Differentiation and integration – 2020 O Level Additional Math

1. Nov/2020/Paper_12/No.7

A curve has equation $y = \dfrac{\ln(3x^2 - 5)}{2x + 1}$ for $3x^2 > 5$.

(a) Find the equation of the normal to the curve at the point where $x = \sqrt{2}$. [6]

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2} + h$, where h is small. [1]

2. Nov/2020/Paper_12/No.9

A curve has equation $y = (2x-1)\sqrt{4x+3}$.

(a) Show that $\frac{dy}{dx} = \frac{4(Ax+B)}{\sqrt{4x+3}}$, where A and B are constants. [5]

(b) Hence write down the x-coordinate of the stationary point of the curve. [1]

(c) Determine the nature of this stationary point. [2]

3. Nov/2020/Paper_12/No.12b

(b) The acceleration, a ms^{-2}, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When $t = 0$ the particle is at point O and is travelling with a velocity of $10\,\text{ms}^{-1}$.

(i) Find the velocity of Q at time t. [3]

(ii) Find the displacement of Q from O at time t. [3]

4. Nov/2020/Paper_13/No.2

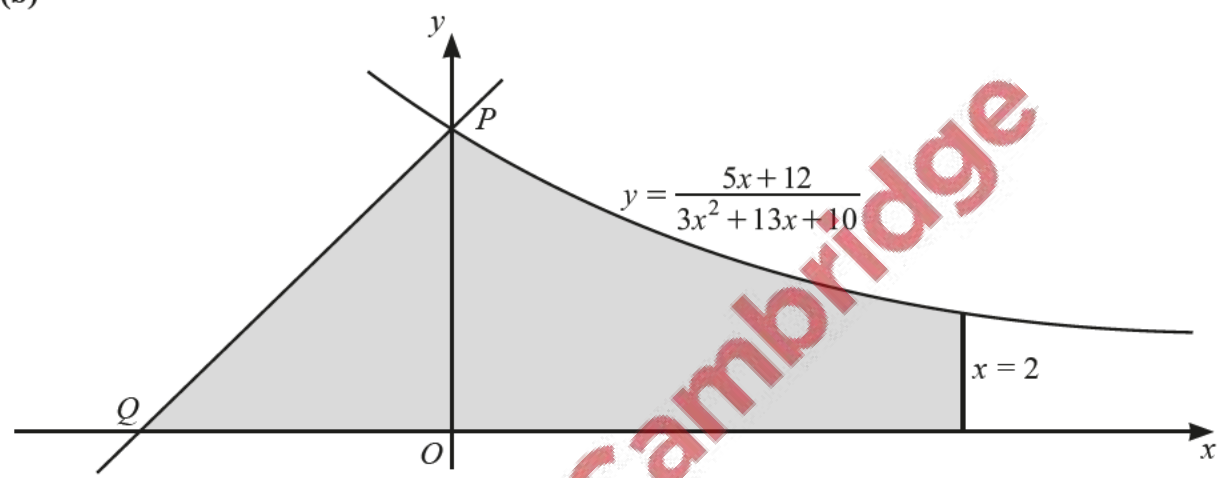
(a) Given that $y = \frac{e^{2x-3}}{x^2+1}$, find $\frac{dy}{dx}$. [3]

(b) Hence, given that y is increasing at the rate of 2 units per second, find the exact rate of change of x when $x = 2$. [3]

5. Nov/2020/Paper_13/No.10

(a) Show that $\frac{1}{x+1}+\frac{2}{3x+10}$ can be written as $\frac{5x+12}{3x^2+13x+10}$. [1]

(b)



The diagram shows part of the curve $y=\frac{5x+12}{3x^2+13x+10}$, the line $x=2$ and a straight line of gradient 1. The curve intersects the y-axis at the point P. The line of gradient 1 passes through P and intersects the x-axis at the point Q. Find the area of the shaded region, giving your answer in the form $a+\frac{2}{3}\ln\left(b\sqrt{3}\right)$, where a and b are constants. [9]

6. Nov/2020/Paper_22/No.5

DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$. [4]

(b) Find the coordinates of the point where this tangent meets the curve again. [5]

7. Nov/2020/Paper_22/No.6

Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$. [6]

8. Nov/2020/Paper_23/No.4

It is given that $y = \ln(1 + \sin x)$ for $0 < x < \pi$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$, giving your answer in the form $\frac{1}{\sqrt{a}}$, where a is an integer. [2]

(c) Find the values of x for which $\frac{dy}{dx} = \tan x$. [5]

9. Nov/2020/Paper_23/No.7

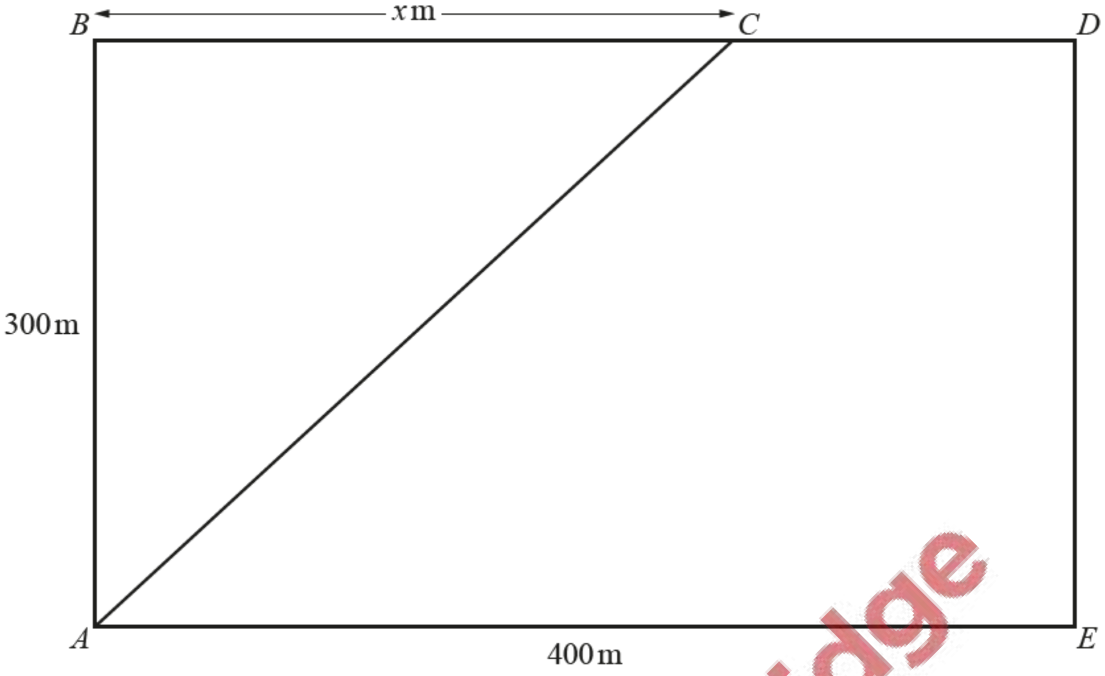
A curve has equation $y = x\cos x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the normal to the curve at the point where $x = \pi$, giving your answer in the form $y = mx + c$. [4]

(c) Using your answer to **part (a)**, find the exact value of $\int_0^{\frac{\pi}{6}} x\sin x \, dx$. [5]

10. Nov/2020/Paper_23/No.9



The rectangle $ABCDE$ represents a ploughed field where $AB = 300\,\text{m}$ and $AE = 400\,\text{m}$. Joseph needs to walk from A to D in the least possible time. He can walk at $0.9\,\text{ms}^{-1}$ on the ploughed field and at $1.5\,\text{ms}^{-1}$ on any part of the path BCD along the edge of the field. He walks from A to C and then from C to D. The distance $BC = x\,\text{m}$.

(a) Find, in terms of x, the total time, T s, Joseph takes for the journey. [3]

(b) Given that x can vary, find the value of x for which T is a minimum and hence find the minimum value of T. [6]

11. June/2020/Paper_11/No.5

Find the equation of the tangent to the curve $y = \frac{\ln(3x^2 - 1)}{x+2}$ at the point where $x = 1$. Give your answer in the form $y = mx + c$, where m and c are constants correct to 3 decimal places. [6]

12. June/2020/Paper_11/No.8b,8c

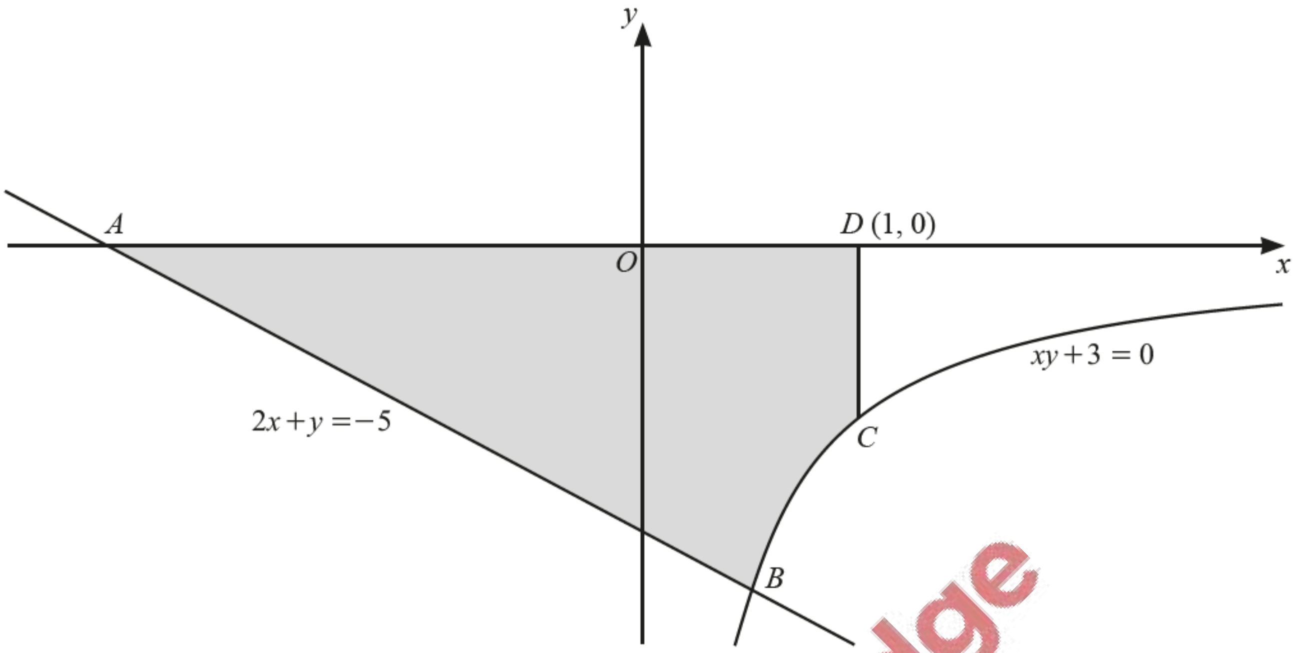
(b) Hence find $\int \frac{12x}{4x^2-9}\,dx$, giving your answer as a single logarithm and an arbitrary constant. [3]

(c) Given that $\int_2^a \frac{12x}{4x^2-9}\,dx = \ln 5\sqrt{5}$, where $a > 2$, find the exact value of a. [4]

13. June/2020/Paper_11/No.11

A curve is such that $\frac{d^2y}{dx^2} = 5\cos 2x$. This curve has a gradient of $\frac{3}{4}$ at the point $\left(-\frac{\pi}{12}, \frac{5\pi}{4}\right)$. Find the equation of this curve. [8]

14. June/2020/Paper_12/No.6



The diagram shows the straight line $2x + y = -5$ and part of the curve $xy + 3 = 0$. The straight line intersects the x-axis at the point A and intersects the curve at the point B. The point C lies on the curve. The point D has coordinates $(1, 0)$. The line CD is parallel to the y-axis.

(a) Find the coordinates of each of the points A and B. [3]

(b) Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers. [6]

15. June/2020/Paper_12/No.7

(a) Given that $y = (x^2 - 1)\sqrt{5x+2}$, show that $\frac{dy}{dx} = \frac{Ax^2 + Bx + C}{2\sqrt{5x+2}}$, where A, B and C are integers. [5]

(b) Find the coordinates of the stationary point of the curve $y = (x^2 - 1)\sqrt{5x+2}$, for $x > 0$. Give each coordinate correct to 2 significant figures. [3]

(c) Determine the nature of this stationary point. [2]

16. June/2020/Paper_12/No.9

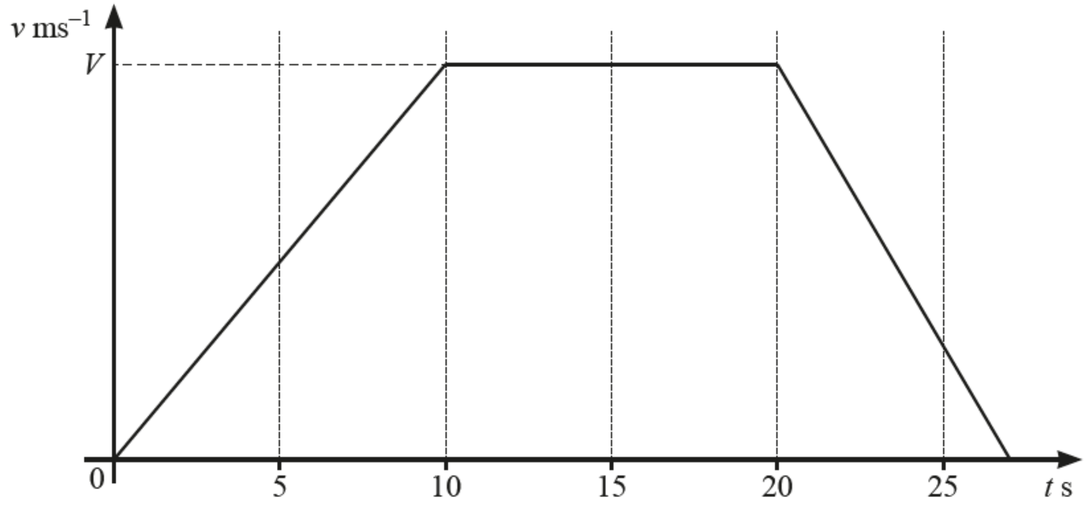
(a) A particle P moves in a straight line such that its displacement, x m, from a fixed point O at time t s is given by $x = 10\sin 2t - 5$.

(i) Find the speed of P when $t = \pi$. [1]

(ii) Find the value of t for which P is first at rest. [2]

(iii) Find the acceleration of P when it is first at rest. [2]

(b)



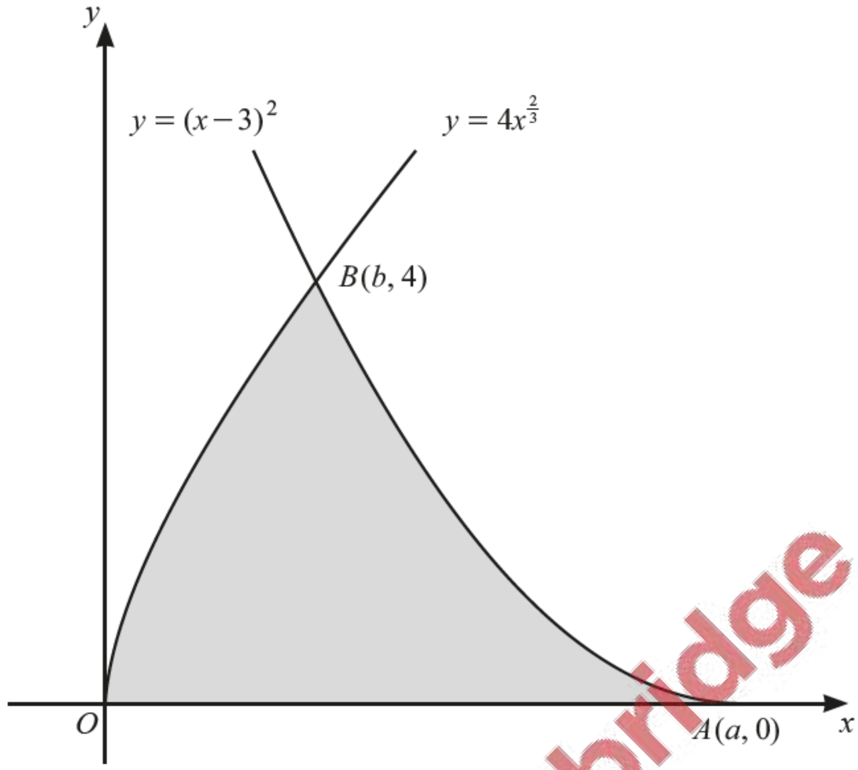
The diagram shows the velocity–time graph for a particle Q travelling in a straight line with velocity $v\,\text{ms}^{-1}$ at time $t\,\text{s}$. The particle accelerates at $3.5\,\text{ms}^{-2}$ for the first $10\,\text{s}$ of its motion and then travels at constant velocity, $V\,\text{ms}^{-1}$, for $10\,\text{s}$. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval $20 \leqslant t \leqslant 25$ is $112.5\,\text{m}$.

(i) Find the value of V. [1]

(ii) Find the velocity of Q when $t = 25$. [3]

(iii) Find the value of t when Q comes to rest. [3]

17. June/2020/Paper_21/No.10



The diagram shows part of the graphs of $y = 4x^{\frac{2}{3}}$ and $y = (x-3)^2$. The graph of $y = (x-3)^2$ meets the x-axis at the point $A(a, 0)$ and the two graphs intersect at the point $B(b, 4)$.

(a) Find the value of a and of b. [2]

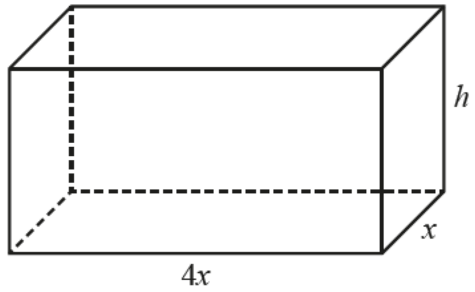
(b) Find the area of the shaded region. [5]

18. June/2020/Paper_21/No.12

(a) Find the x-coordinates of the stationary points of the curve $y = e^{3x}(2x+3)^6$. [6]

(b) A curve has equation $y = f(x)$ and has exactly two stationary points. Given that $f''(x) = 4x - 7$, $f'(0.5) = 0$ and $f'(3) = 0$, use the second derivative test to determine the nature of each of the stationary points of this curve. [2]

(c) In this question all lengths are in centimetres.



The diagram shows a solid cuboid with height h and a rectangular base measuring $4x$ by x. The volume of the cuboid is $40\,\text{cm}^3$. Given that x and h can vary and that the surface area of the cuboid has a minimum value, find this value. [5]

19. June/2020/Paper_22/No.7

Giving your answer in its simplest form, find the exact value of

(a) $\int_0^4 \frac{10}{5x+2}\mathrm{d}x$, [4]

(b) $\int_0^{\ln 2} \left(e^{4x+2}\right)^2 \mathrm{d}x$. [5]