## MATHEMATICS SYLLABUS D

## Paper 4024/11

Paper 11

## Key messages

To do well in this paper, candidates need to

- be familiar with all the syllabus content
- be able to carry out basic calculations without a calculator
- understand and use correct mathematical terminology
- draw and interpret graphs and diagrams
- apply mathematical techniques to solve problems
- set out their work in clear, logical steps.


## General comments

Many candidates presented their work well with workings set out legibly and answers clearly stated on the answer line. Most used the appropriate geometrical instruments correctly to draw and take measurements from diagrams.

Many candidates demonstrated sound basic arithmetic skills, although errors were seen when working with decimals or working with numbers written in standard form. Candidates often demonstrated good algebraic skills, although care should be taken when forming equations in the context of graphs or vectors.

Candidates should be encouraged to recognise the difference between expressions and equations. For example, candidates attempted to solve expressions by adding an ' $=0$ ' to the end of the expression.

Candidates should take care to present work carefully and should ensure that numbers such as 1,4 and 7 can be clearly distinguished. They should cross out and replace work if they have made errors rather than overwriting as this cannot be read clearly. They should show their method for a question in the working space for that question: if work continues elsewhere, this should be referred to in the working space for the question and the additional work must be clearly labelled with the relevant question number.

## Comments on specific questions

## Question 1

(a) Many good responses were seen to this part. One less successful approach seen was the multiplication of the numerator and denominator by 10; often no further working was given in these cases beyond the initial step.
(b) Again, many good responses were seen to this part. Some incorrect answers were 28, 32 and 2400.

## Question 2

(a) Many candidates wrote down the correct area shaded. A variety of incorrect answers were seen including

$$
\frac{1}{9}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3} \text { and } \frac{3}{2} .
$$

(b) This part was also usually well done. Incorrect answers included 0.1 and 2.5.

## Question 3

(a) Many correct answers were seen. Some responses did not include the right angle and some developed an equation with the angles round a point summing to $180^{\circ}$. Some errors in adding up the angles resulted in wrong answers, for example $120+90+40=350$ leading to $x=10$.
(b) Many correct answers were seen. Most responses stated that an equilateral triangle involved angles of $60^{\circ}$ but then gave 60 as the answer for $y$. A few candidates gave the answer 90 .

## Question 4

(a) (i) Many correct expressions for Maryam's age were seen. The most common incorrect response was $t>5$.
(ii) Most correct expressions for Colin's age were seen. The most common incorrect response was $t^{2}$.
(b) Most responses correctly evaluated this expression. A few responses gave the step $15-4$, leading to the answer 11.

## Question 5

(a) Many good responses were seen to this part. A few gave the answer $3+(5 \times 2)-7=9$.
(b) Most candidates answered this correctly, using all three symbols to make the statement correct.

## Question 6

(a) Many candidates completed the pattern correctly. In some cases, an extra triangle at the bottom was shown on the pattern. In other cases, responses showed a vertical or horizontal line of symmetry. In a minority of responses, attempts at rotational symmetry about the centre point of the pattern were seen.
(b) The best responses showed sketches of the hexagons with the lengths of the sides shown clearly. Some responses showed accurate hexagons but a sketch of a hexagon was sufficient here. A few did not include labels of the lengths of the sides while others had incorrect labels, for example all sides labelled 30 cm or all labelled 6 cm or some 3 cm and some 6 cm .

## Question 7

(a) This was answered well by the majority of candidates. The most commonly seen incorrect answer was $-2,-10,-18,-21,17$.
(b) Many candidates gave the correct temperature. Common wrong answers were -13 and 23.

## Question 8

(a) The best responses clearly identified that the length of the shortest piece was 180 cm as the first step in the working. Some responses worked with 180 cm as the total length of the piece of rope, rather than the shortest, going on to calculate $\frac{5}{12}$ of 180 giving the answer 75 cm .
(b) Many good responses were seen to this part, working from the answer to part (a). In a small number of cases, answers were not written in metres as requested, or the conversion from centimetres to metres was attempted by multiplying by 10 rather than 100.

## Question 9

Most responses correctly identified that the transformation was a rotation. Many gave the correct angle of rotation although a few omitted the direction of the rotation. Some gave the wrong centre of rotation or did not include the centre of rotation.

## Question 10

(a) This part was often answered correctly. Some responses gave the answer as a mixed number in its simplest form. Some responses showed the working to reach $\frac{32}{27}$ but then attempts to change this to a mixed number involved slips in arithmetic, giving answers including $1 \frac{5}{9}$ or $1 \frac{1}{27}$.
(b) Many correct responses were seen to this part. In some responses, the first step was to calculate $\frac{1}{4}+\frac{2}{3}$ but then the step involving subtracting this answer from 1 was not given. A few responses began with the incorrect operation when dealing with the two fractions and calculated either $\frac{1}{4} \times \frac{2}{3}$ leading to the answer $\frac{10}{12}$ or $\frac{2}{3}-\frac{1}{4}$ leading to the answer $\frac{7}{12}$.

## Question 11

Many candidates solved the inequality correctly. Most candidates correctly got as far as $x-3 x>5+7$ or $3 x-x<-5-7$. A common incorrect answer was $x>-6$ following on from $-2 x>12$, omitting the step of reversing the inequality when dividing through by a negative number.

## Question 12

(a) The best responses recognised that data for the first 20 games would provide a best estimate of the probability. Some responses included a step of adding or multiplying $\frac{2}{6}$ and $\frac{7}{20}$ while others worked with $\frac{4}{6}$ and $\frac{13}{20}$. Some answers were greater than 1 ; candidates are encouraged to check that any probability stated must be less than 1 in accordance with the probability scale.
(b) Many correct answers were seen to this part. Some responses attempted the step of dividing 14 by 0.2 but made slips in doing so and gave the answer as 0.7 . A common error was to multiply 14 by 0.2 .

## Question 13

(a) The most successful responses here included a sketch showing the relative positions of Mingfield and Lenton. Common incorrect answers were 204 (from 360-156 or $180+24$ ), 24 (from 180 156), 294 (from $270+24$ ) and 156 and -156.
(b) The best responses started with stating that 1 cm represented 4.5 km and worked from there. Some responses used the wrong conversion from km to m . Some did not include the step of converting the units.

## Question 14

(a) This question was answered well by many candidates. A few responses showed slips in the expansion of the second bracket, leading to $-6 x-9$ instead of $-6 x+9$ resulting in the most common incorrect answer of $9 x-19$. Some responses, having shown the correct steps of expansion and simplification, went on to form the equation $9 x=-1$ and gave an answer as $x=-\frac{1}{9}$.
(b) This part was also answered well. Almost all expanded the brackets correctly but some showed slips in collecting terms, for example $2 x^{2}+3 x-14 x-21=2 x^{2}+17 x-21$ or $2 x^{2}-17 x-21$. Other responses gave the answer $2 x^{2}-11-21$, omitting the $x$ from the $x$ term. Some formed the equation $2 x^{2}-11 x-21=0$ and attempted to solve it.

## Question 15

Many responses showed use of the formula $a+(n-1) d$ to find the $n$th term of an arithmetic sequence. Many responses worked with the correct common difference of 6 but in some cases this value was used incorrectly, leading to answers of $n+6, n-6$ or $6+n-1$. Some responses involved slips in simplifying $1+$ $6(n-1)$.

## Question 16

Some good responses were seen to this part, using the correct formulae related to the area and circumference of a sector. Some responses reached the step of writing $30=\frac{x}{360} \times 20^{2}$ but cancelled incorrectly to get $5=\frac{x}{6} \times 20^{2}$. In some cases, the formula for the circumference of a circle was used instead of that for the area of a circle.

## Question 17

This question proved challenging for many. The best responses recognised the different powers of 10 as a first step and converted the mass of Venus (the value with the smallest power) to have the same power of 10 as Saturn. The steps seen were typically writing $5.7 \times 10^{26}$ as $570 \times 10^{24}$, carrying out the subtraction $570-4.90$ and then writing $565.1 \times 10^{24}$ in standard form. A few responses subtracted 4.9 from 5.7 and 24 from 26 giving the answer $0.8 \times 10^{2}$.

## Question 18

Many candidates answered this correctly. The step which proved the most challenging was removing the square root in the equation by squaring both sides. Some common errors seen in this step are given below.

- $\sqrt{ } y=\frac{x+2}{3}$ then $3 \sqrt{ } y=x+2$ then $x=3 \sqrt{ } y-2$
- $3 y^{2}=x+2$ then $x=\frac{3 y^{2}}{2}$
- $3 y=\sqrt{x+2}$ then $3 y-2=\sqrt{ } x$ then $x=(3 y-2)^{2}$
- $\sqrt{ } 3 y=\sqrt{ } x+\sqrt{ } 2$ then $\sqrt{ } x=\sqrt{ } 3 y-\sqrt{ } 2$ then $x=3 y-2$


## Question 19

Many good attempts at this question were seen, clearly stating that the triangles contained equal radii or equal tangents or a common side or a right angle formed between a radius and a tangent. The best responses gave correct statements of equality and reasons and went on to state that the triangles were therefore congruent with the appropriate reason. Some responses stated that angles OTB and OTA were equal or that angles $T O A$ and $T O B$ were equal, without first showing that the triangles are congruent.

## Question 20

The majority of candidates answered this question correctly. Incorrect answers included $\frac{x+10}{7}$ and $\frac{10-x}{7}$ from sign errors.

## Question 21

Many good responses were seen to this part, with a common first step being the calculation of the width of $P Q R S$. Some responses gave a width here of 5 cm , using the incorrect approach of subtracting 3 from the width of $A B C D$ rather than using the ratio of the lengths to find the width of $P Q R S$.

## Question 22

(a) Many good responses were seen to this part. In most cases, an attempt at partial factorisation was successful, with the step $y(7+2 x)$ often seen. Some responses went on to form an equation and then attempt to solve it, for example $y(7+2 x)=21+6 x$ followed by $y=\frac{21+6 x}{7+2 x}$ was seen.
(b) Many correct answers were seen to this part. Many partially correct factorisations were seen, with responses ending with $3\left(a^{2}-4 b^{2}\right)$. This was sometimes followed by $3(a+4 b)(a-4 b)$ or $3(a-2 b)^{2}$. Again some responses went on to include a step in which an equation was formed, for example, $3 a^{2}=4 b^{2}$, with attempts then to solve the equation formed.

## Question 23

(a) This part was answered well by many. A common incorrect response seen was to give the number of people as 35000 .
(b) The best responses clearly identified the minimum number of men and subtracted that from the maximum number of people at the cricket match. Some responses subtracted 21000 from 36000 and then attempted to deal with the bounds by adding 250 to 15000 , giving an answer of 15250. Other responses used the correct bounds for the maximum number of men at the match but then went on to subtract this from 36000 , rather than the maximum attendance at the match of 34999.

## Question 24

Few fully correct responses were seen to this part. The best responses showed working clearly for the calculation of the frequency densities, with many making use of the space below the table to show the results of any calculations.

A common error seen was to determine the height of the bar for the interval $10<t \leqslant 15$ but to then draw this bar across the interval 0 to 15 . The height of the column for $30<t \leqslant 50$ was sometimes drawn at 0.15 instead of 1.5.

## Question 25

Many correct values of $a$ and $b$ were seen. Some responses found a value for $a$ only, with incorrect values of 9 and 27 often seen. Some responses reached $\left(x^{b}\right)^{3}=x^{3 b}=x^{4}$ but did not get to $b=\frac{4}{3}$. Some responses incorrectly dealt with the removal of brackets, with the step $\left(x^{b}\right)^{3}=x^{b+3}$ shown. Candidates are reminded that answers may be best stated as a fraction rather than a rounded decimal.

## Question 26

(a) Many responses gave a correct midpoint. Some responses included a step in which the $x$ and $y$ coordinates were added, giving the answer $(2,10)$. Other responses subtracted the coordinates before dividing by 2 instead of adding i.e. $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$. In a small number of cases, the length of $A B$ was found rather than the midpoint.
(b) Many methods began with the step of identifying that the line required was the perpendicular bisector $A B$ and attempted to find its equation. The best responses went on to find the gradient of the perpendicular line and then find the correct equation through their midpoint, often including a sketch with the calculations. Some responses reached the gradient of $A B$ i.e. $\frac{7-3}{4-2}$ although there were often slips in arithmetic when working out this value. In some cases, responses used the gradient of $A B$ and so found the line through $(1,5)$ with this gradient. This leads to the equation of the line $A B$ and not the required line.

## Question 27

(a) Many of the candidates who attempted this part gave the correct vector. Common incorrect answers were $3 \mathbf{c}-2 \mathbf{a}$ and $2 \mathbf{a}-3 \mathbf{c}$.
(b) (i) Many correct vectors were seen. Some incorrect answers were $3 \mathbf{c}+\mathbf{a}$ and $3 \mathbf{c}-\frac{1}{2} \mathbf{a}$.
(ii) The best responses began by writing down the vector route for the required vector. Many responses gave the correct vector route but did not then find the vector, often due to a slip in the signs indicating the direction. For example, some responses gave the correct vector route of $\overrightarrow{A B}+\overrightarrow{B T}$ but added $\frac{1}{3} \overrightarrow{O B}$ to $3 c$ rather than subtracting $\frac{1}{3} \overrightarrow{O B}$ from $3 c$ resulting in the answer $4 \mathbf{c}+\frac{2}{3} \mathbf{a}$. Some showed errors in the simplification, for example $3 \mathbf{c}-\frac{1}{3}(2 \mathbf{a}+3 \mathbf{c})=3 \mathbf{c}-\frac{2}{3} \mathbf{a}+\mathbf{c}$ leading to the answer $4 \mathbf{c}-\frac{2}{3} \mathbf{a}$.
(c) Few fully correct answers were seen to this part. Most responses explained in words why ATM was a straight line and did not explain the reason by using vectors. Some answers correctly showed that $A T$ was parallel to $T M$ or $A M$ or that $A M$ was parallel to $T M$ but did not also include the step of stating a common point.

## MATHEMATICS SYLLABUS D

## Paper 4024/12

Paper 12

## Key messages

To do well in this paper, candidates need to

- be familiar with all the syllabus content
- be able to carry out basic calculations without a calculator
- understand and use correct mathematical terminology
- draw and interpret graphs and diagrams
- apply mathematical techniques to solve problems
- set out their work in clear, logical steps.


## General comments

In general, candidates were well prepared for most of the topics covered by this paper and most attempted all the questions.

Candidates need to be prepared for questions that assess topics in an unfamiliar way. The questions assessing proportionality, probability and perpendicular lines were found to be the most challenging with few candidates able to apply their knowledge of these topics efficiently to the given problem.

Many candidates presented their work well with workings set out legibly and answers clearly stated on the answer line. Most used the appropriate geometrical instruments correctly to draw and take measurements from diagrams.

Many candidates demonstrated sound basic arithmetic skills, although errors were seen when working with negative numbers or cancelling fractions. Candidates often demonstrated good algebraic skills, particularly in solving simultaneous equations and expanding brackets, although care should be taken with signs when collecting terms. Some candidates were unfamiliar with function notation.

Candidates would benefit from a greater understanding of the mathematical terms used in the syllabus such as mixed number, plane of symmetry and correlation. Where a question asks for an answer in its simplest form such as Question 8 and Question 22, fractions should be written in their lowest terms.

Candidates should aim to present their work in a legible manner and should ensure that numbers such as 1, 4 and 7 can be clearly distinguished. They should cross out and replace work if they have made errors rather than overwriting as this cannot be read clearly. They should show their method for a question in the working space for that question: if work continues elsewhere, this should be referred to in the working space for the question and the additional work must be clearly labelled with the relevant question number.

## Comments on specific questions

## Question 1

(a) Almost all candidates were able to subtract these decimal values correctly. Some arithmetic errors were seen often leading to an incorrect answer of 2.52.
(b) The multiplication of $1.2 \times 1.2$ was more of a challenge than the subtraction but many candidates evaluated it correctly. Common incorrect answers were 14.4, 1.4 and 2.4. Some responses showed
the conversion of the decimal to a fraction before squaring which usually led to the correct equivalent answer of $\frac{36}{25}$.

## Question 2

Most responses correctly identified the tangent. Common incorrect answers were chord, circumference, diameter and straight line.

The term sector was less familiar to candidates. Common incorrect answers were segment, arc, arc sector, or a non-mathematical term such as shaded area or pie slice.

## Question 3

Those responses which started by converting all values to decimals were more likely to then write them in the correct order. The most common error was to list $\frac{5}{8}$ either before 62 per cent or at the end of the list.

## Question 4

(a) This part was usually answered correctly. The most common error was to give the answer 4 rather than -4 .
(b) (i) Most responses showed an understanding that the median is the middle value and it is the fourth value in a list of seven. Those responses that started by writing the values in order usually gave the correct answer of -1 . The most common incorrect answer was -7 , the middle value in the unordered list. A minority of candidates found the mean rather than the median.
(ii) The range is found by subtracting the smallest value from the largest value. Candidates who wrote this calculation as $5-(-7)$ usually found the range correctly as 12 . Some responses reversed the order of the subtraction and included the step -7-5 leading to the incorrect answer of -12 . Another common error was to give an interval such as -7 to 5 as the answer.

## Question 5

Most responses showed the correct method to calculate simple interest. It was common to see the answer 528 , the value of the investment after 5 years, rather than the required answer of 48 , the total amount of interest at the end of 5 years. After writing down the correct calculation $48 \times \frac{2}{100} \times 5$, some responses cancelled fractions incorrectly or made arithmetic errors when multiplying one year's interest of $\$ 9.60$ by 5 . A minority of methods started with an incorrect calculation, commonly $\frac{48 \times \frac{2}{100} \times 5}{100}$ or $48 \times 2 \times 5$. Some methods stated that 48 was one year's interest and multiplied this by 5 to get the answer.

## Question 6

(a) (i) Many candidates measured the line accurately and used the scale to convert the distance to kilometres correctly. Candidates should be advised to write down their measurement in centimetres as they can then gain credit for a correct measurement that is converted incorrectly or for using the scale correctly with an incorrect measurement. Some arithmetic errors were seen when multiplying the distance in centimetres by 2 , but it was more common to see the scale used incorrectly, for example dividing by 2 . Some responses divided the distance by 1000 to convert from cm to km .
(ii) The bearing was often measured correctly. The common incorrect answers were $60^{\circ}$, the result of measuring the bearing of $A$ from $B$ instead of $B$ from $A$, and $120^{\circ}$, the result of measuring the angle anticlockwise from $A$ to $B$.
(b) Many candidates identified that the perpendicular bisector was the construction required in this part. The perpendicular bisector was usually accurately drawn with correct arcs shown. The most common errors were construction of the locus of points a fixed distance from line $A B$ or construction of a triangle with $A B$ as one of the sides.

## Question 7

(a) Many candidates correctly identified positive correlation. In some cases, qualifiers such as weak or strong were included, either of which were accepted. Very few candidates thought it was negative correlation, but incorrect terminology such as increasing, direct proportion, irregular, linear, direct and distance-time were seen.
(b) Some candidates drew a valid line of best fit that covered distances from 3 km to 9 km with an approximately equal number of points above and below their line. Common errors were to draw a short line or to draw a line joining the upper and lower points. Only a small number of candidates used a non-straight line or joined all the individual points.
(c) Most candidates read the correct time for a distance of 5 km from their straight line.

## Question 8

Most candidates converted $1 \frac{3}{4}$ to $\frac{7}{4}$ and used a common denominator of either 12 or 24 to add the fractions correctly leading to $\frac{31}{12}$ or $\frac{62}{24}$. The question required the answer to be given as a mixed number in its simplest form, Some responses gave the answer as an improper fraction, and others simplified the result incorrectly. Other errors included writing $1 \frac{3}{4}$ as $\frac{7}{3}$, multiplying the numerators by incorrect values or adding numerators and denominators to give $\frac{7}{4}+\frac{5}{6}=\frac{12}{10}$.

## Question 9

(a) Few correct answers were seen to this part. The answer 5, the number of faces, was as common as the correct answer of 2.
(b) Some candidates were able to apply the formula for the volume of a prism correctly, often by first calculating the area of the triangular cross-section and then multiplying this by the length of the prism. Many candidates calculated the volume of a $3 \times 3 \times 8$ cuboid as 72 but did not halve their answer to give the volume of the triangular prism. Some candidates used an incorrect volume formula of $\frac{1}{3} \times 3 \times 3 \times 8$. Some candidates attempted to find the surface area instead of the volume.

## Question 10

This question was well answered with an equal spread of substitution and elimination methods seen. The most common errors were arithmetic slips or sign errors, for example $4 y-6 y=11-21$ leading to $-2 y=10$, or not multiplying all terms when equating coefficients, for example $2 x+4 y=7$ rather than $2 x+4 y=14$. Other errors included adding rather than subtracting the two equations after multiplying the first by 2 , leading to $5 x=25$ then $x=5$. Many candidates who had reached an incorrect first solution went on to use this solution correctly to find a pair of values that satisfied one of the equations. Candidates should be advised to check their solutions fit both equations as this will help them to identify any errors in their method.

## Question 11

Some candidates were well prepared for this question: they showed the rounded values clearly and then used an efficient method to reach the correct answer. Some candidates rounded the values correctly, but made place value errors when calculating, usually when dealing with division by the decimal number. The most common error was to round 18.2 to 18 rather than 20 as required by the question. Some candidates used 2 rather than 20 when rounding to 1 significant figure. A small number of candidates used the given numbers to try to work out the exact answer to the calculation rather than an estimated value as required by the question.

## Question 12

(a) Most candidates identified the transformation correctly as an enlargement usually with the correct scale factor of 2. The description should be 'scale factor 2' rather than statements such as 'twice the size' or ' $A$ is double $B$ '. Candidates had more difficulty identifying the correct centre of enlargement $(1,-1)$ and some did not state the centre. Common incorrect centres were $(1,1)$, or $(0,0)$. Some responses incorrectly gave the centre of enlargement as a vector rather than a coordinate. In a small number of cases, responses described an enlargement followed by a translation when the question asked for a single transformation.
(b) Some candidates drew the correct rotation of the shape although others rotated by $180^{\circ}$ about the wrong centre. The most common errors were to reflect the shape in the $x$ - or $y$-axis or to rotate by $90^{\circ}$.

## Question 13

(a) Most responses gave the next term of the sequence correctly. Some candidates made arithmetic errors when calculating $27 \times 3$ and others multiplied 27 by 9 rather than by 3 . Some attempted to find an expression for the $n$th term of the sequence.
(b) Many candidates recognised the common difference of -4 between terms and attempted to use the formula $a+(n-1) d$ to find the expression for the $n$th term. The most common errors were to use $d=4$ instead of -4 leading to an answer of $31+4 n$. Another incorrect method seen omitted the multiplication bracket when substituting $d=-4$ leading to $35+(n-1)-4$ which was then simplified to $39+n$. Some responses reversed the values of $a$ and $d$ in the formula and started with $4+35(n-1)$.

## Question 14

(a) Most candidates used an appropriate method to find the correct product of prime factors. In some cases, errors were made in one of the divisions in a factor ladder, but there were usually at least two correct stages following the error.
(b) This was a challenging question, with only a minority reaching the correct answer. Some responses identified $y^{2}$ as a component of the highest common factor, but few found the $x$ component correctly as $x^{n-1}$. The most common partially correct answer was $x^{n} y^{2}$. A small number of candidates found the lowest common multiple of $P$ and $Q$.

## Question 15

(a) Many responses correctly identified the inequalities $x+y \leqslant 5$ and $y \geqslant-2$. The inequality $y \leqslant 2 x-1$ was often stated incorrectly as $y \geqslant 2 x-1$. It should be noted that solid lines on the diagram indicate that $\geqslant$ and $\leqslant$ are required rather than $>$ and $<$ which were used by many candidates. There was no requirement to rearrange the given equations but correct rearrangements such as $y \leqslant 5-x$ were sometimes seen. A common error was to identify the third boundary line as $x=-2$ instead of $y=-2$. Other errors were to give equations rather than inequalities or to state the coordinates of the vertices of the shaded triangle.
(b) This part was found to be challenging. Those candidates who added the line $x=1$ to the graph often identified the correct region and found the area of the triangle correctly. Some candidates used the incorrect formula area $=$ base $\times$ height for the triangle and others drew the line $y=1$ or found the area of the shaded region.

## Question 16

The majority of candidates demonstrated that they knew the distance travelled is the area under a speedtime graph. Many divided the graph into two or three distinct areas and calculated these areas correctly which led to the correct final answer. Some basic arithmetic errors were seen such as $14 \times 100=140$ or errors when adding the areas. A common incorrect answer was 2100 , which did not include the area of the triangle. Some candidates used speed $\times$ time $=20 \times 150=3000$ which was not sufficient to indicate they were attempting to find the area under the graph. A small number used an incorrect formula such as distance $=$ speed $\div$ time .

## Question 17

(a) Most candidates understood the process to find an inverse function. Some responses gave the final answer in terms of $y$ rather than $x$, although those responses where the first step was writing the function as $x=2-3 y$ did not contain this error. Candidates often made sign errors, for example an incorrect first step of $y+2=3 x$ or a correct first step of $y-2=-3 x$ followed by division by 3 rather than -3 . A small number of candidates confused the inverse function with the reciprocal and gave an answer of $\frac{1}{2-3 x}$.
(b) The first step was critical in this part: those candidates who wrote correct expressions for $\mathrm{f}(x+5)$ and $3 \mathrm{~g}(x)$ often reached the correct answer. Sign errors were common in the expansion with $2-3(x+5)$ often expanded as $2-3 x+15$. Other common errors were to simplify $3 x+3 x$ as $9 x$ or $12-13=1$.

Many candidates did not correctly interpret $f(x+5)$, which was often written as $2-3 x+5$. Some rearranged to give a solution in terms of $f$ and $g$.

## Question 18

(a) Many good responses were seen to this part. The common incorrect error was to find the differences between the total number of items in the two bags and give an answer of 7 , the difference in the total, or (4) the difference in the number of soaps and candles separately.
(b) (i) Those responses which used the correct process for matrix multiplication and reached a $2 \times 1$ matrix usually gave the correct answer. Many candidates found four individual products with the common result of $\left(\begin{array}{cc}720 & 240 \\ 240 & 60\end{array}\right)$. It was rare to see the answer given as a $1 \times 2$ matrix.
(ii) Candidates needed to express their answer clearly to be awarded the mark in this part. It was common for candidates to refer to candles and soaps as well as large and small bags in one sentence which was often unclear. The best answers either referred to each element separately or did not mention the contents of the bag, for example ' 960 g is the mass of soaps and candles in a large bag, 300 g is the mass of soaps and candles in a small bag' or 'the elements represent the mass of a large bag and the mass of a small bag'. Comments such as 'the mass of soaps and candles in each bag' suggest that the soaps and candles are separated. Comments such as 'the mass of soaps and candles in the large and small bags' suggest four separate masses.

## Question 19

(a) This part was found very challenging. Common errors were to shade $\left(A \cap C^{\prime}\right) \cup\left(B \cap C^{\prime}\right)$ or $A \cap B \cap C^{\prime}$. Many candidates drew several of their own diagrams to try and work out the answer.
(b) Some candidates were able to find the correct answer without drawing a Venn diagram, often using a numerical approach. Some candidates formed an equation in $x$, where $x$ represented the number of people who use both a computer and a book. The correct equation $35-x+12-x+8+x=50$ leads to $x=5$ which was often given as the final answer rather than subtracting this from 12 to give the number using a computer only. Errors were common in setting up the initial equation, for example $35+12+8+x=50$ which leads to a negative value for the intersection.

## Question 20

(a) Many candidates expanded the brackets correctly and simplified the result to give the correct 3term expression. Most were able to gain partial credit for getting at least 3 terms correct in their expansion of the brackets. Care needs to be taken to include all necessary powers, to multiply terms correctly and to pay attention to signs when collecting terms. Final answers involving $x$ in place of $x^{2}, y$ in place of $y^{2}$ or $+5 y^{2}$ in place of $-5 y^{2}$ were common. Another common error after correctly expanding the brackets was for candidates to attempt to re-factorise their expression.
(b) The most successful approach in this part was to simplify the expression in two stages leading to the correct answer of $\frac{x^{8}}{4}$. In some cases, the denominator was not fully simplified and left as $2^{2}$ or 2 $8^{\overline{3}}$. Candidates were more successful in applying the indices to the numerator than the denominator and common partially correct answers were $\frac{x^{8}}{8}, \frac{x^{8}}{64}$ and $\frac{x^{8}}{2}$.

## Question 21

Candidates who were able to manipulate algebra confidently generally reached the correct solutions. Most candidates attempted the correct first step of eliminating the fraction and some reached the correct quadratic equation $x^{2}-4 x-12=0$, although some made errors in expanding $(x+4)(x-3)$. Some candidates made errors when factorising the quadratic equation and those who used the quadratic formula often made sign errors when substituting. Other candidates did not rearrange their 3-term quadratic into the form $a x^{2}+b x+c$ prior to substitution leading to the use of incorrect values for $a, b$ and $c$.

As the question did not start with a quadratic equation, some candidates did not use their knowledge of solving quadratic equations to reach a solution. Some attempted to factorise a partially simplified expression or took square roots using a variety of incorrect approaches.

## Question 22

This question involving proportional relationships between three variables was found to be challenging for many and few candidates were able to combine the two proportional relationships to reach a correct relationship between $y$ and $x$ only.

Most candidates began with $y=k w^{2}$ and $x=\frac{k}{w}$. The constants were often correctly evaluated as 0.05 and 4 respectively but many candidates were unable to progress any further. The most successful method was to rearrange to get $w=\frac{4}{x}$ and eliminate $w$ by substituting this expression into $y=0.05 w^{2}$. Occasionally errors were made when simplifying this expression, particularly if $\frac{1}{20}$ was used instead of 0.05 . It was common to see the error $0.4 \times 10=40$ when evaluating the constant in $x=\frac{k}{w}$.

Some candidates used an approach which assumed that the two constants should be the same. They often used $y=k w^{2}$ to evaluate $k$ as 0.05 , substituted $k=0.05$ and $w=10$ in $x=\frac{k}{w}$ beyond which no further progress could be made.

Candidates should be made aware that expressing proportional relationships using the proportion symbol $\propto$ is a first step after which an equation with a constant should then be formed.

Very few candidates used the approach of eliminating $w$ from $y=k w^{2}$ and $x=\frac{c}{w}$ before evaluating constants.

## Question 23

(a) While many candidates clearly understand probability, few were familiar with the information being presented in a table. Some were unable to extract the relevant figures from the table and consequently used incorrect starting fractions. Candidates who identified that there were 7 green shapes often wrote a correct probability of $\frac{7}{10}$. This sometimes led to a correct product $\frac{7}{10} \times \frac{7}{10}$ and the correct answer although some treated this as a 'without replacement' question so used $\frac{7}{10} \times \frac{6}{9}$. A common error was to consider green squares and green triangles separately and to find the product $\frac{3}{10} \times \frac{4}{10}$ or $\frac{3}{10} \times \frac{4}{9}$.
(b) Some candidates recognised that they needed to add the probability of selecting two squares to the probability of selecting two triangles. Some candidates identified that there were 4 squares and 6 triangles and used these values to form two products. A common error was the failure to recognise that the selection was made without replacement and products with a denominator of 10 throughout were often used. Many candidates used numbers from the table rather than combining them to find the total number of squares and the total number of triangles and calculations such as $\frac{3}{10} \times \frac{1}{9}+\frac{4}{10} \times \frac{2}{9}$ were common.

## Question 24

To demonstrate that $L$ is the perpendicular bisector of $A B$ it is necessary to show that the product of the gradients of the two lines is -1 and that line $L$ passes through the midpoint of $A B$. Many candidates showed one of these facts, but few clearly showed both.

It was common to see a correct method to calculate the gradient of $A B$ using the formula (difference in y ) $\div$ (difference in x ) leading to the correct gradient of $A B$ as 2 and identification of the gradient of $L$ as $-\frac{1}{2}$. Many candidates stopped at this point, having identified that the lines were perpendicular but not demonstrating that $L$ was the bisector of $A B$. Candidates who found the midpoint of $A B$ as $(-1,3)$ could use this to demonstrate that $L$ bisects $A B$. The most successful approach was to substitute $(-1,3)$ into $y=-\frac{1}{2} x+c$ to evaluate $c$ and then rearrange this equation to the given equation of $L$ which is $2 y+x=5$. Candidates who instead chose to confirm that $(-1,3)$ was on the line $L$ by substituting $x=-1$ and $y=3$ into $2 y+x=5$ rarely gave a clear explanation about why this result demonstrated that $L$ was the perpendicular bisector.

It was common to see candidates attempt to draw axes and plot points $A$ and $B$ and the line $L$. While a sketch is often helpful in developing a method or communicating understanding, it is not an appropriate way to show that one line is a perpendicular bisector of another line.

## MATHEMATICS SYLLABUS D

## Paper 4024/21

Paper 21

## Key messages

Candidates are encouraged to review their solutions to make sure that they have answered the question asked and that they have given their answer in the required form (Question 5(b)(iii), Question 6(b), Question 6(c) and Question 10(b)(i)).

Candidates should ensure that in 'show that...' type questions they present all working clearly (Question 1(a) and Question 1(d)(i)).

Candidates should take care to write time in a correct form and when converting between different time formats (Question 2(a), Question 2(b)(ii) and Question 8(a)(ii)).

## General comments

In some places, candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. Typically, intermediate values should be rounded to at least one more significant figure than given in the question. It is important that candidates retain sufficient figures in their working and only round their final answer to three significant figures if their answer is not exact.

## Comments on specific questions

## Question 1

(a) Few candidates made meaningful progress here. In some cases this was due to candidates not using the formula for the volume of a cuboid, or not being able to find the surface area. The most common error was a lack of consistent use of brackets around the edge of length $5-x$. It is important in 'show that...' type questions that candidates show all working clearly.
(b) This part was answered well by almost all candidates.
(c) Many good responses were seen to this question. Most candidates plotted the points correctly. The most common error was seen when plotting the $y$ values of 78.75 and 93.75 . The points were then usually joined with a smooth curve.
(d) (i) Few good responses were seen to this question. Some candidates did not use the given formula. Others did not clearly communicate where the base area of 81 came from. It is important in 'show that...' type questions that candidates show all working clearly.
(ii) Some good responses were seen here, with clear method leading to the correct answers. Some responses didn't show a suitable straight line on the grid and instead solved the equation simultaneously.

## Question 2

(a) Many candidates were able to calculate the correct time, but few wrote the answer in the correct form. Common incorrect answers seen were 1.35 or 1335 pm.
(b) (i) Many good responses were seen to this part.
(ii) The percentage work in this part was often done correctly, with the better responses converting 3.6 hours into 3 hours and 36 minutes successfully. Answers of 3 hours and 6 minutes were quite often seen.
(c) More good percentage work was seen here, with few errors. Occasionally candidates divided by 33408 , rather than 32000 . Other errors were to give answers of 0.044 or 104.4.
(d) This percentage question was challenging for many candidates. The main error when setting up the original equation was to omit the initial deposit in the account. Candidates therefore were solving $0.036 x=890.96$ rather than $1.036 x=890.96$. Another common error was to calculate $1.036 \times 890.96$. Some responses used a compound interest formula rather than a simple interest formula.

## Question 3

(a) Successful responses were scaffolded by clear steps accompanied by succinct reasoning. There were few fully correct solutions. Many responses did not provide a reason for each step of their working, showing the calculations only. Some candidates gave arithmetic calculations rather than geometric reasons for their steps.
(b) Very few correct solutions were given for this part. Some candidates made meaningful progress by recognising the right angle for angle OXZ, but few could take the solution any further. The best responses obtained angle $X U O$ in terms of $a$, or angle $U X O$ in terms of $b$, going on to find $b$ in terms of $a$.

## Question 4

(a) (i) Many responses showed lines on the graph supporting the method of finding the median, and correctly stated the value from the $x$-axis. Some responses showed a line which was not drawn at the value of 50 on the $y$-axis.
(ii) Again, many good responses were seen. The common errors seen were responses which found $\frac{1}{4}$ of 100 and $\frac{3}{4}$ of 100 , then gave the interquartile range as $75-25=50$.
(b) Many correct answers were seen with each step clearly shown; 88 people spending less than $\$ 91$ so 12 people spent more than $\$ 91$, giving a fraction of $\frac{12}{100}$.
(c) Many good responses were seen to this part. The main error was in finding the cumulative frequency from the graph rather than using this to find the frequency.

## Question 5

(a) (i) Most candidates showed fully correct responses to this part with occasional slips.
(ii) Some candidates made meaningful progress here. Some wrote the elements, rather than the number of elements as requested.
(iii) Again, many good responses were seen to this part. The most successful responses had clearly annotated the Venn diagram to support the working.
(iv) Few fully correct responses were seen to this part. Success generally depended upon candidates having just 6 and 12 in the required subset, and identifying these as multiples of 6 , or an equivalent description. Describing these as, for example, multiples of 3 was not precise enough due to there being other multiples of 3 not within this subset.
(v) Those candidates who had a correct Venn diagram often gained full marks here.
(b) (i) Few responses made meaningful progress to this question. The better responses identified from the stem that $2 x=8$ and so $x=4$, then went on to identify that $x-y=6$ and therefore $y=-2$. Other responses found an incorrect value for $x$ but used this correctly to find $y$.
(ii) A very small minority of candidates were successful here. The best responses identified the square number as $2^{2 x} \times 3^{4}$, substituted in $x=4$ and evaluated the result.
(iii) Many responses correctly identified the HCF of $M$ and $N$. Few were expressed as a product of prime factors.

## Question 6

(a) The vast majority of candidates gained full marks in this part.
(b) There were few responses that were fully successful in this part. Again, candidates should note the question requires them to form an equation in $m$ and then solve this. Not all responses set up the equation in $m$ correctly, others did not convert the money parts to the same units. Some methods did not form an equation in $m$ and showed instead arithmetical calculations.
(c) The best responses identified that this was a question relating to direct proportionality usually set up an initial equation in terms of $y, k$ and $x$ correctly, and often went on to gain full marks. There were quite a few attempts using inverse proportionality rather than direct, or using square proportionality rather than cube proportionality.
(d) Many fully correct responses were seen for this part. The most common error was not considering the double negative when expanding the second bracket.

## Question 7

(a) (i) This question proved difficult for candidates. The main error was in measuring obtuse angle BAC with quite a large proportion of candidates drawing an acute angle for BAC.
(ii) Few candidates could accurately measure the sides of the triangle. Not all correctly find the perimeter of the triangle. The best responses listed all measurements and then found the sum of these, showing each step in the working.
(b) (i) Roughly half the candidates were successful in this part, making use of the sine rule in finding PS.
(ii)(a) Many candidates were able to find the value of $S R$, in a 'show that...' question of this type. However, they are to be reminded that they should find the answer to at least one more decimal place than that required in the question.
(b) Very few fully correct responses were seen to this part. The best responses used the given value of $S R$ from part (a) and the information about the angles in both triangles together with the cosine rule to find $Q R$.

## Question 8

(a) (i) Most candidates were able to add Maya's stay at the factory for 1.5 hours to the distance-time graph. However, fewer candidates managed to complete the rest of the graph correctly.
(ii) Many responses wrote down the time at which their graph returned to zero on the vertical axis, using the correct format.
(b) Very few responses gained full credit for this part. There were some candidates who obtained one correct expression for the average speed in terms of $d$, but only a small number of candidates found two correct expressions and equated these together with $\frac{4}{5}$.

## Question 9

(a) Almost all candidates gave the correct answer to this part.
(b) (i) Those solutions which correctly set up the simultaneous equations usually then achieved the correct values for $x$ and $y$. Some responses did not reach the step of finding the matrix represented by MN, and hence could not set up the simultaneous equations.
(ii) Some good responses were seen to this part, with a clear description of what $P$ represents by referring to the cost of flights and each family. A common error was to refer to the cost for adults and children rather than each family.

## Question 10

(a) (i) Most candidates wrote a clear explanation. The best responses included a comment about the total frequency without $p$ being 88 , and as $p$ cannot be negative, the total number of bags in the box cannot be 87 .
(ii) Some good responses were seen to this part. Common errors were to omit the term $14 p$ when accumulating the number of sweets, to omit $p$ when accumulating the frequency, or to write the total frequency as $88 p$ rather than $88+p$.
(b) (i) The candidates that successfully set up the multiplication of algebraic fractions usually went on to correctly establish the given result. Some responses found the value of $r$, answering a different question to that on the examination paper.
(ii) Many candidates were successful here, identifying that $r$ had to be a positive value and correctly discarding the negative. Many responses clearly showed the working as requested.

## Question 11

(a) The best responses added a perpendicular line from $C$ to edge $A B$ and used right angled Pythagoras with $28-24$ and 16, then added 90 degrees to find angle $D C B$. Many responses made a good start, but lacked a clear strategy to finding the solution.
(b) Some good responses were seen here. Many responses were not successful in finding the area of a trapezium and so could make only limited progress. Candidates are reminded that all answers are to be given to at least three significant figures.
(c) Very few candidates were successful with this part. The most successful methods used the rightangled triangle $E F H$ to find the length $F H$ and then the right-angled triangled $D F H$ to find angle DFH.

## MATHEMATICS SYLLABUS D

## Paper 4024/22

Paper 22

## Key messages

Candidates generally performed well on this paper. Many good responses were seen, with examples of clear mathematical communication. In some responses, there was evidence of premature rounding and this was particularly noticeable on Questions 8(b)(ii) and 9(c)(ii). Candidates are encouraged to write mathematical expressions and equations carefully such that slips in algebraic notation are avoided. Candidates are also encouraged, once they have found a solution, to revisit the question and ensure they have answered it as requested and that the answer makes sense in the given context. An example of this was seen in Question 8(b)(i) where the answer was frequently not given in terms of $\pi$ as stated in the question.

## General comments

There were many good responses from candidates at the beginning of the paper. Clear communication of method was seen in the better responses and important steps in working shown. Candidates performed well in the first part of Question 3, for example, but not many responses were seen in which an equation was explicitly stated and then solved to find $y$. It was pleasing to note that were more responses involving candidates appreciating the need to give the length to at least 2 decimal places when asked to show a length to 1 decimal place. Candidates should also check that their working leads to answers which are appropriate for the question. In Question 4(d), for example, responses were seen in which the points that had been determined did not form a straight line when plotted.

## Comments on specific questions

## Question 1

(a)(i) Many correct solutions were seen, indicating a good understanding of the calculations involved. If an error was made, such as premature rounding for $q$, then most candidates gained a correct follow through value for $r$. Lastly, candidates are to be reminded that they should give some consideration to whether their answers are reasonable in the context of the question.
(ii) Many good responses were shown in calculating the percentage profit. However, some candidates need to remember it is the percentage that is needed to gain full credit. Furthermore, candidates are to be reminded to not round too early in their response.
(b) (i) (a) Many candidates were able to calculate the required angle accurately, however, some rounding issues resulted in incorrect answers. Some candidates found 9520.7 as a decimal fraction or percentage of 34974 , sometimes rounding it before continuing, and in some cases giving their answer as a percentage rather than an angle. Another common error was to find the difference between the fruit sales and the total sales and then dividing this by 360, leading to a wrong answer of $70.7^{\circ}$.
(b) Many good responses to this question considered the total sales and calculated the frozen food sales using the fraction of the whole circle. Others chose to consider the fruit sales and calculate the frozen food sales using the fraction of the fruit sales. Rounding issues were seen, usually rounding the decimal value of $\frac{46}{360}$ before multiplying by 34974 . Other errors included subtracting the fruit sales from the total sales before multiplying by $\frac{46}{360}$.
(ii) Many good responses were noted using the standard method. Incorrect responses typically involved increasing or decreasing 34974 by 4.4 per cent. Candidates are encouraged to consider if their answer is appropriate, such as an answer being given for 2021 that is greater than the given value for 2022 which cannot be correct when there has been an increase.

## Question 2

(a) Candidates were usually able to give the correct answer of 62 cents but many were not able to set up a correct equation. It was common for candidates to either set up an equation with inconsistent units, $5 p+6=9.10$, leading to $p=0.62$ and then multiply by 100 to convert this to cents or to obtain 62 from correct calculations without forming an equation. Some did not appreciate that the value of $p$ was required in cents and gave an answer in dollars. Other errors included arithmetic errors, or using $5 p+600=9.10$ and thus a negative answer of -118.18 , or setting up simultaneous equations.
(b) Many candidates gained maximum marks for their fully factorised response. However, candidates are reminded to be careful not to continue to further incorrect factorisation, usually involving two brackets.
(c) Many responses demonstrated the need to both multiply the two fractions as well as cancelling where possible. The most common errors included only simplifying one or two of the three parts correctly, for example $\frac{15 m n^{2}}{90}$ or $\frac{m^{2} n^{2}}{6 m}$. Candidates are reminded that even if the expression was written as a single fraction as required, cancelling will still be needed.
(d) Many candidates displayed a correct order of operations needed to rearrange the equation to make $x$ the subject. Candidates are to be reminded that correct mathematical notation is required to gain full marks; such as when writing the square root to ensure that it covered all of $\frac{5 y}{3}$. Answers such as $\frac{\sqrt{5} y}{3}, \frac{\sqrt{5 y}}{3}, \sqrt{\frac{5}{3}} y$ and $\sqrt{5 y} / 3$ were all common, although these answers usually followed the equation $x^{2}=\frac{5 y}{3}$.
(e) Few fully correct answers were seen to this part. Many candidates gave a correct expression for the total of the original numbers, $56.8 k$, although this was often seen as an incidental term showing up in a variety of attempts. Fewer were able to give a correct expression for the total with the added number, either $56.5(k+1)$ or $56.8 k+52$. A common method used was to produce and then solve two equations, one for the $k$ numbers and one for the $k+1$ numbers, namely, $\frac{x}{k}=56.8$ and $\frac{x+52}{k+1}=56.5$ where $x$ is the total of the $k$ numbers. Errors with the methods used included using a denominator of $k$ or $k+52$ rather than $k+1$, or errors in subsequent rearranging. Some candidates correctly reached the 15 numbers, however, they did not realise this was the value of $k$ and then multiplied by 56.8 to arrive at the answer for the total of the $k$ numbers.

## Question 3

(a) (i) Most correct responses were shown when the angles 165 and 60 were labelled in the diagram. The most common incorrect response was 15 where candidates worked out the exterior angle of polygon $A$.
(ii) Most candidates that attempted to answer this question by calculating the exterior angle of polygon $A$ and using this to find the number of sides had more success with this question. The most
common approach to answer this was to attempt to use $\frac{180(n-2)}{n}=165$ to find the number of sides. Errors were seen either when solving this or when setting up the equation as frequently it was set equal to either 135 or 360 or the denominator of the equation was incorrect or omitted.
(b) (i) Some correct answers were seen, with the relevant angles and reasons being clearly stated. Candidates are to be reminded that they need to be able to state the relevant circle theorem to gain full marks. The most common errors were to state that angle AOC was $x$ with the reason of angles in the same segment, or to think that angle $A B C$ is a right angle thinking that $B C$ was a diameter and so using angles in a semicircle. Those who obtained an angle for AOC were not always able to show the relevant working with the reason being an isosceles triangle to arrive at the angle OAC.
(ii) Candidates who were most successful here usually drew the line $O B$ and correctly calculated $x$ as $54-11$. Some set up and solved a correct equation to find $x$, usually $x+54+90-x+90-x-11$ $=180$. Common errors included assuming that angle BAC was 90 degrees or the angle between $O A$ and $B C$ were 90 degrees. Candidates rarely named the angles they were attempting to use or labelled these angles in the correct place on the diagram.

## Question 4

(a) Nearly all candidates completed the table correctly, with some arithmetic slips seen.
(b) The vast majority of candidates plotted the points correctly and joined these with a smooth curve. Occasionally the points were joined by ruled straight lines. The two points that were most frequently plotted incorrectly were $(-1,0.1)$ and $(0,0.2)$ sometimes with transposition between the $x$ and $y$ axes or with $(-1,0.1)$ plotted at $(-0.9,0.1)$.
(c) Some candidates managed to draw a suitable line on the grid to solve the given equation. Those who correctly drew the line $y=1.2$ were usually able to read the intersection accurately to solve the equation. Candidates must make sure that they use the method required when answering this type of question. Having found the solution of, say $x=1.6$, many then drew that line on their graph. About a tenth of the candidates omitted this part.
(d) (i) Generally the table was completed correctly with the most common mistake being to omit the minus sign before 0.25 .
(ii) Over half the candidates were able to draw a correct ruled line. Candidates lost marks mostly through drawing a freehand line or plotting one of the points incorrectly, usually ( $1,-0.25$ ).
(iii) Those who had a ruled straight line were usually able to correctly read the $x$-coordinate of the two points of intersection. Some candidates did not have two points of intersection to read from. Occasionally candidates omitted the minus sign when reading one of the values.
(iv) Some candidates were able to correctly equate the two equations and use this equation to state the correct integer values. Occasionally non-integer values were given for the required constants. A common misunderstanding seen was to write that $4 \times 2^{x}=8^{x}$. Other errors included rearranging the equation to $4 \times 2^{x}=10 x+5$ and not going on to obtain an equation set equal to 0 , resulting in three positive values being given for the constants. It was not uncommon to see equations involving both $x$ and $y$, usually containing both a $4 y$ and a $y$ term.

## Question 5

(a) (i) A correct answer was given by most candidates. The common errors were either to give the name of the country with the smallest population or to give the number with the smallest digit of 2.17 without consideration being given to the effect that the power of 10 has to the size of the population.
(ii) Many candidates gave the correct answer. However, candidates are reminded to give their answer in the form requested. A common error was subtracting the population values instead of the area values. Other candidates subtracted 6.56 from 8.82 without dealing with the powers of 10 first, resulting in an answer of 2.26 with a power of 10 .
(iii) Many responses demonstrated how to calculate a population density and give the largest one. The most common wrong population density given was 246 , usually without any evidence of the other population densities. The most commonly seen error was to multiply the population and the area and to give the answer as the largest of these values.

Cambridge Ordinary Level<br>4024 Mathematics Syllabus D June 2023<br>Principal Examiner Report for Teachers

(b) (i) About a third of candidates were able to write $A$ and $B$ with a common power of 10 before subtracting and then ensuring the final answer was given in standard form. The most common error was to subtract 1.5 from 8.6 before dealing with the powers resulting in an answer of 7.1 times a power of 10 . Some candidates chose to select a value for $n$ and then give the answer when these numbers were subtracted.
(ii) More candidates gave correct responses than to (i), however not all were able to write $12.9 \times 10^{2 n-1}$ in standard form with many writing $1.29 \times 10^{2 n-2}$. The most common misconception was to multiply the powers as well as the numbers, giving $12.9 \times 10^{n(n-1)}$. Candidates are encouraged to ensure their notation is mathematically correct as the power $n 2$ was common and it was not clear whether this was intended as $n \times 2$ or $n$ squared.

## Question 6

(a) Many candidates were successful in simplifying the ratio of three distances. Most took the option to change the distances to metres. A significant number opted to change into kilometres with the result that many ended up with non-integer values in their final ratio, usually 6.3:3:1.8 or 2.1:1:0.6. Some responses did not deal with the inconsistent units and showed no meaningful progress towards a correct answer. Errors were seen when some candidates attempted to simplify a correct ratio while others used an incorrect conversion, usually $1 \mathrm{~km}=100 \mathrm{~m}$.
(b) Most candidates gave fully correct responses to this part, with clear steps shown to their solutions. These were firstly, using the correct distance and the speed with consistent units to find the time for walking. Secondly, converting this time to minutes or hours and minutes. Thirdly, using this time to find a starting time. The most common errors usually involved the units of time. It was common to see 4500 seconds converted to 75 minutes and/or 1.25 hours and finally quoting that this was 1 hour 25 minutes leading to the common wrong answer of 0940 . Less common errors involved 4500 being used as a time in hours, for example 4.5 hours, or multiplying the distance by the speed in an attempt to find the time or adding the journey length to the finish time. Very occasionally the wrong distance was used, usually the total of the three distances.
(c) Most candidates showed good knowledge of the values of the bounds for the individual variables. Over half of the candidates correctly worked with the upper bound for the distance and the lower bound for the time to obtain the upper bound for the cycling speed. Some were less successful in matching up the bounds needed to maximise the result. The most common error was to use the upper bound for both the distance and the time. Other errors included calculating the speed with the given values and then attempting to deal with the bounds later or using incorrect values from the bounds, for example 3010, 3050 or 3000.5 for the distance and $449.5,450.5$ or 460 for the time or using the wrong formula of time divided by the distance in an attempt to calculate the speed.

## Question 7

(a) (i) The majority of candidates reached the correct answer of 15. The common wrong answers were 5 and 0.5.
(ii) Many candidates were able to complete the histogram correctly, with both the correct heights and widths of the bars. Those who did not have all three bars correct, usually drew the first two bars correctly. The common error was to draw the final bar with a height of 2.4.
(iii) This was the most challenging part of this question for candidates. Many succeeded in adding 19 and 24 but not all went on to use this to obtain a probability. Others that appreciated the need for a probability sometimes used the numerator as 19 or 24 or $24+19+24$.
(b) (i) Around three fifths of the candidates were able to calculate the estimated mean using the midvalues of the intervals. Of those who used the correct method, there were occasionally one of the mid-values incorrect or an error in the multiplication in the numerator or having arrived at $\frac{4065}{95}$ the answer was then truncated as 42.7. Other errors included using either the upper bounds of the intervals or the lower bounds or assuming the total frequency was 100. The most common error was to use the class widths instead of the mid-values to calculate the estimated mean.

Cambridge Ordinary Level<br>4024 Mathematics Syllabus D June 2023<br>Principal Examiner Report for Teachers

(ii) Few responses stated that 41 was in the interval $40<m \leqslant 50$ and so a mid-value of 45 would be used. Those who did get this far were usually able to relate this to the current estimated mean although a few did not quite go far enough in their explanation. Nearly all candidates referred just to the total increasing and saying this was greater than the increase in frequency or that the frequency for this interval was now 25 so the estimated mean was higher.

## Question 8

(a) (i) Over half of the candidates were able to calculate the correct length and width, however some transposed their answers. Some expressed the length and width correctly using algebra but others didn't succeed in formulating the correct expression for the perimeter, as a result leading to the incorrect length. Common wrong methods led to a length of 13.6 and a width of 6.8 or a length of 10.2 and a width of 5.1. Some candidates gave algebraic expressions for the length and width.
(ii) Candidates had most success with this part of the question with many obtaining either the correct length or were able to obtain the length of rectangle $S$ that was 1.5 times their length of rectangle $R$. Two approaches were used by the candidates. The most successful was to take their length answer from (a)(i) and scale it. The other approach was to use the same method in (a)(ii) as used in (a)(i) to find the length. The most common error seen when attempting the method of scaling was to introduce a power, usually a power of 2.
(b) (i) Around half the candidates were able to find a correct expression for the arc length of the sector in terms of $\pi$. The two common errors were either to work out the area of the sector or to evaluate the arc length.
(ii) This question proved difficult for many candidates. It was common to see attempts at using the arc length as the diameter of the cone or using trigonometry to find the radius with an angle of $37.5^{\circ}$. Others did not use the information already given about the piece of card but instead assumed the radius was half of the slant height. Having obtained a radius, many were able to use Pythagoras to calculate a height for the cone and then use both the radius and height correctly in the formula for the volume of the cone. It was not uncommon to see attempts at the volume of the cone with either the radius or the height as 8 .

## Question 9

(a) Many candidates realised the need to use the cosine rule and nearly three fifths of them did so successfully. Knowing the size of angle BAC was essential and many obtained the correct value of $133^{\circ}$. Some candidates made errors with the bearings given, resulting in angles of 72,205 or 83 being commonly used in the cosine rule. Mistakes were seen when candidates chose to start with the cosine rule in the form $\cos A=\ldots$ and then attempted to rearrange this into the form $a^{2}=\ldots A$ significant number of candidates took angle $B A C$ to be a right angle and so the use of Pythagoras was common. A smaller number attempted to use the sine rule but without success.
(b) Generally, there were many relevant correct angles seen on the diagram, however not all of these candidates were able to follow this through to give the bearing required. Candidates are to be reminded of the appropriateness of their answer by noting that the angle required was an acute one here.
(c) (i) Few candidates were able to show the value of $A D$ was 86.1 correct to 1 decimal place. Significantly more wrote a correct calculation but did not give a value of $A D$ to more than 1 decimal place to show that it rounded to 86.1. There were a significant number of candidates who used a circular argument where they used $A D$ to find either $C D$ or one of the angles and then used this value to get back to the value of $A D$.
(ii) Very few candidates gave fully correct solutions. Those candidates that drew diagrams tended to be more successful. Many of the correct solutions opted to use tangent ratio twice with a small number opting for less efficient methods by calculating sides $B X$ or $D X$. Some candidates used premature approximation for intermediate stages which resulted in an inaccurate final angle of elevation. It was not uncommon for candidates not to realise that triangle $B A X$ is a right angle and so use of 101 and 83 was not uncommon in their calculations for $A X$. These candidates usually went on to calculate the angle of elevation using their value of $A X$.

## Question 10

(a) Many candidates performed well on this question. Many made a good start by either locating $H$ correctly or using their $H$ or their $\overrightarrow{F H}$ in a correct method for finding the length of $F H$ using Pythagoras' theorem. The most successful candidates were those who either drew a diagram or used statements such as $\overrightarrow{F H}=\overrightarrow{F G}+\overrightarrow{G H}$. The most common error was to add the coordinates of $F$ and $G$ and the values in the vector $\overrightarrow{G H}$, giving $\overrightarrow{F H}=\binom{3}{-3}$. Candidates are to be reminded that correct mathematical notation is required to gain full credit. Some responses gave unlabelled vectors or coordinates. Other common mistakes included errors in squaring or signs in the formula $\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$ and attempts to use vectors as products e.g. $F \times H$, leading to complicated calculations.
(b) (i) A good number of candidates answered this vector question correctly. A common incorrect answer was $\mathbf{a}+k \mathbf{b}+m O C$ or longer expressions.
(ii) A significant number of candidates were able to use a correct vector route to obtain the correct equation $\frac{3}{5} \mathbf{a}-\frac{1}{2} \mathbf{b}=-\mathbf{b}+m \mathbf{a}+m k \mathbf{b}$. The best responses went on to find the value of $k$ correctly by separately equating the vectors in terms of $\mathbf{a}$ to find $m$ and then in terms of $\mathbf{b}$ to find $k$. Some candidates equated $\overrightarrow{B X}$ to their vector in part (b)(i), while others had an incorrect vector route for $\overrightarrow{B X}$, for example $\overrightarrow{O B}+\overrightarrow{O X}$. Some responses did not make the connection with part (b)(i), which limited meaningful progress.

