Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

2625652817

MATHEMATICS (SYLLABUS D) **4024/22**

Paper 2 **October/November 2019**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

DC (LK/TP) 172992/2

Cambridge Assessment International Education

1 Tanya owns a small business.

(a) Tanya has 4 employees.
Every week, the employees each work for $7\frac{3}{4}$ hours each day for 5 days.
Each employee is paid $15.20 per hour.

Calculate the total amount Tanya pays her 4 employees in one week.

Each employee works = 7.75 hrs × 5 days
= 38.75 hr

1 hour → 15.20
38.75 hr — ?
= 589

1 employee earns $589 in 5 days.
4 employees — ?
4 × 589 = 2356

$ 2356 [2]

(b) The business made a profit of $25 700 in 2017 compared with $22 102 in 2018.

Calculate the percentage decrease in profit from 2017 to 2018.

Profit 2017 = 25,700
" 2018 = 22102
decrease = 25700 − 22102
= 3,598

Percentage decrease = $\frac{3598}{25700} \times 100\%$
= 14

14 % [2]

(c) Tanya must add 8% sales tax to the initial cost of a job.
She then adds 15% to the cost, including sales tax, to find the amount to charge a client.
Tanya charges one client a total of $465.75 for a job.

Calculate the initial cost of this job.

Let Initial cost be = x
8% + 100 = 108
$\frac{x}{100} \times 108 = 1.08x$
15% + 100% = 115% $= \frac{1.08x \times 115}{100}$
$= 1.15(1.08x)$

$1.15(1.08x) = 465.75$
$\frac{\cancel{1.242}x}{\cancel{1.242}} = \frac{465.75}{1.242}$
$x = 375$

$ 375 [3]

(d) Tanya invests $8500 in an account paying 3.1% per year compound interest.
At the end of 5 years she takes $9300 from the account to buy new equipment for the business.

Calculate how much money is left in the account after buying the new equipment.

$A = P\left(1 + \frac{r}{100}\right)^n$
$= 8500\left(1 + \frac{3.1}{100}\right)^5$
$= 8500(1.031)^5$
$= 9901.757$

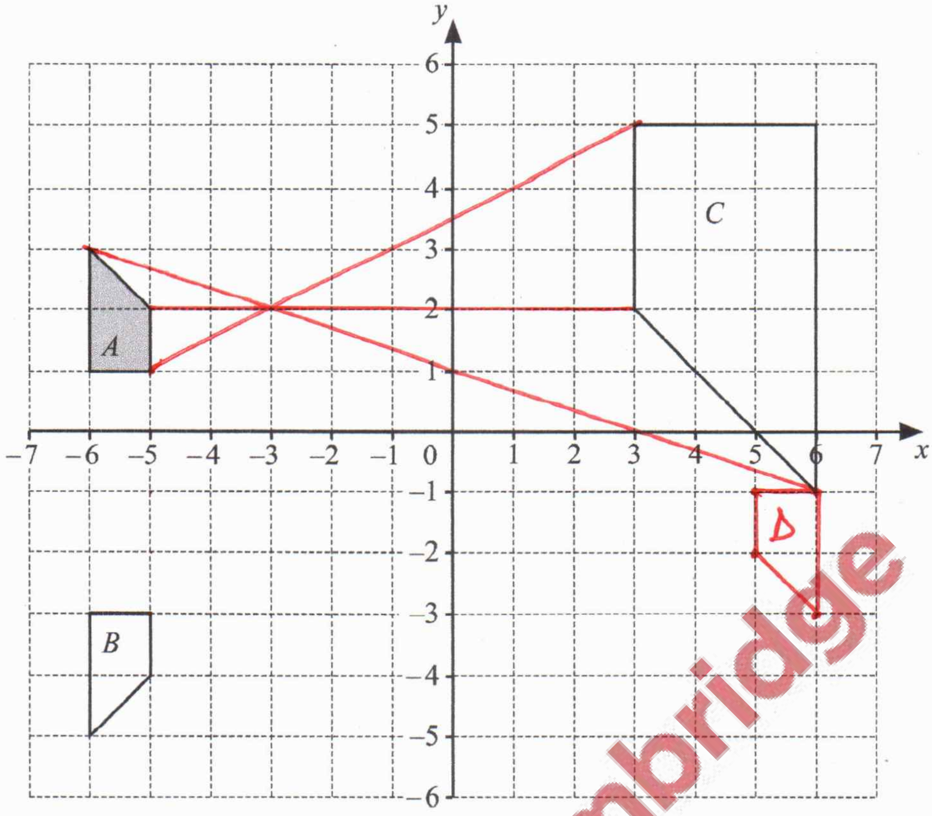
Remaining balance
= 9901.757 − 9300
= 601.757
≃ 601.76

$ 601.76 [3]

2

scale factor = -3

(-3, 2)



Shapes A, B and C are drawn on the grid.

(a) Shape A is mapped onto shape B by a reflection.

Write down the equation of the line of reflection.

$y = -1$ .... [1]

(b) Describe fully the **single** transformation that maps shape A onto shape C.

It is enlargement with scale factor -3 and centre (-3, 2) .... [3]

(c) Transformation T is represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

T maps shape A onto shape D.

Draw and label shape D. [2]

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -5 & -6 & -6 & -5 \\ 1 & 1 & 3 & 2 \end{pmatrix}$$

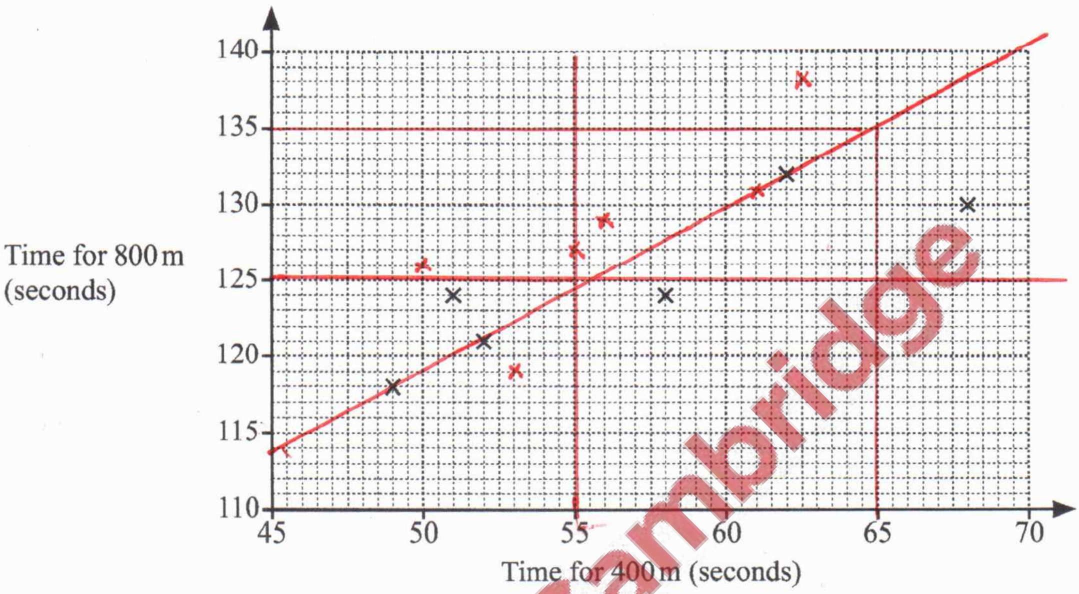
To = I

$$= \begin{pmatrix} 5 & 6 & 6 & 5 \\ -1 & -1 & -3 & -2 \end{pmatrix}$$

3 (a) The table shows the times, in seconds, taken for each of 12 members of an athletics club to run 400 metres and 800 metres.

Time for 400 m (seconds)	49	62	58	52	51	68	56	63	50	61	53	55
Time for 800 m (seconds)	118	132	124	121	124	130	129	138	126	131	119	127

(i) On the grid, complete the scatter diagram.
The first six points have been plotted for you.



[2]

(ii) Runners who took less than 55 seconds to run 400 metres **and** less than 125 seconds to run 800 metres are selected to enter an athletics competition.

How many of these runners are selected for the competition?

4 .... [1]

(iii) What type of correlation does the scatter diagram show?

Positive .... [1]

(iv) Draw a line of best fit on the scatter diagram. [1]

(v) Another runner took 65 seconds to run 400 metres.

Use your line of best fit to estimate how long they would take to run 800 metres.

135 .... s [1]

$p+10+15+13+q=50$ $p+q=50-38$

$38+p+q=50$ $p+q=12$

(b) 50 members of the athletics club attempted the high jump.
The table summarises the heights, in centimetres, of their jumps.

Height (h cm)	$140 < h \leqslant 145$	$145 < h \leqslant 150$	$150 < h \leqslant 155$	$155 < h \leqslant 160$	$160 < h \leqslant 165$
Frequency	p	10	15	13	q
Midpoint (x)	142.5	147.5	152.5	157.5	162.5
fx	142.5p	1475	2287.5	2047.5	162.5q

The athletics coach uses the mid-interval values to calculate an estimate of the mean height jumped by the 50 athletes.
His estimate of the mean is 153.6 cm.

(i) Explain why $p+q=12$.

The total number of member of the club is 50 and If we add all the frequency we find $p+q=12$. [1]

(ii) Show that $142.5p+162.5q=1870$.

$142.5p+1475+2287.5+2047.5+162.5q$

$\frac{142.5p+162.5q+5810}{50} = 153.6$ ×50

$142.5p+162.5q+5810 = 7680$

$142.5p+162.5q = 1870$

[2]

(iii) Find the value of p and the value of q.
Show your working.

$p+q=12$

$142.5p+162.5q=1870$

$p=12-q$

$142.5(12-q)+162.5q=1870$

$1710-142.5q+162.5q=1870$

$20q = 1870-1710$

$\frac{20q}{20} = \frac{160}{20}$

$q=8$

$p=12-q$

$p=12-8$

$p=4$

$p =$ 4

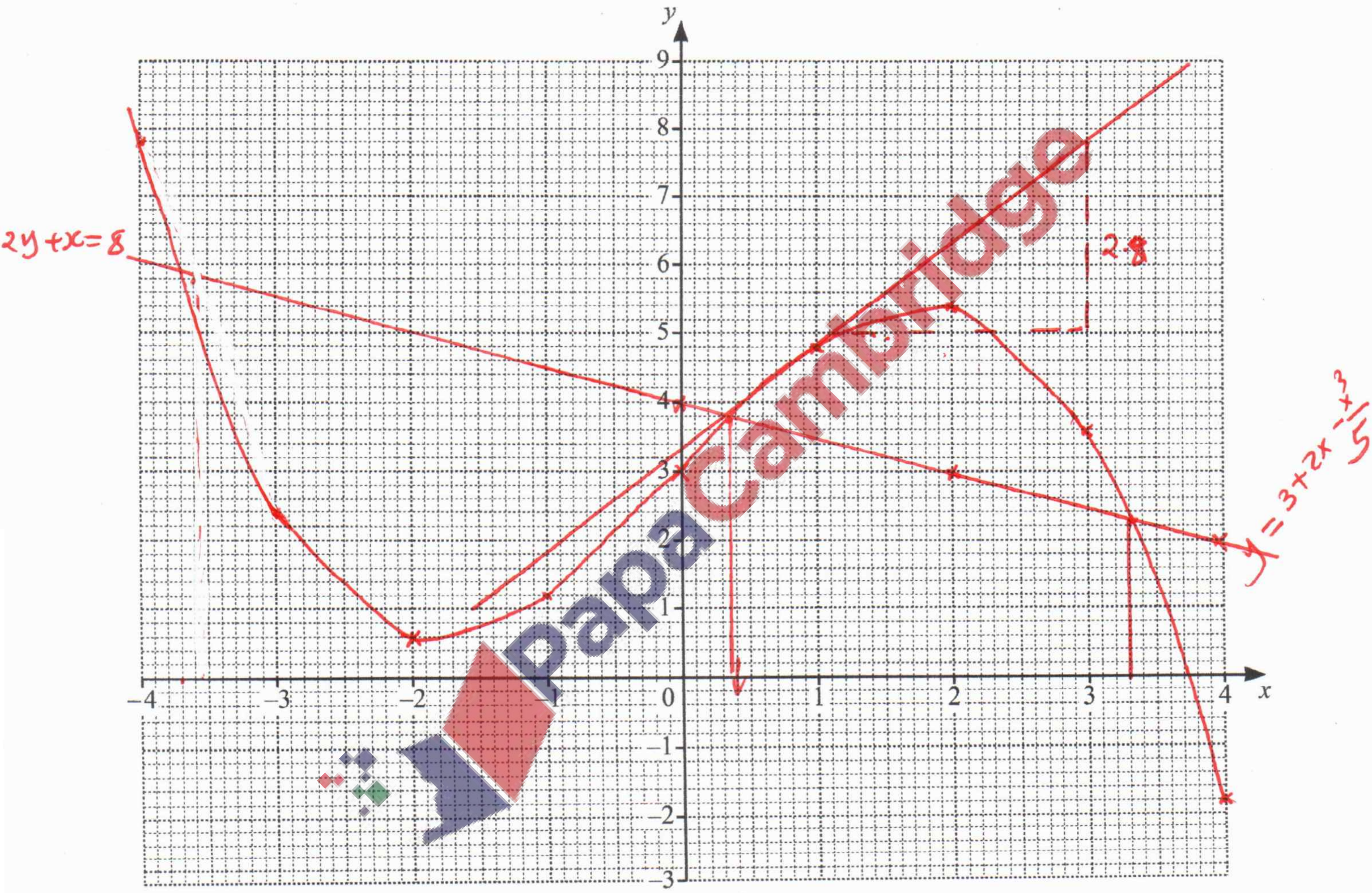
$q =$ 8 [3]

4 **(a)** Complete the table for $y = 3 + 2x - \frac{x^3}{5}$.

x	-4	-3	-2	-1	0	1	2	3	4
y	7.8	2.4	0.6	1.2	3	4.8	5.4	3.6	-1.8

[1]

(b) Draw the graph of $y = 3 + 2x - \frac{x^3}{5}$ for $-4 \leqslant x \leqslant 4$.



$x = -3.$ $\quad$ $x = 0.4$ $\quad$ $x = 3.3$

[3]

(c) By drawing a tangent, estimate the gradient of the graph of $y = 3 + 2x - \frac{x^3}{5}$ at (1, 4.8).

Gradient $= \frac{2.8}{1.85}$

$= \underline{1.5}$

1.5 .... [2]

(d) **(i)** On the grid, draw the line $2y + x = 8$.

$2y + x = 8$

x	0	8	4	2
y	4	0	2	3

[2]

(ii) Write down the x-coordinates of the points where the line intersects the graph of $y = 3 + 2x - \frac{x^3}{5}$.

−3.7, 0.4, 3.3 .... [2]

(iii) These x-coordinates are the solutions of the equation $2x^3 + Ax + B = 0$.

Find the value of A and the value of B.

$2y + x = 8$

$y = 3 + 2x - \frac{x^3}{5}$

$2\left(3 + 2x - \frac{x^3}{5}\right) + x = 8$

$6 + 4x - \frac{2x^3}{5} + x = 8$ (×5)

$30 + 20x - 2x^3 + 5x = 40$

$30 + 25x - 2x^3 = 40$

$-2x^3 + 25x = 40 - 30$

$-2x^3 + 25x = 10$

$-2x^3 + 25x - 10 = 0$

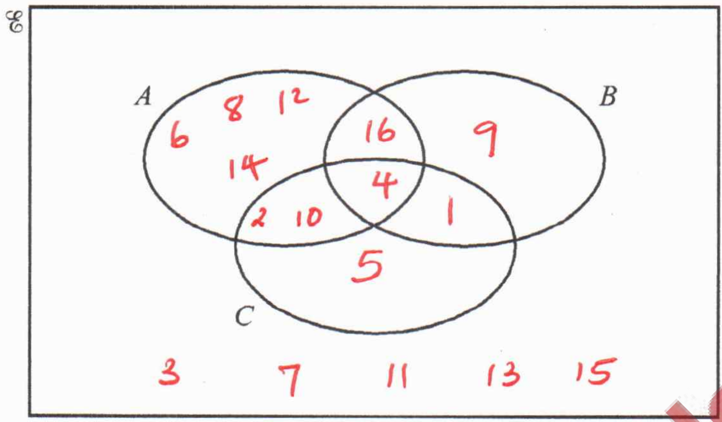
$\underline{2x^3 - 25x + 10 = 0}$

$A =$ −25 ....

$B =$ 10 .... [3]

5 **(a)** $\mathscr{E} = \{x : x \text{ is an integer } 1 \leqslant x \leqslant 16\}$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}
$A = \{x : x \text{ is an even number}\}$ A {2, 4, 6, 8, 10, 12, 14, 16,}
$B = \{x : x \text{ is a square number}\}$ B {1, 4, 9, 16}
$C = \{x : x \text{ is a factor of } 100\}$ C {1, 2, 4, 5, 10}

(i) Complete the Venn diagram.



[3]

(ii) Find $n(A' \cup B)$.

10 .... [1]

(iii) $p \in A \cap C$

Write down all the possible values of p.

2, 4, 10 .... [1]

(b) Ateeq has a set of 16 cards numbered from 1 to 16.

1, 4, 9, 16

(i) He takes a card from the set at random.

Find the probability that the card shows an odd square number.

P(odd Numbers) = $\frac{2}{16}$

$= \frac{1}{8}$

$\frac{1}{8}$ .... [1]

(ii) Ateeq takes two cards at random from the set of 16 cards.

Find the probability that both cards show even numbers that are factors of 100.

1, 2, 4, 5, 10

P(even numbers) = $\frac{3}{16}$

$\frac{3}{16} \times \frac{2}{15}$

$= \frac{6}{240}$

$= \frac{1}{40}$

$\frac{1}{40}$ .... [2]

6 (a) Rearrange the formula $v = \frac{3}{p+5}$ to make p the subject.

$\frac{V}{1} = \frac{3}{p+5}$

$V(p+5) = 3$

$Vp + 5V = 3$

$p + 5 = \frac{3}{V}$

$p = \frac{3}{V} - 5$

$p = \frac{3-5V}{V}$ [2]

(b) Express as a single fraction in its simplest form $\frac{x}{2x-5} + \frac{3}{x-6}$.

$\frac{x}{2x-5} + \frac{3}{x-6}$

$\frac{x(x-6) + 3(2x-5)}{(2x-5)(x-6)}$

$x^2 - 6x + 6x - 15$

$\frac{x^2 \cancel{-6x} \cancel{+6x} - 15}{(2x-5)(x-6)}$

$\frac{x^2 - 15}{(2x-5)(x-6)}$

$= \frac{x^2-15}{(2x-5)(x-6)}$

.... [3]

(c) Solve $\frac{4x+5}{1-3x} = \frac{2}{1}$.

$4x + 5 = 2(1-3x)$

$4x + 5 = 2 - 6x$

$4x + 6x = 2 - 5$

$\frac{10x}{10} = \frac{-3}{10}$

$x = -3/10$

$x = -3/10$ [3]

(d) Solve $5(x+3) = 2x(2x-1)$.
Show your working.

$5(x+3) = 2x(2x-1)$

$5x + 15 = 4x^2 - 2x$

$7x + 15 = 4x^2$

$4x^2 - 7x - 15 = 0$

$a = 4$
$b = -7$
$c = -15$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Quadratic equation Formula)

$\dfrac{7 \pm \sqrt{(-7)^2 - 4 \times 4 \times -15}}{8}$

$\dfrac{7 \pm \sqrt{49 - (-240)}}{8}$

$\dfrac{7 \pm \sqrt{289}}{8}$

$x = \dfrac{7 \pm 17}{8}$

$x = \dfrac{7 + 17}{8}$

$x = \dfrac{24}{8}$

$x = \underline{\underline{3}}$

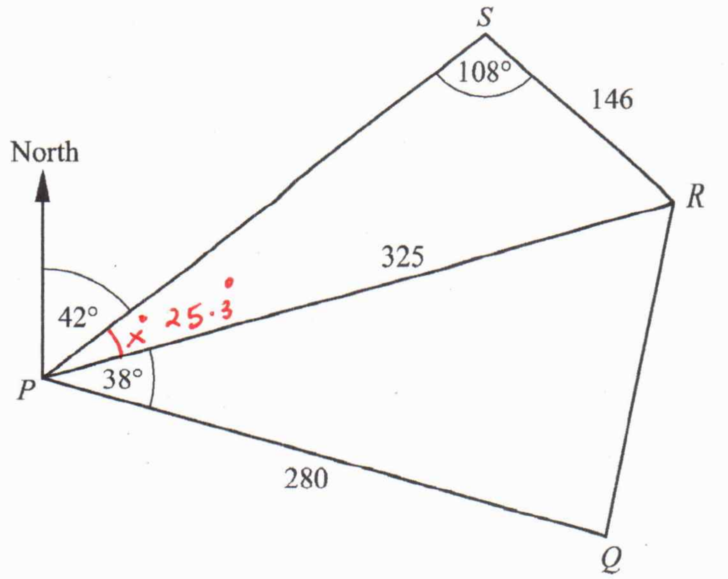
$\dfrac{7 - 17}{8}$

$x = -10/8$

$x = -5/4$ or $\underline{\underline{-1.25}}$

$x =$ ......−1.25...... or $x =$ ......3...... [4]

7



NOT TO SCALE

A field is in the shape of a quadrilateral *PQRS*.
A path crosses the field from *P* to *R*.
$PQ = 280$ m, $RS = 146$ m and $PR = 325$ m.
S is on a bearing of 042° from *P*, $P\hat{S}R = 108°$ and $R\hat{P}Q = 38°$.

(a) Calculate the bearing of *R* from *P*.

$\frac{146}{\sin x°} = \frac{325}{\sin 108}$

$\sin x° = \frac{146 \sin 108}{325}$

$\sin x°^{-1} = 25.29$

$\simeq 25.3°$

$42° + 25.3$

$= 67.3$

$67.3°$ .... [4]

(b) **(i)** Show that $QR = 202$ m, correct to the nearest metre.

$QR = 325^2 + 280^2 - 2 \times 325 \times 280 \cos 38°$

$QR^2 = 105{,}625 + 78400 - 182{,}000 \cos 38°$

$QR^2 = 184025 - 182000 \cos 38$

$QR^2 = 184025 - 143{,}417.957$

$QR^2 = \sqrt{40607.0428}$

$QR = 201.51$

$QR \simeq 202$ M

[3]

(ii) Mia walks at a constant speed of 5.5 km/h.

Calculate the time it takes her to walk from Q to R.
Give your answer in minutes and seconds, correct to the nearest second.

Q to R = $\frac{202\text{ m}}{1000}$ = 0.202 km

(60 mins) 1 hr → 5.5 km (5500 m)

1 min → $\frac{5500}{60}$ = $91\frac{2}{3}$ m

1 min → $91\frac{2}{3}$ m

? — 202 m

= $\frac{202 \times 1}{91\frac{2}{3}}$

= $2\frac{56}{275}$ mins

= 2 mins

= 2 mins 12 seconds

$\frac{56}{275} \times 60$

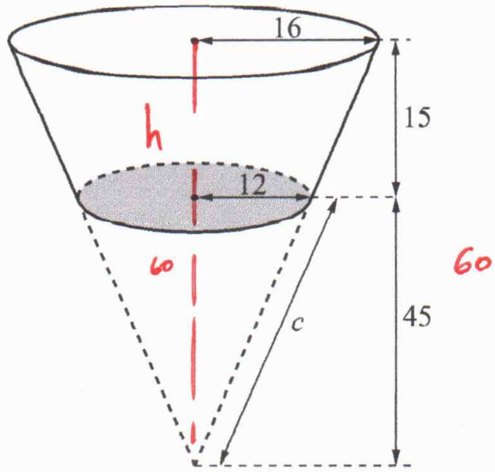
= 12.21818

= 12 seconds

...2... minutes ...12... seconds [3]

8 [Volume of cone $= \frac{1}{3}\pi r^2 h$]

[Curved surface area of a cone $= \pi r l$]



The diagram shows a bowl with a circular base.
The curved surface of the bowl is formed by removing a cone with radius 12 cm and height 45 cm from a larger cone as shown in the diagram.
The radius of the top of the bowl is 16 cm and its height is 15 cm.

(a) Calculate the volume of the bowl.

Volume of Bowl = Volume of Bigger − Volume of smaller

$= \left(\frac{1}{3} \times \pi \times 16 \times 16 \times 60\right) - \frac{1}{3} \times \pi \times 12 \times 12 \times 45$

$= 16084.95 - 6785.840$

$= 9299.11$

$= \underline{9299}$

.........9299......... cm³ [3]

(b) The slant height of the cone that has been removed is c cm.

Show that $c = 46.6$, correct to 3 significant figures.

$c^2 = 12^2 + 45^2$

$c^2 = \sqrt{144 + 2025}$

$c^2 = \sqrt{2169}$

$C = 46.572$

$C = \underline{46.6 \text{ cm}}$

[2]

(c) The bowl is completely filled with water.

Calculate the total surface area of the bowl that is in contact with the water.

l (slant height of the whole cone

$l^2 = 16^2 + 60^2$

$l^2 = 256 + 3600$

$l^2 = 3856$

$l = \sqrt{3856}$

$l = 62.096$

Curved surface area:

Surface area of bigger cone $= \pi r l$

$= \pi \times 16 \times 62.096$

$= 3121.285 \text{ cm}^2$

Smaller cone $= \pi r l$

$= \pi \times 12 \times 46.6$

$= 1756.7786 \text{ cm}^2$

$= 3121.285 - 1756.7786$

$= 1364.506$

$A = \pi r^2 = \pi \times 12 \times 12$

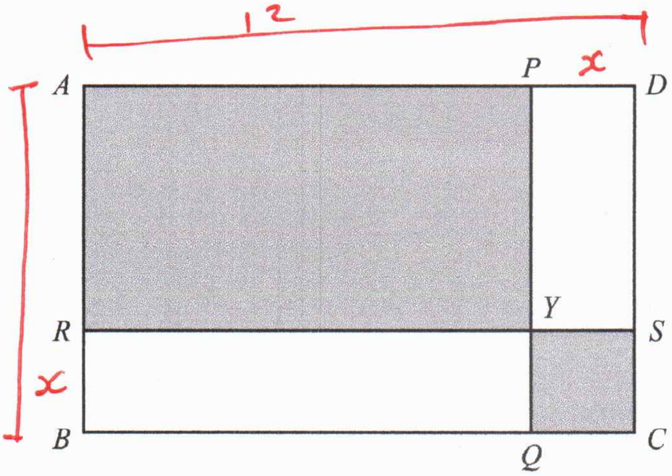
$= 452.389 \text{ cm}^2$

Total surface area $= 1364.506 + 452.389$

$= 1816.90 \text{ cm}^2$

1816.90 .... cm^2 [4]

9



NOT TO SCALE

$ABCD$ is a rectangle with $AB = 10$ cm and $AD = 12$ cm.
PQ is parallel to AB and RS is parallel to AD.
Y is the point of intersection of PQ and RS.
$BR = DP = x$ cm.
A smaller rectangle and a square are shaded.

(a) Write down an expression, in terms of x, for

(i) AR,

$AR = 10 - x$

$10 - x$ .... cm [1]

(ii) AP.

$12 - x$

$12 - x$ .... cm [1]

(b) $\dfrac{\text{Area of shaded rectangle } ARYP}{\text{Area of rectangle } ABCD} = \dfrac{3}{4}$

Show that $x^2 - 22x + 30 = 0$.

$A = L \times W$
$= (12 - x)(10 - x)$
$= 12(10 - x) - x(10 - x)$
$= 120 - 12x - 10x + x^2$
$= 120 - 12x - 10x + x^2$
$= 120 - 22x + x^2$

$\dfrac{120 - 22x + x^2}{120} \cancel{=} \; 3/4$

$4(120 - 22x + x^2) = 360$
$480 - 88x + 4x^2 = 360$
$4x^2 - 88x = 360 - 480$
$4x^2 - 88x = -120$
$\dfrac{4x^2}{4} - \dfrac{88x}{4} + \dfrac{120}{4} = 0$
$x^2 - 22x + 30 = 0$

[3]

(c) Solve the equation $x^2 - 22x + 30 = 0$.
Show your working and give your answers correct to 2 decimal places.

$a = 1$
$b = -22$
$c = 30$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{22 \pm \sqrt{(-22)^2 - 4 \times 1 \times 30}}{2}$

$\frac{22 \pm \sqrt{484 - 120}}{2}$

$\frac{22 \pm \sqrt{364}}{2}$

$\frac{22 \pm 19.078}{2}$

$x = \frac{22 + 19.078}{2}$ or $\frac{22 - 19.078}{2}$

$= 20.539$ $= 1.461$
$= \underline{20.54}$ $= \underline{1.46}$

$x =$ 1.46 or $x =$ 20.54 [3]

(d) Find the total **unshaded** area in rectangle *ABCD*.

$x(12 - x) + x(10 - x)$

$1.46(12 - 1.46) + 1.46(10 - 1.46)$

$= 1.46(10.54) + 12.4684$

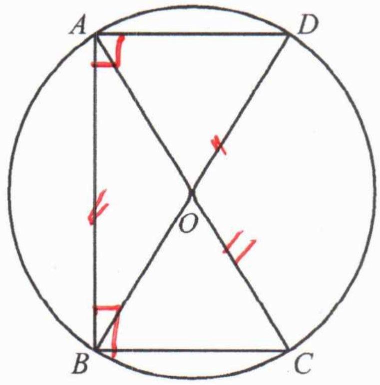
$= 15.3884 + 12.4684$

$= 27.8568$

$= \underline{27.86}$

27.86 cm^2 [2]

10 (a)



NOT TO SCALE

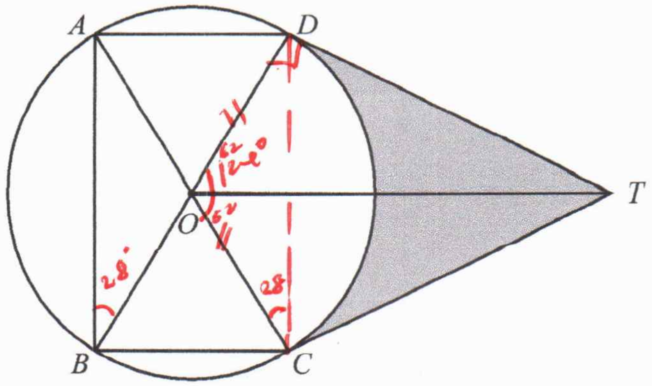
AC and *BD* are diameters of the circle, centre *O*.

Show that triangle *ABC* is congruent to triangle *BAD*.
Give a reason for each statement you make.

AC = BD, they are diameters of the Centre of the Circle and are same. ∠ DAB and ∠ ABC is 90°. (Angles in a semi-circle equals to 90°. AB = AB same distance in both triangles. [3]

∠ BAC = ∠ ABC Angles of Isosceles Triangle are equal.

(b)



NOT TO SCALE

Two tangents, TC and TD, are drawn to the circle in **part (a)**.
The diameter of the circle is 8 cm and $A\hat{B}D = 28°$.

(i) Find $C\hat{O}D$.

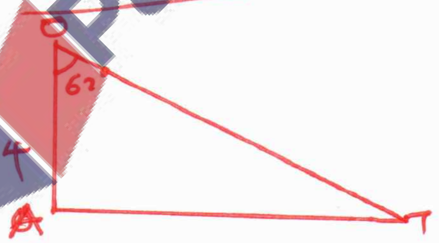
$\angle COD = 180° - (28 + 28)$
$= 180 - 56$
$= 124°$

$C\hat{O}D =$ ....124°.... [2]

(ii) Calculate the area shown shaded in the diagram.

Area of sector DOC $= \frac{62°}{360} \times \pi \times 4 \times 4$
$= 8.6568 \text{ cm}^2$

OT =



$OT \times \cos 62° = \frac{4}{OT} \times OT$

$OT = \frac{4}{\cos 62}$

$OT = 8.52$

$DT^2 = 8.52^2 - 4^2$
$= 72.5904 - 16$
$= \sqrt{56.5904}$
$= 7.523$

Area of Triangle ODT
$= \frac{1}{2} \times 4 \times 7.523$
$= 15.046 \text{ cm}^2$

Shaded area
$= 15.046 - 8.6568$
$= 6.3892 \times 2$
$= 12.7784$
≈ 12.778

....12.778.... cm^2 [5]