FURTHER MATHEMATICS

Paper 1348/01
Further Pure Mathematics

Key messages

There are significant points that candidates need to bear in mind before they sit this paper, many of which are highlighted again in the later comments on individual questions. Firstly, this is a long and tough paper and few candidates complete attempts at all questions. Thus, for each individual candidate, it is very important to choose a suitable order in which to attempt the questions, one that maximises the opportunities to gain high marks. Also, there is consequently little time in which candidates will have an opportunity to review or correct working, so care must be taken to ensure that work is done without the need for re-evaluation. Oftentimes, candidates are best served by a thoughtful approach to each question that considers – in particular – how the question is structured (where appropriate); many candidates frequently ignore the very careful signposts offered within the question. In a similar vein, candidates must be aware of how appropriate the working they are writing down is to the demand of that question or question-part; producing up to a page-and-a-half of working for a result that has been assigned just 1 or 2 marks is clearly inappropriate, and candidates should allow themselves to be guided by the number of marks.

Another key point is that many candidates seem to lack confidence when deploying prior learning in the further maths setting – almost as if they have forgotten about them altogether – and they should be aware that understanding and fluency in GCSE-level and (single) Maths-9794-level techniques are taken as 'background/assumed knowledge' within the further mathematics papers. Moreover, when such work is required within one of the further pure mathematics questions, they are often assigned relatively few marks compared to those assigned to the higher level topics of work that are primarily being tested here.

Finally, a consistent shortfall in the scoring of marks arises whenever an explanation, justification or proof is required of candidates: or, in a similar vein, a diagram with all appropriate points/lines/asymptotes, etc., clearly marked on it. Such opportunities arose this year in **Questions 3, 6, 9, 10, 11** (especially) and **12** and candidates were not averse to skipping through them without taking sufficient care with the 'small details'.

General comments

Overall, the quality of candidates' work continues to be of a very high standard and, despite the very off-putting appearance of the final question this year, almost all candidates made some attempt at every question on the paper. Marks for the first nine questions were very high indeed, falling (on average) only 1 or 2 short of the maximum mark available. The work produced within scripts for these questions revealed a cohort of capable and well-prepared candidates.

It was **Questions 10–12** that provided the real challenge to candidates, with a very long and tough **Question 12** the primary culprit in this respect: this probably accounts for the absence of anyone scoring full marks this year.

Comments on specific questions

Question 1

This was a gentle introduction to the paper and was well done by almost all candidates. The only slips of any consequence related to carelessness with minus signs and in forgetting to make sure that the values for a and b were paired correctly.

Answer: $a = \pm 5$, $b = \mp 2$

Question 2

This question was also found to be quick and easy by almost all candidates. Most took the expected route via $\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$, although a small number opted to find the equation with roots α^2, β^2 and γ^2 instead and then pick out the coefficient of the quadratic term. Most comments correctly spotted that (at least) one of the roots had to be complex in order for a sum of squares to be negative, although rather too many candidates seemed to be unaware of the difference between the terms 'complex' and 'imaginary' – they were not punished for any lack of clarity in this respect, however. Many candidates went further still and explained that exactly one root was real and the other two were complex conjugates; though this, of course, follows from the fact that the original equation has all of its coefficients real.

Answer: –2

Question 3

This question was also well-handled by almost everyone, although quite a few candidates overlooked the need to show some sort of scale on their sketch-graph (the point (1, 0) would have sufficed). Apart from a small number of minor mistakes in the integration work, **part (ii)** went very smoothly too.

Answer: **(ii)** $\dfrac{\pi}{1+2\pi}$

Question 4

For the most part, this was all done very capably by almost all candidates. The few exceptions seemed to be those who either couldn't recall the correct formula or who had difficulty interpreting the result that appears in the examination's accompanying Formula Booklet. Following that, the key is to realise that $1 + \left(\dfrac{dy}{dx}\right)^2$ needs to be a perfect square. Those who managed to clear these hurdles gained 6 or 7 of the marks; those who did not got 1 or 2 at best.

Answer: 96π

Question 5

This question was also done very well overall. **Part (i)** was intended to be a standard piece of bookwork, requiring candidates to turn the statement $y = \tanh^{-1}x$ into $\tanh y = x$ and use the given definition to deduce the required result. However, some preferred to simply verify by substitution instead and this works equally well. **Part (ii)** was open to a variety of approaches – the mark scheme for the paper has all three of those that were seen, with the first two appearing for the most part and with approximately equal frequency.

Answer: **(ii)** $\tfrac{1}{4}\ln 3$

Question 6

Questions on rational functions and their graphs have proved to be immensely popular with candidates over recent years and this year's question proved no exception. However, there were still a few hiccups to be found on the scripts, the principal ones involving oversights or miscalculations regarding the horizontal asymptote, errors in using the *Quotient Rule of Differentiation* in **part (ii)** and incorrectly drawn graphs in **part (iii)**, where the left-hand 'half' was occasionally missing altogether or, more commonly, drawn below the x-axis.

Answers: **(i)** $x = -1$, $y = 1$ **(ii)** $\dfrac{dy}{dx} = \dfrac{2(x-1)}{(x+1)^3}$ $(1, \tfrac{1}{2})$

Question 7

This was a relatively routine second-order differential equation question and it was dealt with appropriately by almost everyone. Just occasionally, a candidate would overlook the need to produce an answer *without* complex coefficients: it is expected that a complementary function of the form $Ae^{2ix} + Be^{-2ix}$ will be turned into one of the form $C\cos 2x + D\sin 2x$ in order to gain full marks.

Answers: **(i)** $k = 2$ **(ii)** $y = \cos 2x + (2x + \frac{1}{2})\sin 2x$

Question 8

This vectors question was handled very well and with an interesting variety of approaches. Almost everyone ignored the straightforward trigonometric approach in **part (i)(b)**, for instance, with the most usual alternative involving the well-known formula quoted in the Formula Booklet. In **part (i)(a)**, a very small number of candidates confused the angle between **d** and **n** with the angle between the line and the plane; otherwise, things progressed nicely, apart from those candidates who worked out the distance between the two planes in **part (iii)** in bits and muddled up some scale-factor or other along the way.

This is a good point at which to mention another unhelpful feature of many candidates' working, where the tendency is simply to write down a whole load of numbers with little indication as to what they represent; this may be easy to interpret positively by the marker when the right answer appears at the end of it all, but it is often of little help otherwise when there is no apparent structure or process to what has been committed to writing. Candidates need to be aware that an examiner is unable to reward what they think the candidate might be doing, and method marks require some visible structure that can be followed.

Answers: **(i)(a)** $\frac{20}{21}$ **(b)** $\frac{7}{4}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ **(c)** 5 **(ii)** $\frac{13}{2}$

Question 9

This question on a mixed bag of tricky calculus matters, involving both inverse trigonometric functions and hyperbolic functions, was expected to be found very tough, since little like it has appeared on the papers before now. However, it was handled almost universally successfully by candidates. The only possible point of interest arose with the taking of square-roots and the need to justify that only the positive one applied in each of **parts (i)** and **(ii.**

Answer : **(iii)** $\dfrac{-\sec^{-1}x}{x} + \dfrac{\sqrt{x^2-1}}{x}$ (+ C)

Question 10

This was the point at which things got significantly more demanding on this paper. Part of the reason for the extra difficulty on this question is, firstly, that it is almost entirely algebraic. Secondly, a numerical or sign error early on naturally led to the loss of several of the following marks. Finally, **part (iv)** required a little bit of constructive thought from candidates and the time constraint factor, for the paper as a whole, was probably starting to tell.

To begin with, there were quite a few candidates who did not appreciate the standard 'partial fraction form' that was being demanded of them; these few candidates were thus barred from any chance of successful progress with either of **parts (i)** or **(ii)**. Candidates did not deal with the partial fractions using the '*Cover-Up Method*', which would have meant that the terms in **part (i)** could simply have been written straight down. Beyond that small detail, however, it was noticeable that most candidates preferred to work with

$\dfrac{1}{2(k-1)} - \dfrac{1}{k} + \dfrac{1}{2(k+1)}$ rather than $\dfrac{\frac{1}{2}}{k-1} - \dfrac{1}{k} + \dfrac{\frac{1}{2}}{k+1}$, and keeping the $\frac{1}{2}$s well away from the denominators makes the working much less awkward. The other helpful hint (for future reference) would be that it is easier to cancel terms in **part (ii)** when the two positive terms appear side-by-side *before* attempting to subtract the remaining one; in this way, the reciprocals of the integers in the summation generally appear once as a positive and once as a negative and the cancelling of terms is then far more readily identified (and justified).

Part (iv) was usually rather poorly handled, as most candidates were unable able to figure out where the $\frac{27}{24}$ and the $\frac{29}{24}$ had to come from.

Answer : **(i)** $\dfrac{\frac{1}{2}}{k-1} - \dfrac{1}{k} + \dfrac{\frac{1}{2}}{k+1}$

Question 11

Groups questions have traditionally been found difficult in all cases except the most straightforward ones. This one was a mixture of the easy and the tough and, although almost all candidates scored some marks, they still managed to miss out on several more. While **parts (i)(a)**, **(b)** and **(iii)(a)**, **(b)** provided easy access to 6-8 marks for most, parts **(ii)** and **(iii)(c)** were sources of considerable misunderstanding as to the true nature of the task.

The Principal Examiner would direct future candidates and their Teachers to the mark-scheme for **part (ii)** for guidance, as there were lots of marks unacquired by those who undoubtedly thought they had scored most, if not all, of the 5 available to them. The real issue here was that the elements of G were not just 2×2 matrices, but rather those whose determinants were equal to 1. It was, therefore, insufficient (for example) merely to point out that the inverse of a given matrix existed (which is a known result for all such matrices with non-zero determinant); it was important to justify that it was actually **in** G because its determinant was 1. A similar statement was also required to justify the assertion that the 2×2 identity matrix **I** was in G. The whole point of **part (i)** was to supply candidates with the means to justify the '*closure*' property, though this fact was overlooked by rather a lot of them.

By the time it came to **part (iii)(c)**, most candidates seemed to have decided it was time to move on to the final question as there were very few serious suggestions offered here. Even among this minority, most did not notice the requirement that the second subgroup needed to be in G also and they merely proposed another cyclic group of order 6. The anticipated subgroup was that of the rotations through multiples of 60° but there were alternatives involving a variation on the theme of **K**; however, as these were about fifty-fifty for correctness, it was not entirely clear whether some candidates were just luckier guessers than their peers. Even so, it was not apparent to all but a few that they should make any attempt to justify that their proposal was indeed both a subgroup of G **and** isomorphic to H.

Answers: **(i)(a)** $\mathbf{AB} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$ det $\mathbf{A} = ad - bc$, det $\mathbf{B} = eh - fg$ **(iii)(b)** $n = 6$

Question 12

This was the most demanding question on the paper. However, for any candidate prepared to hold their nerve there were plenty of carefully directed marks available, and it was only the induction in **part (iii)** that was actually especially tough; even then, there are always 2 or 3 marks available on any induction question for starting off the process.

Oddly enough, one of the biggest hurdles to steady progress lay with a lot of candidates' refusal to adopt the 'c' for $\cos\theta$, 's' for $\sin\theta$ abbreviation suggested at the outset. The difficulties that followed revolved around the extraordinary amount of extra writing that these candidates had to undertake and the difficulty that naturally arose from trying to sort out the sea of letters that they were then confronted with. The next factor that largely went unnoticed is that key parts of **parts (i)**, **(ii)(b)** *and* **(iii)** all fall nicely out when using the most basic of trigonometric identities, namely $c^2 + s^2 = 1$. Fortunately, all parts of this question could be successfully tackled by any one of several methods, and with the necessary contributory bits applied in a variety of orders.