Differentiation and integration – 2022 O Level Additional Math

1. June/2022/Paper_11/No.2

A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time t seconds, $t \geqslant 0$, is given by $s = (1+3t)^{-\frac{1}{2}}$.

(a) Find the exact speed of the particle when $t = 1$. [3]

(b) Show that the acceleration of the particle will never be zero. [2]

2. June/2022/Paper_11/No.10

The normal to the curve $y = \tan\left(3x + \frac{\pi}{2}\right)$ at the point P with coordinates $(p, -1)$, where $0 < p \leqslant \frac{\pi}{6}$, meets the x-axis at the point A and the y-axis at the point B. Find the exact coordinates of the mid-point of AB. [10]

3. June/2022/Paper_12/No.7(b)

(b) Find $\int_2^a \frac{8-3x}{(x-1)^2(2x+3)}\mathrm{d}x$ where $a > 2$. Give your answers in the form $c+\ln d$, where c and d are functions of a. [6]

4. June/2022/Paper_12/No.9

The normal to the curve $y = \dfrac{\ln(3x^2+2)}{x+1}$, at the point A on the curve where $x = 0$, meets the x-axis at point B. Point C has coordinates $(0, 3\ln 2)$. Find the gradient of the line BC in terms of $\ln 2$. [9]

5. June/2022/Paper_21/No.7

Variables x and y are such that $y = \dfrac{(1+\sin 3x)^4}{\sqrt{x}}$. Use differentiation to find the approximate change in y when x increases from 1.9 to $1.9+h$, where h is small. [6]

6. June/2022/Paper_21/No.10

(a) Differentiate $x\ln x - 2x$ with respect to x. Simplify your answer. [2]

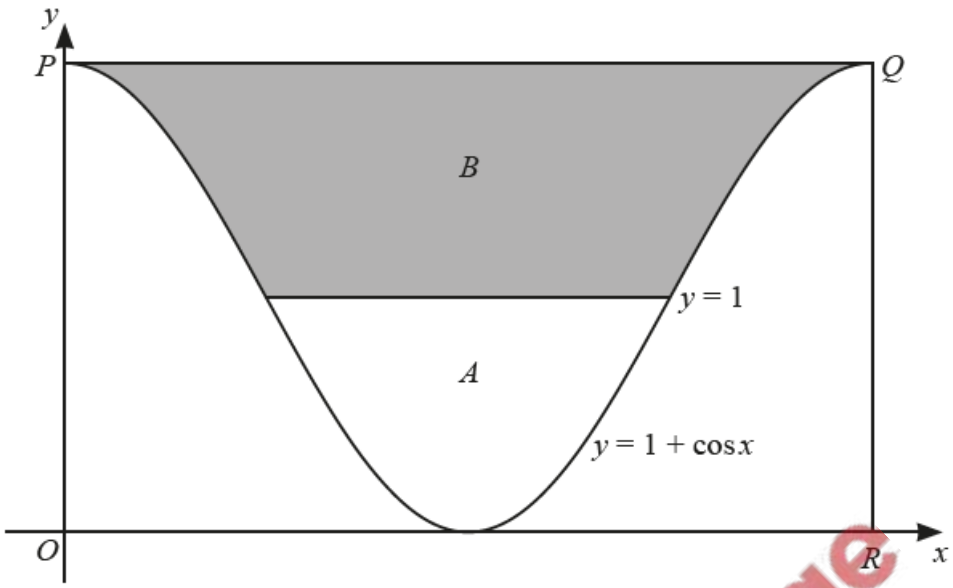
(b) A curve is such that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\dfrac{x+1}{\sqrt{x}}\right)^2$. It is given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{e}^2}{2} + 2\mathrm{e}$ at the point $\left(\mathrm{e}, \dfrac{\mathrm{e}^3}{6} + \mathrm{e}^2\right)$.

Using your answer to **part (a)**, find the exact equation of the curve. [8]

7. June/2022/Paper_22/No.7

Differentiate $y = \dfrac{e^{4x}\tan x}{\ln x}$ with respect to x. [4]

8. June/2022/Paper_22/No.11



The diagram shows part of the line $y = 1$ and one complete period of the curve $y = 1 + \cos x$, where x is in radians. The line PQ is a tangent to the curve at P and at Q. The line QR is parallel to the y-axis. Area A is enclosed by the line $y = 1$ and the curve. Area B is enclosed by the line $y = 1$, the line PQ and the curve.

Given that area A : area B is $1 : k$ find the exact value of k. [9]

9. June/2022/Paper_22/No.11

A curve is such that $\frac{d^2y}{dx^2} = \left(\frac{\sqrt{x}+1}{\sqrt[4]{x}}\right)^2$. Given that the gradient of the curve is $\frac{4}{3}$ at the point $(1, -1)$, find the equation of the curve. [7]