Factors of polynomials – 2020 O Level Additional Math

1.

The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $x-2$.

(a) Given that $p(1) = -2p(0)$, find the value of a and of b. [4]

(b) Using your values of a and b,

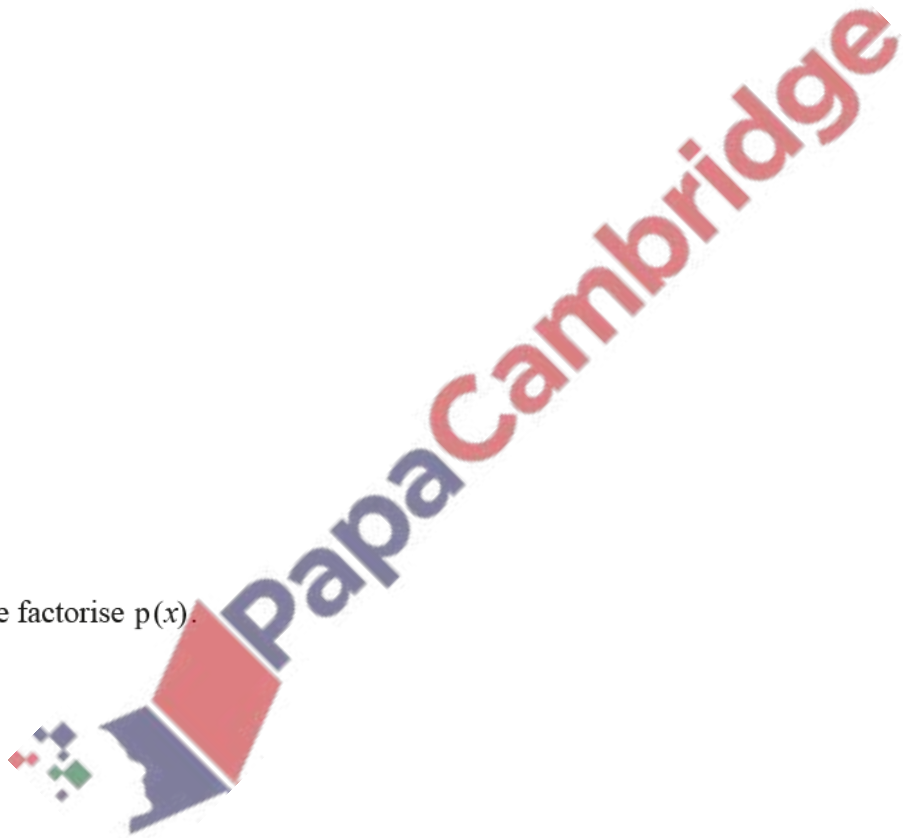(i) find the remainder when $p(x)$ is divided by $2x - 1$, [2]

(ii) factorise $p(x)$. [2]

The polynomial $p(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x + 4$ and is such that $2p(1) = 5p(0)$.

(a) Show that $p(x) = (x+4)(Ax^2 + Bx + C)$, where A, B and C are integers to be found. [6]

(b) Hence factorise $p(x)$. [1]

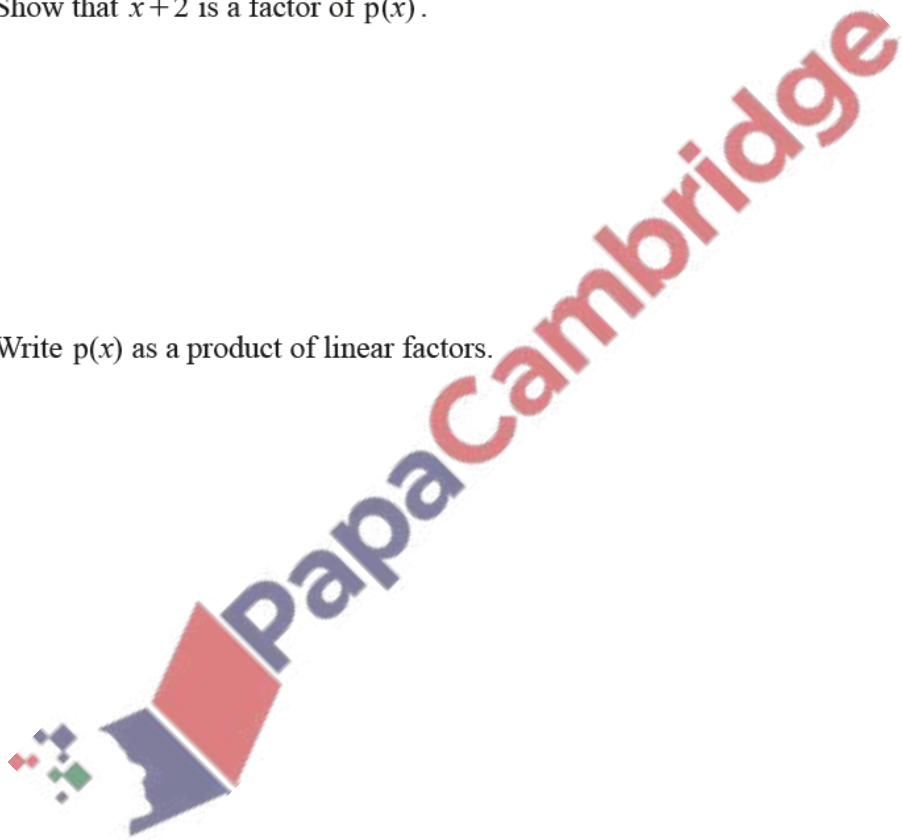**(c)** Find the remainder when $p'(x)$ is divided by x. [1]

DO NOT USE A CALCULATOR IN THIS QUESTION.

$$p(x) = 15x^3 + 22x^2 - 15x + 2$$

(a) Find the remainder when $p(x)$ is divided by $x+1$. [2]

(b) (i) Show that $x+2$ is a factor of $p(x)$. [1]

(ii) Write $p(x)$ as a product of linear factors. [3]

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4.

The three roots of $p(x) = 0$, where $p(x) = 2x^3 + ax^2 + bx + c$ are $x = \frac{1}{2}$, $x = n$ and $x = -n$, where a, b, c and n are integers. The y-intercept of the graph of $y = p(x)$ is 4. Find $p(x)$, simplifying your coefficients. [5]