  $B$  is where  $y = \frac{1}{2}x + 5$  &  $y = 3x - 15$  intersect.

$$B: \frac{1}{2}x + 5 = 3x - 15$$

$$5 + 15 = 3x - \frac{1}{2}x$$

$$20 = \frac{5}{2}x$$

$$20 \div \frac{5}{2} = x$$

$$x = 8$$

$$y = \frac{1}{2}(8) + 5$$

$$y = 9$$

$$B: (8, 9)$$

D: Perpendicular to  $y = \frac{1}{2}x$ , Find line equation for  $AD$ .

$$y = mx + c$$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{1}{2}} = -2$$

$$y = 2x + c$$

\* Substitute  $A$  coords

$$6 = 2(2) + c$$

$$6 + 4 = c$$

$$c = 10$$

$$y = -2x + 10$$

let  $-2x + 10$  equal to  $\frac{1}{2}x$ , intersection of those lines is point  $D$ .

$$-2x + 10 = \frac{1}{2}x$$

$$10 = \frac{1}{2}x + 2x$$

$$10 = \frac{5}{2}x$$

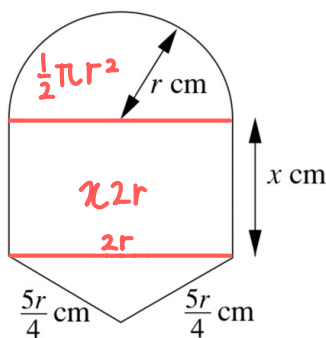
$$4 = x \quad D: (4, 2)$$

$$4 = x$$

$$y = \frac{1}{2}(4) \quad y = 2$$

12 Answer only **one** of the following two alternatives.

**EITHER**



A piece of wire, 125 cm long, is bent to form the shape shown in the diagram. This shape encloses a plane region, of area  $A \text{ cm}^2$ , consisting of a semi-circle of radius  $r \text{ cm}$  and an isosceles triangle having two equal sides of length  $\frac{5r}{4} \text{ cm}$ .

(i) Express  $x$  in terms of  $r$  and hence show that  $A = 125r - \frac{\pi r^2}{2} - \frac{7r^2}{4}$ . [6]

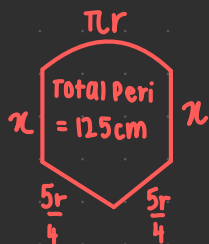
Given that  $r$  can vary,

(ii) calculate, to 1 decimal place, the value of  $r$  for which  $A$  has a maximum value. [4]

i.  $\pi r + 2x + \frac{5r}{2} = 125$

$2x = 125 - \frac{5r}{2} - \pi r$

$x = \frac{125}{2} - \frac{5r}{4} - \frac{\pi r}{2}$



ii. Differentiate the Area equation with respect to  $r$

$\frac{dA}{dr} = 125 - \pi r - \frac{7}{2}r$

To find the maximum, let

$\frac{dA}{dr} = 0$

$125 - \pi r - \frac{7}{2}r = 0$

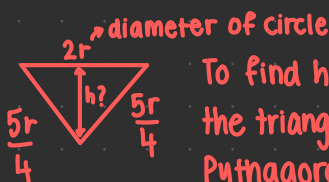
$125 = \pi r + \frac{7}{2}r$

$125 = r(\pi + \frac{7}{2})$

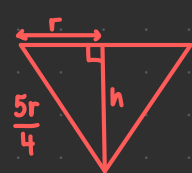
$\frac{125}{\pi + \frac{7}{2}} = r$

$r = 18.8 \text{ cm}$

Final Answer!



To find  $h$ , you can split the triangle in half to use Pythagoras theorem.



$h = \sqrt{(\frac{5r}{4})^2 - r^2}$

$h = \frac{3}{4}r$

Area of triangle:  $\frac{1}{2}bh$

$= \frac{1}{2}(2r)(\frac{3}{4}r)$

$= \frac{3}{4}r^2$

$A = \frac{1}{2}\pi r^2 + 2rx + \frac{3}{4}r^2$   
 Semicircle      rectangle      triangle

Substitute  $2x = 125 - \frac{5r}{2} - \pi r$  into rectangle area

$A = \frac{1}{2}\pi r^2 + (125 - \frac{5r}{2} - \pi r)r + \frac{3}{4}r^2$   
 $= \frac{1}{2}\pi r^2 + 125r - \frac{5r^2}{2} - \pi r^2 + \frac{3}{4}r^2$

$A = 125r - \frac{\pi r^2}{2} - \frac{7}{4}r^2$

Hence, Shown.

Final Answer!

- 8 A curve has the equation  $y = (ax + 3) \ln x$ , where  $x > 0$  and  $a$  is a positive constant. The normal to the curve at the point where the curve crosses the  $x$ -axis is parallel to the line  $5y + x = 2$ . Find the value of  $a$ . [7]

$$5y + x = 2$$

$$y = -\frac{1}{5}x + \frac{2}{5}$$

$$m_1 = -\frac{1}{5}$$

Line is perpendicular to normal

gradient of tangent =  $-\frac{1}{m_1}$

$$m_2 = 5$$

$$y = (ax + 3) \ln x$$

Crosses  $x$ -axis,  $y = 0$

$$0 = (ax + 3) \ln x$$

$$\ln x = 0$$

$$x = e^0$$

$$x = 1$$

to find gradient equation at  $x=1$ :

$$y = (ax + 3) \ln x$$

$$u = ax + 3 \quad v = \ln x$$

$$u' = a \quad v' = \frac{1}{x}$$

$$\frac{dy}{dx} \Big|_{x=1} = (ax + 3) \times \frac{1}{x} + a \times \ln x$$

$$\frac{dy}{dx} \Big|_{x=1} = \frac{ax + 3}{x} + a \ln x$$

$$= \frac{a(1) + 3}{(1)} + a \ln(1)$$

$$= a + 3 + a \ln(1)$$

$$= a + 3$$

↳ gradient at  $x=1$

gradient at tangent is parallel, so you can equate them to each other:

$$a + 3 = 5$$

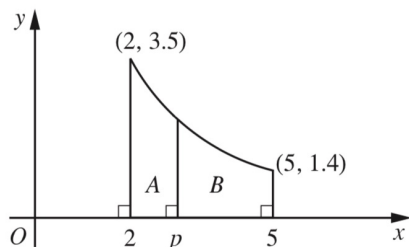
$$a = 5 - 3$$

$$a = 2 \#$$

↑ Final answer!



11



The diagram shows part of a curve, passing through the points (2, 3.5) and (5, 1.4). The gradient of the curve at any point (x, y) is  $-\frac{a}{x^3}$ , where a is a positive constant.

(i) Show that  $a = 20$  and obtain the equation of the curve. [5]

The diagram also shows lines perpendicular to the x-axis at  $x = 2$ ,  $x = p$  and  $x = 5$ . Given that the areas of the regions A and B are equal,

(ii) find the value of p. [5]



i. Gradient equal is differentiated version of y  
So, integrate  $-\frac{a}{x^3}$  to find what the equation of the line is

$$\frac{dy}{dx} = -\frac{a}{x^3}$$

$$y = \frac{a}{2x^2} + C$$

Substitute 2 points (2, 3.5) & (5, 1.4) to find C & a

$$-① \quad 3.5 = \frac{a}{2(2)^2} + C \quad -② \quad 1.4 = \frac{a}{2(5)^2} + C$$

$$3.5 = \frac{a}{8} + C$$

$$1.4 = \frac{a}{50} + C$$

$$3.5(8) = a + 8C$$

$$1.4(50) = a + 50C$$

$$28 = a + 8C$$

$$70 = a + 50C$$

$$a + 8C = 28$$

$$- \quad a + 50C = 70$$

$$\hline -42C = -42$$

$$C = 1 \#$$

$$28 = a + 8(1)$$

$$28 - 8 = a$$

$$a = 20 \#$$

$$y = \frac{20}{2x^2} + 1$$

$$y = \frac{10}{x^2} + 1$$

Final Answer!

ii. Integrate  $y = \frac{10}{x^2} + 1$  then use  $p-2$  &  $5-p$  as limits to find the value of p

$$\int_2^p (10x^{-2} - 1) dx = \int_p^5 (10x^{-2} + 1) dx$$

$$\left[-\frac{10}{x} + x\right]_2^p = \left[-\frac{10}{x} + x\right]_p^5$$

$$-\frac{10}{p} + p - (-5 + 2) = -2 + 5 - \left(-\frac{10}{p} + p\right)$$

$$-\frac{10}{p} + p + 5 - 2 = -2 + 5 + \frac{10}{p} - p$$

$$\frac{p^2 - 10}{p} + 3 = 3 + \frac{10 - p^2}{p}$$

$$p^2 - 10 = 10 - p^2$$

$$p^2 + p^2 = 10 + 10$$

$$2p^2 = 20$$

$$p^2 = 10$$

$$p = \sqrt{10} \#$$

Final Answer!

# 4037\_w04\_qp\_01

- 11 The line  $4y = 3x + 1$  intersects the curve  $xy = 28x - 27y$  at the point  $P(1, 1)$  and at the point  $Q$ . The perpendicular bisector of  $PQ$  intersects the line  $y = 4x$  at the point  $R$ . Calculate the area of triangle  $PQR$ . [9]

$$4y = 3x + 1$$

$$x = \frac{4y-1}{3} \rightarrow \text{Substitute into } xy = 28x - 27y$$

$$\left(\frac{4y-1}{3}\right)y = 28\left(\frac{4y-1}{3}\right) - 27y$$

$$\frac{4y^2 - y}{3} = \frac{112y - 28}{3} - 27y$$

$$\frac{4y^2 - y}{3} - \frac{112y - 28}{3} = -27y$$

$$\frac{4y^2 - y - 112y - 28}{3} = -27y$$

$$4y^2 - 113y - 28 = -27y \quad (3)$$

$$4y^2 - 113y + 81y + 28 = 0$$

$$4y^2 - 32y + 28 = 0$$

$$y_1 = 7 \quad y_2 = 1$$

$$x = \frac{4y-1}{3}$$

$$x_1 = 9 \quad x_2 = 1$$

$$P(1, 1) \quad Q(9, 7)$$

To Find the perpendicular bisector  
- Find gradient & midpoint

$$m_1 = \frac{3}{4} \rightsquigarrow m_2 = -\frac{4}{3}$$

$$\text{Midpoint} = (5, 4)$$

$$y = mx + c$$

$$m = -\frac{4}{3}$$

$$(5, 4)$$

$$4 = \left(-\frac{4}{3}\right)(5) + c$$

$$4 = -\frac{20}{3} + c$$

$$4 + \frac{20}{3} = c$$

$$c = \frac{32}{3}$$

$$y = -\frac{4}{3}x + \frac{32}{3}$$

$$3y = -4x + 32$$

$$3y + 4x = 32$$

$$y = 4x$$

$$3(4x) + 4x = 32$$

$$12x + 4x = 32$$

$$16x = 32$$

$$x = 2$$

$$y = 4(2)$$

$$y = 8$$

To Find the Area, Use the Shoe lace method:

$$P(1, 1) \quad Q(9, 7) \quad R(2, 8)$$

$$\text{Area} = \frac{1}{2} \left| \begin{array}{cc} 1 & 9 \\ 1 & 7 \\ 1 & 8 \end{array} \right|$$

$$\text{Area} = \frac{1}{2} | 7 + 72 + 2 - 9 - 14 - 8 |$$

$$\text{Area} = \frac{1}{2} | 50 |$$

$$\text{Area} = 25 \text{ units}^2$$

12 Answer only **one** of the following two alternatives.

**EITHER**

- (a) At the beginning of 1960, the number of animals of a certain species was estimated at 20 000. This number decreased so that, after a period of  $n$  years, the population was

$$20\,000e^{-0.05n}.$$

Estimate

- (i) the population at the beginning of 1970, [1]  
 (ii) the year in which the population would be expected to have first decreased to 2000. [3]
- (b) Solve the equation  $3^{x+1} - 2 = 8 \times 3^{x-1}$ . [6]

i.  $20000e^{-0.05(10)} = 12130$   $\hookrightarrow N=10$  because  $1970-1960=10$  year difference.

ii.  $20000e^{-0.05n} = 2000$

$$e^{-0.05n} = \frac{2000}{20000}$$

$$e^{-0.05n} = \frac{1}{10}$$

$$-0.05n = \ln\left(\frac{1}{10}\right)$$

$$n = \frac{(\ln \frac{1}{10})}{-0.05}$$

$$n = 46.1 \text{ years}$$

$$n = 47 \text{ years} - \text{rounded up!}$$

$$1960 + 47 = \boxed{2007}$$

$\hookrightarrow$  Final answer!

(b) Let  $3^x = y$

$$(3^x)(3^1) - 2 = 8 \times \frac{(3^x)}{(3^1)}$$

$$3y - 2 = 8 \times \frac{1}{3}y$$

$$3y - 2 = \frac{8}{3}y$$

$$-2 = \frac{8}{3}y - 3y$$

$$+2 = +\frac{1}{3}y$$

$$2 = \frac{1}{3}y$$

$$2 \div \frac{1}{3} = y$$

$$y = 6$$

$$3^x = 6 \quad \text{index} \rightarrow \log \text{ form}$$

$$\log_3(6) = x$$

# 4037\_s05\_qp\_01

A piece of wire, of length 2 m, is divided into two pieces. One piece is bent to form a square of side  $x$  m and the other is bent to form a circle of radius  $r$  m.

(i) Express  $r$  in terms of  $x$  and show that the total area,  $A$  m<sup>2</sup>, of the two shapes is given by

$$A = \frac{(\pi + 4)x^2 - 4x + 1}{\pi} \quad [4]$$

Given that  $x$  can vary, find

(ii) the stationary value of  $A$ , [4]

(iii) the nature of this stationary value. [2]

Find the perimeter & equal it to the Length

$$\underbrace{2\pi r}_{\text{Circle}} + \underbrace{4x}_{\text{Square}} = 2$$

$$2\pi r = 2 - 4x$$

$$r = \frac{2 - 4x}{2\pi} \text{ (simplify)}$$

$$r = \frac{1 - 2x}{\pi}$$

Area Formula :

$$\underbrace{\pi r^2}_{\text{circle}} + \underbrace{x^2}_{\text{square}}$$

-Substitute ( $r = \frac{1-2x}{\pi}$ )

$$\pi \left( \frac{1-2x}{\pi} \right)^2 + x^2$$

$$\pi \left( \frac{(1-2x)(1-2x)}{\pi^2} \right) + x^2$$

$$\pi \left( \frac{1+4x+4x^2}{\pi^2} \right) + x^2$$

$$\left[ \frac{1+4x+4x^2}{\pi} + x^2 \right] \text{ make denominator same}$$

$$\frac{1+4x+4x^2}{\pi} + \frac{x^2\pi}{\pi} = \frac{1+4x+(4+\pi)x^2}{\pi} \quad \#$$

ii. Differentiate  $A$  and equal it to 0.

$$\frac{dA}{dx} = \frac{(\pi+4)x^2 - 4x + 1}{\pi}$$

$$\frac{dA}{dx} = \frac{2(\pi+4)x - 4}{\pi}$$

$$\frac{2(\pi+4)x - 4}{\pi} = 0$$

$$2(\pi+4)x - 4 = 0$$

$$2(\pi+4)x = 4$$

$$2(\pi+4)x = 4$$

$$x = \frac{4}{2\pi+8}$$

$$x = \frac{2}{\pi+4}$$

Substitute into  $A$ .

$$A = \frac{(x+4)x^2 - 4x + 1}{\pi}$$

$$A = \frac{\left( \left( \frac{2}{\pi+4} \right) + 4 \right) \left( \frac{2}{\pi+4} \right)^2 - 4 \left( \frac{2}{\pi+4} \right) + 1}{\pi}$$

$$A = 0.14$$

iii) find  $\frac{d^2A}{dx^2}$

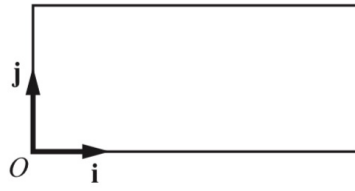
$$\frac{2(\pi+4)x - 4}{\pi}$$

$$\frac{d^2A}{dx^2} = \frac{2\pi+8}{\pi}$$

$\frac{2\pi+8}{\pi} > 0$ , hence minimum point

## 4037\_w05\_qp\_01

- 5 The diagram, which is not drawn to scale, shows a horizontal rectangular surface. One corner of the surface is taken as the origin  $O$  and  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors along the edges of the surface.



A fly,  $F$ , starts at the point with position vector  $(\mathbf{i} + 12\mathbf{j})$  cm and crawls across the surface with a velocity of  $(3\mathbf{i} + 2\mathbf{j})$  cm  $\text{s}^{-1}$ . At the instant that the fly starts crawling, a spider,  $S$ , at the point with position vector  $(85\mathbf{i} + 5\mathbf{j})$  cm, sets off across the surface with a velocity of  $(-5\mathbf{i} + k\mathbf{j})$  cm  $\text{s}^{-1}$ , where  $k$  is a constant. Given that the spider catches the fly, calculate the value of  $k$ . [6]

Find Position Vector of  $F$  &  $S$  :

$$R_F = (\hat{i} + 12\hat{j}) + t(3\hat{i} + 2\hat{j})$$

$$R_F = \hat{i} + 12\hat{j} + 3t\hat{i} + 2t\hat{j}$$

$$R_F = (3t+1)\hat{i} + (12+2t)\hat{j}$$

$$R_S = (85\hat{i} + 5\hat{j}) + t(-5\hat{i} + k\hat{j})$$

$$R_S = 85\hat{i} + 5\hat{j} - 5t\hat{i} + kt\hat{j}$$

$$R_S = (85-5t)\hat{i} + (5+kt)\hat{j}$$

$$R_F = R_S, \text{ Solve for } k \text{ \& } t$$

$$3t+1 = 85-5t \quad 5+kt = 12+2t$$

$$3t+5t = 85-1 \quad kt-2t = 12-5$$

$$8t = 84 \quad k(10.5) - 2(10.5) = 7$$

$$t = \frac{84}{8}$$

$$10.5k - 21 = 7$$

$$t = 10.5$$

$$k = \frac{7+21}{10.5}$$

$$k = 2\frac{2}{3} \quad \# \text{ Final Answer!}$$



# 4037\_w05\_qp\_01

- 9 (a) Find all the angles between  $0^\circ$  and  $360^\circ$  which satisfy the equation

$$3 \cos x = 8 \tan x. \quad [5]$$

- (b) Given that  $4 \leq y \leq 6$ , find the value of  $y$  for which

$$2 \cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0. \quad [3]$$

(a)  $3 \cos x = 8 \tan x.$

$$3 \cos x = 8 \left( \frac{\sin x}{\cos x} \right)$$

$$3 \cos^2 x = 8 \sin x$$

↑  
identities

$$3(1 - \sin^2 x) = 8 \sin x$$

$$3 \sin^2 x + 8 \sin x - 3 = 0$$

$$\sin x = \frac{1}{3}$$

$$x = \sin^{-1}\left(\frac{1}{3}\right)$$

$$x_1 = 19.47$$

$$x_2 = 180 - (19.47)$$

$$x_2 = 160.52$$

$$2 \cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0$$

$$2 \cos\left(\frac{2y}{3}\right) = -\sqrt{3}$$

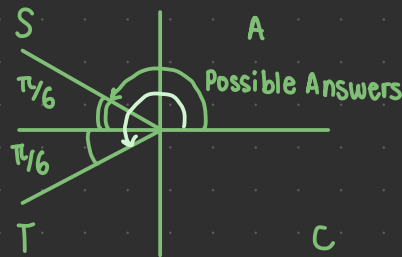
$$\cos\left(\frac{2y}{3}\right) = \frac{-\sqrt{3}}{2}$$

Basic angle:

$$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{6}$$

$$\cos\left(\frac{2y}{3}\right) = -\frac{\sqrt{3}}{2} \quad \therefore \text{S, T quadrants}$$



$$\frac{2y}{3} = \frac{5\pi}{6}$$

$$y = \frac{5\pi}{6} \div \frac{2}{3}$$

$$y = \frac{5}{4}\pi \quad [\text{REJECT}]$$

$$4 \leq y \leq 6$$

$$\frac{2y}{3} = \frac{7\pi}{6}$$

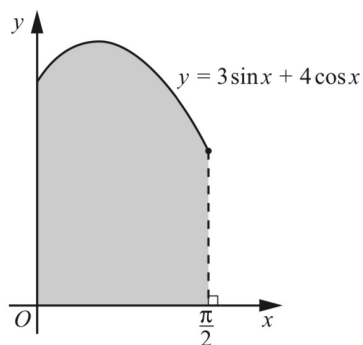
$$y = \frac{7\pi}{6} \div \frac{2}{3}$$

$$y = \frac{7\pi}{4} \quad \text{or} \quad 5.5$$

# 4037\_s07\_qp\_01

11 Answer only **one** of the following two alternatives.

**EITHER**



The graph shows part of the curve  $y = 3 \sin x + 4 \cos x$  for  $0 \leq x \leq \frac{\pi}{2}$  radians.

- (i) Find the coordinates of the maximum point of the curve. [5]  
 (ii) Find the area of the shaded region. [5]

i. differentiate the  $y$  equation and let it equal to 0.

$$\frac{dy}{dx} = 3 \cos x - 4 \sin x$$

$$3 \cos x - 4 \sin x = 0$$

$$3 \cos x = 4 \sin x$$

$$\frac{3}{4} = \frac{\sin x}{\cos x}$$

$$\frac{3}{4} = \tan x$$

$$x = \tan^{-1}\left(\frac{3}{4}\right)$$

$$x = 0.644$$

$$y = 3 \sin x + 4 \cos x$$

$$y = 3 \sin(0.644) + 4 \cos(0.644)$$

$$y = 4.99$$

$$y = 5.00$$

$$\text{maximum coordinate} = (0.644, 5.00)$$

ii. Integrate  $y$  equation with limits 0 to  $\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} 3 \sin x + 4 \cos x \, dx$$

$$= \left[ -3 \cos x + 4 \sin x \right]_0^{\frac{\pi}{2}}$$

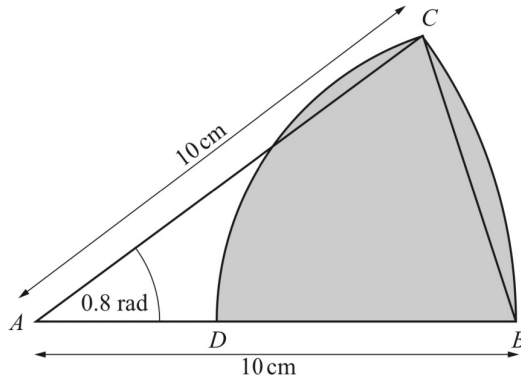
$$= \left( -3 \cos\left(\frac{\pi}{2}\right) + 4 \sin\left(\frac{\pi}{2}\right) \right) - \left( -3 \cos(0) + 4 \sin(0) \right)$$

$$= (0 + 4) - (-3 + 0)$$

$$= 4 + 3$$

$$= 7 \text{ units}^2$$

↑ Final Answer!



The diagram shows a sector  $ABC$  of the circle, centre  $A$  and radius  $10\text{ cm}$ , in which angle  $BAC = 0.8$  radians. The arc  $CD$  of a circle has centre  $B$  and the point  $D$  lies on  $AB$ .

- (i) Show that the length of the straight line  $BC$  is  $7.79\text{ cm}$ , correct to 2 decimal places. [2]  
 (ii) Find the perimeter of the shaded region. [4]  
 (iii) Find the area of the shaded region. [4]

i. Use the Cosine rule to find  $BC$ .

$$BC^2 = AB^2 + AC^2 - 2ABAC \cos(A)$$

$$BC = \sqrt{AB^2 + AC^2 - 2ABAC \cos(A)}$$

$$BC = \sqrt{(10)^2 + (10)^2 - 2(10 \times 10) \cos(0.8)}$$

[Solve in calculator in RADIAN MODE]

$$= \sqrt{100 + 100 - 200 \cos(0.8)}$$

$$= 7.7883$$

$$= 7.79$$

ii. Find arc length  $BC$

$$\begin{aligned} BC &= r\theta \\ &= 10(0.8) \\ &= 8 \end{aligned}$$

$BD = BC$  — radius of circle Centre point  $B$

$$BD = 7.79$$

$$DC = r\theta$$

$$r = 7.79$$

to find  $\theta$  use Sine rule!

$$\frac{\sin A}{a} = \frac{\sin \theta}{b} \quad \frac{\sin 0.8}{7.79} = \frac{\sin \theta}{10}$$

$$10 \left( \frac{\sin 0.8}{7.79} \right) = \sin \theta \quad \sin^{-1} \left( 10 \left( \frac{\sin 0.8}{7.79} \right) \right) = \theta$$

$$\theta = 1.17 \text{ rad.}$$

$$DC = r\theta$$

$$= (7.79)(1.17)$$

$$= 9.11$$

$$\text{Perimeter} = BC + DB + DC$$

$$= 8 + 7.79 + 9.11$$

$$= 24.9$$

iii. Area =

Sector  $DBC$  + (Sector  $CAB$  - triangle  $CAB$ )

$$\begin{aligned} \text{Sector } DBC &= \frac{1}{2}r^2\theta = \frac{1}{2}(7.79)^2(1.17) \\ &= 35.50 \end{aligned}$$

$$\begin{aligned} \text{Sector } CAB &= \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2(0.8) \\ &= 40 \end{aligned}$$

$$\begin{aligned} \text{Triangle } CAB &= \frac{1}{2}r^2 \sin \theta = \frac{1}{2}(10)^2 \sin 0.8 \\ &= 35.87 \end{aligned}$$

$$\begin{aligned} \text{Area} &= 35.50 + (40 - 35.87) \\ &= 39.6 \text{ unit}^2 \end{aligned}$$

# 4037\_s08\_qp\_01

10 In this question,  $\mathbf{i}$  is a unit vector due east and  $\mathbf{j}$  is a unit vector due north.

At 0900 hours a ship sails from the point  $P$  with position vector  $(2\mathbf{i} + 3\mathbf{j})$  km relative to an origin  $O$ . The ship sails north-east with a speed of  $15\sqrt{2}$  km  $\text{h}^{-1}$ .

(i) Find, in terms of  $\mathbf{i}$  and  $\mathbf{j}$ , the velocity of the ship. [2]

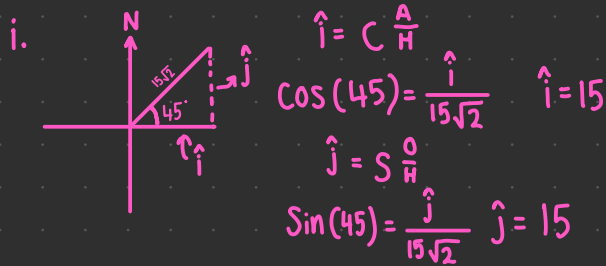
(ii) Show that the ship will be at the point with position vector  $(24.5\mathbf{i} + 25.5\mathbf{j})$  km at 1030 hours. [1]

(iii) Find, in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $t$ , the position of the ship  $t$  hours after leaving  $P$ . [2]

At the same time as the ship leaves  $P$ , a submarine leaves the point  $Q$  with position vector  $(47\mathbf{i} - 27\mathbf{j})$  km. The submarine proceeds with a speed of  $25$  km  $\text{h}^{-1}$  due north to meet the ship.

(iv) Find, in terms of  $\mathbf{i}$  and  $\mathbf{j}$ , the velocity of the ship relative to the submarine. [2]

(v) Find the position vector of the point where the submarine meets the ship. [2]



$$\vec{v} = 15\hat{i} + 15\hat{j}$$

ii. Position vector =  $a + tv$

$$r = a + tv$$

$$r = (2\hat{i} + 3\hat{j}) + (1.5)(15\hat{i} + 15\hat{j})$$

$$r = (2\hat{i} + 3\hat{j}) + (22.5\hat{i} + 22.5\hat{j})$$

$$r = 24.5\hat{i} + 25.5\hat{j}$$

iii. Position vector =  $a + tv$

$$r = a + tv$$

$$r = (2\hat{i} + 3\hat{j}) + (t)(15\hat{i} + 15\hat{j})$$

iv.

$$\vec{v}_{sub} = 25\hat{j}$$

$$\vec{v}_{ship} = 15\hat{i} + 15\hat{j}$$

Relative velocity =  $\vec{v}_{ship} - \vec{v}_{sub}$

$$= 15\hat{i} + 15\hat{j} - 25\hat{j}$$

$$= 15\hat{i} - 10\hat{j}$$

v. Let the position vector of ship & submarine equal to each other.

$$r_{ship} = (2\hat{i} + 3\hat{j}) + (t)(15\hat{i} + 15\hat{j})$$

$$r_{sub} = a + tv$$

$$= (47\hat{i} - 27\hat{j}) + t(25\hat{j})$$

$$2\hat{i} + 3\hat{j} + 15\hat{i}t + 15\hat{j}t = 47\hat{i} - 27\hat{j} + 25\hat{j}t$$

$$(2 + 15t)\hat{i} + (3 + 15t)\hat{j} = 47\hat{i} + (25t - 27)\hat{j}$$

$$2 + 15t = 47$$

$$15t = 47 - 2$$

$$15t = 45$$

$$t = \frac{45}{15}$$

$$t = 3$$

Substitute into position vector of either Ship or Submarine.

$$R = (47\hat{i} - 27\hat{j}) + 3(25\hat{j})$$

$$= (47\hat{i} - 27\hat{j}) + 75\hat{j}$$

$$= 47\hat{i} + 48\hat{j}$$

12 OR

A particle moves in a straight line such that its displacement,  $s$  m, from a fixed point  $O$  at a time  $t$  s, is given by

$$s = \ln(t+1) \quad \text{for } 0 \leq t \leq 3,$$

$$s = \frac{1}{2} \ln(t-2) - \ln(t+1) + \ln 16 \quad \text{for } t > 3.$$

Find

- (i) the initial velocity of the particle, [2]  
 (ii) the velocity of the particle when  $t = 4$ , [2]  
 (iii) the acceleration of the particle when  $t = 4$ , [2]  
 (iv) the value of  $t$  when the particle is instantaneously at rest, [2]  
 (v) the distance travelled by the particle in the 4th second. [2]

i. Differentiate  $S = \ln(t+1)$  and let  $t = 0$

$$\begin{aligned} \frac{ds}{dt} &= \frac{1}{t+1} \\ &= \frac{1}{0+1} \\ &= 1 \text{ m/s} \end{aligned}$$

ii. Differentiate  $S = \frac{1}{2} \ln(t-2) - \ln(t+1) + \ln 16$  and let  $t = 4$

$$\frac{ds}{dt} = \frac{1}{2} \left( \frac{1}{t-2} \right) - \left( \frac{1}{t+1} \right)$$

$$\frac{ds}{dt} = \frac{1}{2t-4} - \frac{1}{t+1}$$

Substitute  $t = 4$ 

$$\frac{1}{2(4)-4} - \frac{1}{4+1} = \frac{1}{20}$$

iii. 
$$V = \frac{1}{2t-4} - \frac{1}{t+1}$$

$$\frac{dv}{dt} = -1(2t-4)^{-2} \times 2 - (-1(t+1)^{-2})$$

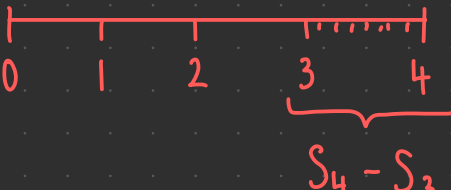
$$\frac{dv}{dt} = -2(2t-4)^{-2} + 1(t+1)^{-2}$$

$$\frac{dv}{dt} = \text{acceleration}$$

$$= -2(2(4)-4)^{-2} + 1(4+1)^{-2}$$

$$= -\frac{17}{200}$$

iv. 
$$\begin{aligned} \frac{1}{2t-4} - \frac{1}{t+1} &= 0 \\ \frac{1}{2t-4} &= \frac{1}{t+1} \\ t+1 &= 2t-4 \\ 2t-t &= 1+4 \\ t &= 5 \end{aligned}$$

v. 

$$S_3 = \ln(3+1) \quad S_3 = \ln(4)$$

$$S_4 = \frac{1}{2} \ln(4-2) - \ln(4+1) + \ln 16$$

$$S_4 = \frac{1}{2} \ln(2) - \ln(5) + \ln 16$$

$$S_4 = \ln \sqrt{2} - \ln 5 + \ln 16$$

$$S_4 = \ln \frac{\sqrt{2}}{5} \times 16$$

$$S_4 = \ln \frac{16\sqrt{2}}{5}$$

$$\begin{aligned} S_4 - S_3 &, \quad \ln \frac{16\sqrt{2}}{5} - \ln 4 \\ &= \ln \left( \frac{16\sqrt{2}}{5} \div 4 \right) \\ &= \ln \frac{4\sqrt{2}}{5} \end{aligned}$$



# 4037\_w08\_qp\_01

8 A curve is such that  $\frac{d^2y}{dx^2} = 4e^{-2x}$ . Given that  $\frac{dy}{dx} = 3$  when  $x = 0$  and that the curve passes through the point  $(2, e^{-4})$ , find the equation of the curve. [6]

#1: integrate  $\frac{d^2y}{dx^2}$  to find  $\frac{dy}{dx}$

$$\int 4e^{-2x} dx$$

$$= -2e^{-2x} + C \rightarrow \frac{dy}{dx}$$

$$-2e^{-2(0)} + C = 3$$

(Substitute  $x=0$   $\frac{dy}{dx}=3$ )

$$C = 3 + 2e^{-2(0)}$$

$$C = 5$$

$$\int -2e^{-2x} + 5 dx$$

$$= e^{-2x} + 5x + c$$

$$y = e^{-2x} + 5x + c$$

$$y = e^{-2x} + 5x + c$$

(Substitute  $(2, e^{-4})$ )

$$e^{-4} = e^{-2(2)} + 5(2) + c$$

$$e^{-4} - (e^{-2(2)} + 10) = c$$

$$-10 = c$$

$$y = e^{-2x} + 5x - 10 \#$$

Final Answer!!

12 Answer only **one** of the following two alternatives.

**EITHER**

A curve has equation  $y = \frac{x^2}{x+1}$ .

(i) Find the coordinates of the stationary points of the curve. [5]

The normal to the curve at the point where  $x = 1$  meets the  $x$ -axis at  $M$ . The tangent to the curve at the point where  $x = -2$  meets the  $y$ -axis at  $N$ .

(ii) Find the area of the triangle  $MNO$ , where  $O$  is the origin. [6]

(i) Stationary point:  $\frac{dy}{dx} = 0$

$$\frac{x^2}{x+1} \quad u = x^2 \quad v = x+1$$

$$u' = 2x \quad v' = 1$$

$$\frac{v u' - u v'}{v^2} = \frac{(x+1)(2x) - (x^2)(1)}{(x+1)(x+1)}$$

$$= \frac{2x^2 + 2x - x^2}{x^2 + 2x + 1}$$

$$\frac{dy}{dx} = \frac{x^2 + 2x}{x^2 + 2x + 1}$$

$$0 = \frac{x^2 + 2x}{x^2 + 2x + 1}$$

$$x^2 + 2x = 0$$

$$x = -2, x = 0 \quad y = 0 \quad y = -4$$

Substitute into curve equation for  $y$

$$(0, 0)$$

$$(-2, -4)$$

Final Answer!!

(ii) Find coordinate M:

$$\text{at } x=1, y = \frac{1}{2}$$

$$\frac{dy}{dx} \text{ where } x=1$$

$$= \frac{3}{4}$$

$$\text{gradient} = -\frac{4}{3}$$

$$y - \frac{1}{2} = -\frac{4}{3}(x-1)$$

$$y = -\frac{4}{3}x + \frac{11}{6}$$

at  $x$  axis so  $y=0$

$$0 = -\frac{4}{3}x + \frac{11}{6} \quad x = \frac{11}{8} \quad M: (\frac{11}{8}, 0)$$

Find coordinate of N:

$$x = -2, y = -4$$

$$\frac{dy}{dx} \text{ where } x = -2$$

$$\text{gradient} = 0, y = -4$$

at  $y$  axis so  $x=0$   $N: (0, -4)$

Shoelace method:

$\Delta MNO$

$$M: (\frac{11}{8}, 0)$$

$$N: (0, -4)$$

$$O: (0, 0)$$

$$M: (\frac{11}{8}, 0)$$

$$A = \frac{1}{2} \begin{vmatrix} \frac{11}{8} & 0 & \frac{11}{8} \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |(\frac{11}{8} \times -4)|$$

$$= \frac{1}{2} |-\frac{11}{2}|$$

$$= \frac{1}{2} (\frac{11}{2})$$

$$\text{Area} = \frac{11}{4} \text{ unit}^2$$

# 4037\_s09\_qp\_01

9 At 10 00 hours, a ship  $P$  leaves a point  $A$  with position vector  $(-4\mathbf{i} + 8\mathbf{j})$  km relative to an origin  $O$ , where  $\mathbf{i}$  is a unit vector due East and  $\mathbf{j}$  is a unit vector due North. The ship sails north-east with a speed of  $10\sqrt{2}$  km  $\text{h}^{-1}$ . Find

(i) the velocity vector of  $P$ , [2]

(ii) the position vector of  $P$  at 12 00 hours. [2]

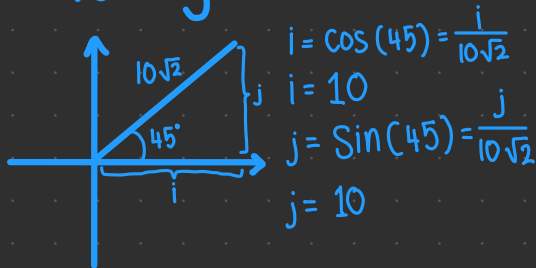
At 12 00 hours, a second ship  $Q$  leaves a point  $B$  with position vector  $(19\mathbf{i} + 34\mathbf{j})$  km travelling with velocity vector  $(8\mathbf{i} + 6\mathbf{j})$  km  $\text{h}^{-1}$ .

(iii) Find the velocity of  $P$  relative to  $Q$ . [REMOVED] [2]

(iv) Hence, or otherwise, find the time at which  $P$  and  $Q$  meet and the position vector of the point where this happens. [3]

(i) Speed:  $10\sqrt{2}$   
North East

Velocity vector:



Velocity vector =  $10\mathbf{i} + 10\mathbf{j}$

(ii) Position vector

$$R = a + tv$$

$$a: -4\mathbf{i} + 8\mathbf{j}$$

$$t: 10.00 \rightarrow 12.00$$

(2 hours)

$$v: 10\mathbf{i} + 10\mathbf{j}$$

$$R = -4\mathbf{i} + 8\mathbf{j} + 2(10\mathbf{i} + 10\mathbf{j})$$

$$= -4\mathbf{i} + 8\mathbf{j} + 20\mathbf{i} + 20\mathbf{j}$$

$$= 16\mathbf{i} + 28\mathbf{j}$$

$$(iv) r = a + tv$$

$$P: \text{at } 12:00 \quad Q: \text{at } 12:00$$

$$: 16\mathbf{i} + 28\mathbf{j} \quad : 19\mathbf{i} + 34\mathbf{j}$$

$$a + tv = a + tv$$

$$16\mathbf{i} + 28\mathbf{j} + t(10\mathbf{i} + 10\mathbf{j}) = 19\mathbf{i} + 34\mathbf{j} + t(8\mathbf{i} + 6\mathbf{j})$$

$$16\mathbf{i} + 28\mathbf{j} + 10t\mathbf{i} + 10t\mathbf{j} = 19\mathbf{i} + 8t\mathbf{i} + 34\mathbf{j} + 6t\mathbf{j}$$

$$(16 + 10t)\mathbf{i} + (28 + 10t)\mathbf{j} = (19 + 8t)\mathbf{i} + (34 + 6t)\mathbf{j}$$

$$16 + 10t = 19 + 8t \quad 28 + 10t = 34 + 6t$$

$$10t - 8t = 19 - 16$$

$$10t - 6t = 34 - 28$$

$$t = \frac{3}{2}$$

$$t = \frac{6}{4} \quad t = \frac{3}{2}$$

$$\frac{3}{2} = 1.5 \text{ h} \quad 12:00 + 1.5 = 13:30$$

$$r = a + tv$$

$$r = (19\mathbf{i} + 34\mathbf{j}) + \frac{3}{2}(8\mathbf{i} + 6\mathbf{j})$$

$$r = 19\mathbf{i} + 34\mathbf{j} + 12\mathbf{i} + 9\mathbf{j}$$

$$r = 31\mathbf{i} + 43\mathbf{j}$$

$$\text{Position Vector} = 31\mathbf{i} + 43\mathbf{j}$$

$$\text{Time of meet} = 13:30$$

↑ Final Answer!!

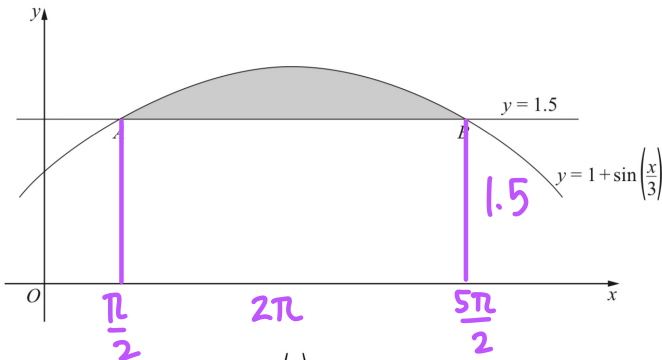
# 4037\_s09\_qp\_01

12 Answer only **one** of the following two alternatives.

**EITHER**

(i) State the amplitude of  $1 + \sin\left(\frac{x}{3}\right)$ . [1]

(ii) State, in radians, the period of  $1 + \sin\left(\frac{x}{3}\right)$ . [1]



The diagram shows the curve  $y = 1 + \sin\left(\frac{x}{3}\right)$  meeting the line  $y = 1.5$  at points  $A$  and  $B$ . Find

(iii) the  $x$ -coordinate of  $A$  and of  $B$ , [3]

(iv) the area of the shaded region. [6]

i.  $y = a \sin bx + c$   
where  $a = \text{amplitude}$

So amplitude = 1

ii. Period =  $\frac{2\pi}{b} = \frac{2\pi}{1/3}$

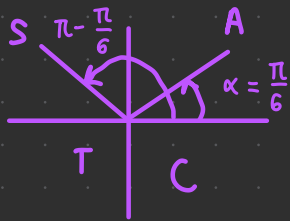
Period =  $6\pi$

ii Let  $y = 1 + \sin\frac{x}{3}$  equal to  $y = 1.5$

$$1 + \sin\frac{x}{3} = 1.5$$

$$\sin\frac{x}{3} = 1.5 - 1$$

$$\frac{x}{3} = \sin^{-1}(0.5)$$



$$\frac{x}{3} = \frac{\pi}{6} \quad \frac{x}{3} = \frac{5\pi}{6}$$

$$x = 3\left(\frac{\pi}{6}\right) \quad x = 3\left(\frac{5\pi}{6}\right)$$

$$x = \frac{3\pi}{6} \quad x = \frac{15\pi}{6}$$

$$x = \frac{\pi}{2} \quad x = \frac{5\pi}{2}$$

$$A = \frac{\pi}{2}$$

$$B = \frac{5\pi}{2}$$

Final Answer!!

iv) integrate  $1 + \sin\left(\frac{x}{3}\right)$  with  $A$  &  $B$  as limits to find area under the graph:

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} 1 + \sin\left(\frac{x}{3}\right)$$

$$\frac{x - 3\cos\left(\frac{x}{3}\right)}{\frac{1}{3}}$$

$$\left[ x - 3\cos\left(\frac{x}{3}\right) \right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$$

$$\frac{5\pi}{2} - 3\cos\frac{5\pi}{3} - \left( \frac{\pi}{2} - 3\cos\frac{\pi}{3} \right)$$

$$= 2\pi + 3\sqrt{3}$$

↑  
minus Area of rectangle ( $2\pi \times 1.5$ )

$$= 2\pi + 3\sqrt{3} - 3\pi$$

$$= 3\sqrt{3} - \pi$$

↑  
Final Answer!!

11 (a) Show that  $\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$ .

[3]

(b) Solve the equation

(i)  $\tan x = 3 \sin x$  for  $0^\circ < x < 360^\circ$ ,

[4]

(ii)  $2\cot^2 y + 3 \operatorname{cosec} y = 0$  for  $0 < y < 2\pi$  radians.

[5]

(a)  $\tan \theta + \cot \theta$

$$\tan \theta + \frac{1}{\tan \theta}$$
$$= \frac{\tan^2 \theta + 1}{\tan \theta}$$

$$\frac{\sec^2 \theta}{\tan \theta}$$

$$= \frac{1}{\cos^2 \theta} \div \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta \sin \theta}$$

$$\operatorname{cosec} \theta \sec \theta = \text{RHS!}$$

(b)  $\tan x = 3 \sin x$

$$\frac{\sin x}{\cos x} = 3 \sin x$$

$$\frac{\sin x}{\cos x} - 3 \sin x = 0$$

$$\sin \left( \frac{1}{\cos x} - 3 \right) = 0$$

$$\sin x = 0 \quad \frac{1}{\cos x} - 3 = 0$$

$$x = \sin^{-1}(0)$$

$$\cos x = \frac{1}{3}$$

$$x = \cancel{0}, 180, \cancel{360}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = 180^\circ$$

$$x = 70.5$$

$$x = 360 - 70.5$$

$$x = 70.5, 289.5$$

$$x = 70.5, 180, 289.5$$

↑ Final Answer !!

iii.)  $2\cot^2 y + 3 \operatorname{cosec} y = 0$

$$2(\operatorname{cosec}^2 y - 1) + 3 \operatorname{cosec} y = 0$$

$$2 \operatorname{cosec}^2 y - 2 + 3 \operatorname{cosec} y = 0$$

$$2 \operatorname{cosec}^2 y + 3 \operatorname{cosec} y - 2 = 0$$

$$\text{Let } \operatorname{Cosec} y = x$$

$$2x^2 + 3x - 2 = 0$$

$$x = \frac{1}{2} \quad x = -2$$

$$\operatorname{Cosec} y = \frac{1}{2} \quad \operatorname{cosec} y = -2$$

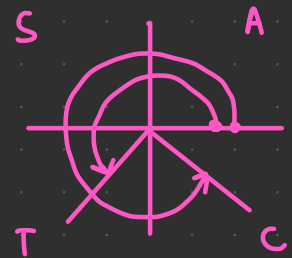
$$\frac{1}{\sin y} = \frac{1}{2}$$

$$\frac{1}{\sin y} = -2$$

$$\sin y = 2$$

$$\sin y = -\frac{1}{2}$$

[REJECT]



$$y = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$y = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$y = \frac{7\pi}{6}, \frac{11\pi}{6}$$

↑ Final Answer !!

# 4037\_s10\_qp\_11

11 Answer only **one** of the following two alternatives.

**EITHER**

A curve has equation  $y = \frac{\ln x}{x^2}$ , where  $x > 0$ .

- (i) Find the exact coordinates of the stationary point of the curve. [6]
- (ii) Show that  $\frac{d^2y}{dx^2}$  can be written in the form  $\frac{a \ln x + b}{x^4}$ , where  $a$  and  $b$  are integers. [3]
- (iii) Hence, or otherwise, determine the nature of the stationary point of the curve. [2]

i)  $\frac{dy}{dx} = 0 \implies \frac{v u' - u v'}{v^2} = 0$

$\frac{\ln x}{x^2} \implies \frac{x^2(\frac{1}{x}) - \ln x(2x)}{(x^2)^2} = 0$

$u = \ln x$   
 $u' = \frac{1}{x}$   
 $v = x^2$   
 $v' = 2x$

$x - 2x \ln x = 0$   
 $-2x \ln x = -x$

$\ln x = \frac{-x}{-2x}$   
 $\ln x = \frac{1}{2}$   
 $x = e^{\frac{1}{2}}$   
 $y = \frac{\ln(e^{\frac{1}{2}})}{(e^{\frac{1}{2}})^2}$   
 $y = \frac{1}{2e}$

Coordinate:  $(e^{\frac{1}{2}}, \frac{1}{2e})$   
 Final Answer!!

ii.)  $\frac{x^2(\frac{1}{x}) - \ln x(2x)}{(x^2)^2} = \frac{x - 2x \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3}$

$\frac{d}{dx} \left( \frac{1 - 2 \ln x}{x^3} \right)$

$u = 1 - 2 \ln x$   
 $u' = -\frac{2}{x}$   
 $v = x^3$   
 $v' = 3x^2$

$\frac{v u' - u v'}{v^2} = \frac{x^3(-\frac{2}{x}) - (1 - 2 \ln x)(3x^2)}{(x^3)^2}$

$= \frac{-2x^2 - 3x^2(1 - 2 \ln x)}{x^6} = \frac{x^2(-2 - 3(1 - 2 \ln x))}{x^6} = \frac{2 + 3 + 6 \ln x}{x^4} = \frac{6 \ln x + 5}{x^4}$

Final Answer!!

iii.  $\frac{d^2y}{dx^2} > 0$  minimum       $6 \ln x + 5 = 0$

$\frac{d^2y}{dx^2} < 0$  maximum       $6 \ln x = -5$

$\frac{6 \ln x + 5}{x^4} = 0$        $\ln x = -\frac{5}{6}$

$x = e^{-\frac{5}{6}}$        $e^{-\frac{5}{6}} < 0$ , maximum point!



# 4037\_s10\_qp\_11

OR

A curve is such that  $\frac{dy}{dx} = 6 \cos\left(2x + \frac{\pi}{2}\right)$  for  $-\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$ . The curve passes through the point  $\left(\frac{\pi}{4}, 5\right)$ .

Find

- (i) the equation of the curve, [4]
- (ii) the  $x$ -coordinates of the stationary points of the curve, [3]
- (iii) the equation of the normal to the curve at the point on the curve where  $x = \frac{3\pi}{4}$ . [4]

[NEXT PAGE]

i.) integrate  $\frac{dy}{dx}$  to  
find curve equation

$$\int 6 \cos\left(2x + \frac{\pi}{2}\right) dx$$

$$y = \frac{6 \sin\left(2x + \frac{\pi}{2}\right)}{2} + C$$

$$y = 3 \sin\left(2x + \frac{\pi}{2}\right) + C$$

[Substitute  $\left(\frac{\pi}{4}, 5\right)$ ]

$$5 = 3 \sin\left(2\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\right) + C$$

$$5 - 3 \sin\left(2\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\right) = C$$

$$C = 5$$

$$y = 3 \sin\left(2x + \frac{\pi}{2}\right) + 5 \quad \#$$

↑ Final Answer!!

ii.)  $\frac{dy}{dx} = 0$

$$6 \cos\left(2x + \frac{\pi}{2}\right) = 0$$

$$\cos\left(2x + \frac{\pi}{2}\right) = 0$$

$$\left(2x + \frac{\pi}{2}\right) = \cos^{-1}(0)$$



$$\cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$2x + \frac{\pi}{2} = \frac{\pi}{2}$$

$$2x = 0$$

$$x = 0$$

$$2x + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$2x = \frac{3\pi}{2} - \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$2x + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$2x = \frac{5\pi}{2} - \frac{\pi}{2}$$

$$x = \pi$$

(iii) the equation of the normal to the curve at the point on the curve where  $x = \frac{3\pi}{4}$ .

[4]

iii) Find gradient at  $x = \frac{3\pi}{4}$  by substituting into  $\frac{dy}{dx}$

$$6 \cos\left(2\left(\frac{3\pi}{4}\right) + \frac{\pi}{2}\right) = 6$$

$m_1 = 6 \rightarrow$  Gradient of tangent

gradient of normal =  $-\frac{1}{m_1}$

$$m_2 = -\frac{1}{6}$$

$$y = -\frac{1}{6}x + c$$

To get a set of coordinates, substitute  $x = \frac{3\pi}{4}$  into curve equation.

$$y = 3 \sin\left(2\left(\frac{3\pi}{4}\right) + \frac{\pi}{2}\right) + 5$$

$$y = 5$$

$$y = -\frac{1}{6}x + c \quad \left(\frac{3\pi}{4}, 5\right)$$

$$5 = -\frac{1}{6}\left(\frac{3\pi}{4}\right) + c$$

$$5 + \frac{1}{6}\left(\frac{3\pi}{4}\right) = c$$

$$c = 5 + \frac{\pi}{8}$$

$$y = -\frac{1}{6}x + 5 + \frac{\pi}{8}$$

$\uparrow$  Final Answer!!

# 4037\_s10\_qp\_12

11 A particle moves in a straight line such that its displacement,  $x$  m, from a fixed point  $O$  on the line at time  $t$  seconds is given by  $x = 12\{\ln(2t + 3)\}$ . Find

- (i) the value of  $t$  when the displacement of the particle from  $O$  is 48 m, [3]
- (ii) the velocity of the particle when  $t = 1$ , [3]
- (iii) the acceleration of the particle when  $t = 1$ . [3]

i.)  $48 = x$   
Solve for  $t$

$$48 = 12(\ln(2t+3))$$

$$\frac{48}{12} = \ln(2t+3)$$

$$e^4 = 2t+3$$

$$e^4 - 3 = 2t$$

$$t = \frac{e^4 - 3}{2} \rightarrow \text{Final Answer!!}$$

ii.)  $\frac{dx}{dt}$  and let  $t=1$

$$12\left(\frac{2}{2t+3}\right) \text{ and let } t=1$$

$$12\left(\frac{2}{2+3}\right) = 12\left(\frac{2}{5}\right)$$

$$= \frac{24}{5} \text{ m/s} \rightarrow \text{Final Answer!!}$$

ii.)  $\frac{d^2x}{dt^2}$  and let  $t=1$

$$12\left(\frac{2}{2t+3}\right)$$

$$\frac{d}{dt}\left(\frac{24}{2t+3}\right) 24(2t+3)^{-1}$$

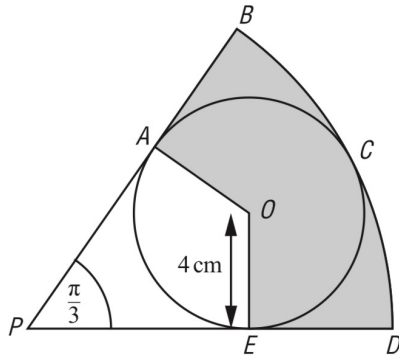
$$-24(2t+3)^{-2} (2)$$

$$= -48(2t+3)^{-2}$$

$$= -48(2+3)^{-2}$$

$\rightarrow$  Final Answer!!

$$= -\frac{48}{25} \text{ m/s}^2$$



The diagram shows a circle, centre  $O$ , radius 4 cm, enclosed within a sector  $PBCDP$  of a circle, centre  $P$ . The circle centre  $O$  touches the sector at points  $A$ ,  $C$  and  $E$ . Angle  $BPD$  is  $\frac{\pi}{3}$  radians.

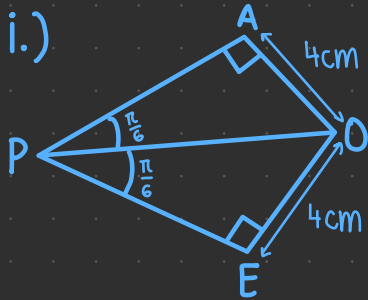
(i) Show that  $PA = 4\sqrt{3}$  cm and  $PB = 12$  cm. [2]

Find, to 1 decimal place,

(ii) the area of the shaded region, [4]

(iii) the perimeter of the shaded region. [4]

$\triangle POA \cong \triangle POE$   
↗



$$\tan\left(\frac{\pi}{6}\right) = \frac{4}{PA}$$

$$PA = \frac{4}{\tan\left(\frac{\pi}{6}\right)}$$

$$PA = 4\sqrt{3}$$

$PB = PC$  ↗ 4, radius

$PC = PO + OC$

$$\sin\left(\frac{\pi}{6}\right) = \frac{4}{PO}$$

$$PO = \frac{4}{\sin\left(\frac{\pi}{6}\right)}$$

$PO = 8$  cm

$PB = 4$  cm +  $8$  cm  
=  $12$  cm

ii.) Area of Sector BPD - 2 triangles

$$\begin{aligned} \text{Area of sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} (12)^2 \left(\frac{\pi}{3}\right) \\ &= 24\pi \end{aligned}$$

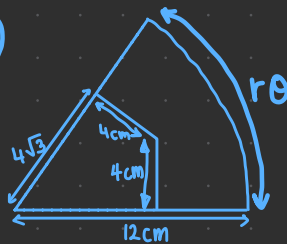
Area of triangle =  $\frac{1}{2} bh$

$$\begin{aligned} &= \frac{1}{2} (4\sqrt{3})(4) \\ &= 8\sqrt{3} \end{aligned}$$

$$24\pi - 2(8\sqrt{3}) = \boxed{47.69 \text{ cm}^2}$$

↗ Final Answer!!

iii.)



$$4 + 4 + 2(12 - 4\sqrt{3}) + 4\pi$$

$$8 + 24 - 8\sqrt{3} + 4\pi$$

$$= \boxed{30.7 \text{ cm}}$$

$$12\left(\frac{\pi}{3}\right) = 4\pi$$

↗ Final Answer!!

# 4037\_w10\_qp\_11

11 (i) Find  $\int \frac{1}{\sqrt{1+x}} dx$ . [2]

(ii) Given that  $y = \frac{2x}{\sqrt{1+x}}$ , show that  $\frac{dy}{dx} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{(\sqrt{1+x})^3}$ , where  $A$  and  $B$  are to be found. [4]

(iii) Hence find  $\int \frac{x}{(\sqrt{1+x})^3} dx$  and evaluate  $\int_0^3 \frac{x}{(\sqrt{1+x})^3} dx$ . [4]

i.)  $\int \frac{1}{\sqrt{1+x}} dx$   
 $\int (1+x)^{-\frac{1}{2}} dx$

$$= 2(1+x)^{\frac{1}{2}} + C$$

← Final Answer!!

ii.)  $\frac{d}{dx} \left( \frac{2x}{\sqrt{1+x}} \right)$

$$2x(1+x)^{-\frac{1}{2}}$$

$$u = 2x$$

$$uv' + vu'$$

$$u' = 2$$

$$v = (1+x)^{-\frac{1}{2}}$$

$$v' = -\frac{1}{2}(1+x)^{-\frac{3}{2}}$$

$$2x \left( -\frac{1}{2}(1+x)^{-\frac{3}{2}} \right) + (1+x)^{-\frac{1}{2}}(2)$$

$$-x(1+x)^{-\frac{3}{2}} + 2(1+x)^{-\frac{1}{2}}$$

$$= \frac{2}{(1+x)^{\frac{1}{2}}} - \frac{x}{(1+x)^{\frac{3}{2}}}$$

$$A = 2$$

$$B = -1$$

← Final Answer!!

iii.)  $\int \frac{2}{(1+x)^{\frac{1}{2}}} - \frac{x}{(1+x)^{\frac{3}{2}}} dx = \frac{2x}{\sqrt{1+x}}$

$$\int \frac{2}{(1+x)^{\frac{1}{2}}} dx - \int \frac{x}{(1+x)^{\frac{3}{2}}} dx = \frac{2x}{\sqrt{1+x}}$$

$$\int \frac{2}{(1+x)^{\frac{1}{2}}} dx - \frac{2x}{\sqrt{1+x}} = \int \frac{x}{(1+x)^{\frac{3}{2}}} dx$$

$$\int 2(1+x)^{-\frac{1}{2}} = 1(1+x)^{\frac{1}{2}}$$

$$\left[ 1(1+x)^{\frac{1}{2}} - \frac{2x}{\sqrt{1+x}} \right]_0^3$$

insert limits:

$$(1+3)^{\frac{1}{2}} - \frac{2(3)}{\sqrt{1+3}} - \left( (0+1)^{\frac{1}{2}} - \frac{2(0)}{\sqrt{0+1}} \right)$$

$$= 1$$

← Final Answer!!

12 OR

A curve has the equation  $y = Ae^{2x} + Be^{-x}$  where  $x \geq 0$ . At the point where  $x = 0$ ,  $y = 50$  and  $\frac{dy}{dx} = -20$ .

- (i) Show that  $A = 10$  and find the value of  $B$ . [5]
- (ii) Using the values of  $A$  and  $B$  found in part (i), find the coordinates of the stationary point on the curve. [4]
- (iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

i.) Substitute  $x=0$  &  $y=50$   
into  $y = Ae^{2x} + Be^{-x}$

$$50 = Ae^{2(0)} + Be^{-(0)}$$

$$A + B = 50 \text{ --- (1)}$$

$$\frac{dy}{dx} = -20$$

$$\frac{d}{dx}(Ae^{2x} + Be^{-x})$$

$$2Ae^{2x} - Be^{-x} \text{ when } x=0, \frac{dy}{dx} = -20$$

$$2Ae^{2(0)} - Be^{-(0)} = -20$$

$$2A - B = -20 \text{ --- (2)}$$

Solve (1) & (2) simultaneously

Elimination Method

$$\begin{array}{r} 2A - B = -20 \\ + \quad A - B = 50 \\ \hline \end{array}$$

$$3A = 30$$

$$A = 10$$

Substitute  $A = 10$  into  $A + B = 50$

$$10 + B = 50$$

$$B = 40$$

$$\text{ii.) } \frac{dy}{dx} = 0$$

$$2Ae^{2x} - Be^{-x} = 0$$

$$2(10)e^{2x} - 40e^{-x} = 0$$

$$20e^{2x} = 40e^{-x}$$

$$\frac{e^{2x}}{e^{-x}} = \frac{40}{20}$$

$$e^{3x} = 2$$

$$3x = \ln 2$$

$$x = \frac{\ln 2}{3}$$

To find  $y$ , substitute  $x$  into  $y$  equation

$$10e^{2(\frac{\ln 2}{3})} + 40e^{-(\frac{\ln 2}{3})} = y$$

$$y = 47.6$$

$$\boxed{\left(\frac{\ln 2}{3}, 47.6\right)} \rightarrow \text{Final Answer!!}$$

$$\text{iii.) } \frac{d}{dx}(20e^{2x} - 40e^{-x})$$

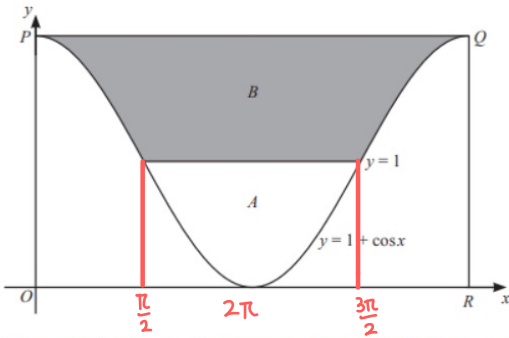
if  $\frac{d^2y}{dx^2} > 0$  minimum &  $< 0$ , maximum

$$40e^{2x} + 40e^{-x} > 0 \text{ [minimum]}$$



# 4037\_S22\_qp\_22

11



The diagram shows part of the line  $y = 1$  and one complete period of the curve  $y = 1 + \cos x$ , where  $x$  is in radians. The line  $PQ$  is a tangent to the curve at  $P$  and at  $Q$ . The line  $QR$  is parallel to the  $y$ -axis. Area  $A$  is enclosed by the line  $y = 1$  and the curve. Area  $B$  is enclosed by the line  $y = 1$ , the line  $PQ$  and the curve.

Given that area  $A$  : area  $B$  is  $1 : k$  find the exact value of  $k$ .

[9]

① Find area  $PQOR = 2 \times 2\pi = 4\pi$

④ Find  $B$

② Area under the graph

$A + B = 2\pi$

$$= \int_0^{2\pi} 1 + \cos x \, dx$$

$A = 2$

$B = 2\pi - 2$

$$= [x + \sin x]_0^{2\pi}$$

⑤ Write  $A : B$  as ratio & simplify

$$= 2\pi + \sin(2\pi) - (0 + \sin(0))$$

$$(2 : 2\pi - 2) \div 2$$

$$= 2\pi$$

$$1 : \pi - 1$$

③ Find  $A$  &  $B$

$$k = \pi - 1$$

$$A + B = 4\pi - 2\pi$$

$$= 2\pi$$

Final Answer!!

Let  $1 + \cos x$  equal to 0 to find intersects ( $\frac{3\pi}{2}$  &  $\frac{\pi}{2}$ )

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 + \cos x$$

$$[x + \sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

Area of Rectangle

$$\frac{3\pi}{2} + \sin \frac{3\pi}{2} - \left( \frac{\pi}{2} + \sin \frac{\pi}{2} \right) = \pi - 2$$

Area of  $A = \pi - (\pi - 2)$

$A = 2$

6 The points  $A(5, -4)$  and  $C(11, 6)$  are such that  $AC$  is the diagonal of a square,  $ABCD$ .

(a) Find the length of the line  $AC$ .

[2]

$$AC = \sqrt{(5-11)^2 + (-4-6)^2}$$

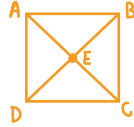
$$AC = 2\sqrt{74} \#$$

(b) (i) The coordinates of the centre,  $E$ , of the square are  $(8, y)$ . Find the value of  $y$ .

[1]

$$\text{Midpoint } AC : \left( \frac{5+11}{2}, \frac{-4+6}{2} \right)$$

$$= (8, 1) \quad y = 1 \#$$



(ii) Find the equation of the diagonal  $BD$ .

[3]

ii.)  $BD =$  perpendicular bisector of  $AC$

$$\text{gradient } AC : \frac{-4-6}{5-11} \quad \text{midpoint } AC [8, 1]$$

$$: \frac{5}{3}$$

$$\text{Perpendicular gradient} = -\frac{1}{\frac{5}{3}} = -\frac{3}{5}$$

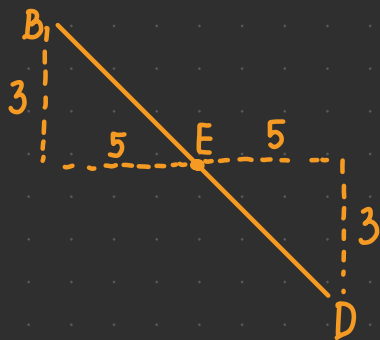
↗ Final Answer!!

$$BD : y - 1 = -\frac{3}{5}(x - 8) \quad BD : y = -\frac{3}{5}x + \frac{24}{5} + 1$$

$$y = -\frac{3}{5}x + \frac{29}{5}$$

(iii) Given that the  $x$ -coordinate of  $B$  is less than the  $x$ -coordinate of  $D$ , write  $\vec{EB}$  and  $\vec{ED}$  as column vectors.

[2]



use gradient to see translation  $-\frac{3}{5}$

$$\vec{EB} : \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

$$\vec{ED} : \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

13 The functions  $f$  and  $g$  are defined, for  $x > 0$ , by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(b) (i) Given that  $f^{-1}$  exists, write down the range of  $f^{-1}$ . [1]

(ii) Show that  $f^{-1}(x) = \frac{px + \sqrt{qx^2 + r}}{4}$ , where  $p, q$  and  $r$  are integers. [4]

b i.) range of  $f^{-1}(x) = \text{domain of } f(x)$

$$\boxed{f^{-1}(x) > 0} \rightarrow \text{Final Answer!!}$$

$$\text{b ii.) } \frac{2x^2 - 1}{3x} = y$$

$$2x^2 - 1 = 3xy$$

$$2x^2 - 3xy - 1 = 0$$

$$a = 2, \quad b = -3y, \quad c = -1$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3y \pm \sqrt{9y^2 + 8}}{4} \quad (f^{-1}(x) > 0)$$

$$\boxed{f^{-1}(x) = \frac{3x + \sqrt{9x^2 + 8}}{4}}$$

↑ Final Answer!!