CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2014 series

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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CAMBRIDGE
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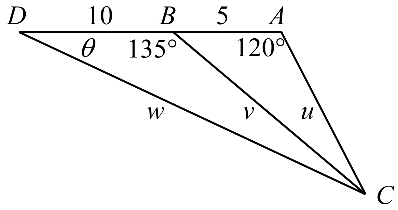
1 **(i)**	$X - Y \sim N(10, 5)$ $z_1 = \frac{8-10}{\sqrt{5}}$ [= –0.894…]; $z_2 = \frac{11-10}{\sqrt{5}}$ [= 0.447…] P (8 < Breadth < 11) = **0.487**	 Either. cc or σ^2: M1A0 Both. FT on σ cwo	B1 M1A1 A1 [4]
(ii)	$X_1 + X_2 + Y_1 + Y_2 \sim N(80, 10)$ $z = \frac{75-80}{\sqrt{10}}$ [= –1.58…] P (Perimeter > 75) = **0.943**	 As above [e.g. N(80, 20), 0.8681: 2/4] cwo	 B1 M1A1√ A1 [4]
2 **(i)**	$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$ stated or implied $\frac{\mu + 0.5\sigma - \mu}{\sigma / \sqrt{n}} > 1.96$ $\Rightarrow 0.5\sqrt{n} > 1.96 \Rightarrow n > 15.355 \Rightarrow$ Least n is 16	 1.96 	B1 M1A1 M1A1 [5]
(ii)	$z = \frac{\mu - 0.1\sigma - \mu}{\sigma / 4} = -0.4$ $\Rightarrow P(\overline{X} > \mu - 0.1\sigma) = 0.655$	FT on their integer n, and also –0.392: M1A1A0	M1 A1√ A1 [3]
3 **(i)**	Let N be the estimated number; $\frac{400}{N} = \frac{20}{400} \Rightarrow N = 8000$		B1 [1]
(ii)	98% confidence: $z = \pm 2.326$ $p_s = 0.05$ $0.05 \pm 2.326\sqrt{\frac{0.05 \times 0.95}{400}}$ = **(0.0246(5), 0.0753(5))**	 Need $\sqrt{\frac{pq}{n}}$ 0.05 and variance correct Both, correct to 3 SF	B1 B1 M1A1 A1 [5]
(iii)	400 ÷ 0.07535 = 5309 and 400 ÷ 0.02465 = 16228	400 ÷ CV Integer answers, a.r.t. 5310, 16200	M1 A1 [2]

4 **(i)**	$\left[-3e^{-x}\right]_0^k = 1 \quad \Rightarrow -3e^{-k}+3=1 \quad \Rightarrow \ e^{-k} = \frac{2}{3}$ **[AG]**		M1A1 [2]
(ii)	$M_X(t) = \int_0^k 3e^{(t-1)x}dx = \left[3\frac{e^{(t-1)x}}{(t-1)}\right]_0^k$ $= 3\frac{e^{(t-1)k}}{(t-1)} - \frac{3}{(t-1)} = \frac{3}{(t-1)}\left(1-e^{-k}e^{kt}\right)$ $= \frac{3}{(t-1)}\left(1-\frac{2}{3}e^{kt}\right)$ **[AG]**		M1 A1 M1 A1 [4]
(iii)	$M_X(t) = 3\left(1+t+t^2\right)\left(1-\frac{2}{3}\left[1+kt+\frac{1}{2}k^2t^2\right]\right)$ $= 1 + (1-2k)t + (1-2k-k^2)t^2$ **[AG]**	Both series correct Deals with $\frac{2}{3}$	B1 B1 B1 [3]
(iv)	$E(X) = 1 - 2k = 1 - 2\ln\left(\frac{3}{2}\right)$ **[AG]**	Allow longer methods if correct	B1 [1]
5 **(i)**	$G(t) = e^{\lambda(t-1)}$ $G'(t) = \lambda e^{\lambda(t-1)}$ $\Rightarrow \mu = G'(1) = \lambda$ $G''(t) = \lambda^2 e^{\lambda(t-1)}$ $\Rightarrow G'(1) = \lambda^2$ $\therefore \sigma^2 = E(X^2) + \mu - \mu^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$	In tables	M1 A1 M1 A1 A1 [5]
(ii)	$z_1 = \frac{229.5-250}{\sqrt{250}} = -1.297$ $z_2 = \frac{260.5-250}{\sqrt{250}} = 0.664$ $P(230 \le X \le 260) =$ **0.649**	One cc correct Both ccs, √250 Correct handling of tails Answer in range [0.649, 0.650]	M1A1 A1 M1 A1 [5]
(iii)	We are approximating to B (250, 0.1). Using N (25,22.5) or Using Po (25) $z = \frac{30.5-25}{\sqrt{22.5}}(=1.1595)$ $z = \frac{30.5-25}{\sqrt{25}}(=1.1)$ P(>30) = **0.123** P(>30) = **0.136**	Distribution stated or implied Correct CC and √ Exact binomial 0.125: M0 Exact Poisson 0.136691: 3	M1 A1 A1 [3]

6 **(i)**	$\frac{4}{\pi}\left[\tan^{-1}x\right]_0^{0.41} = 0.495\ldots, \frac{4}{\pi}\left[\tan^{-1}x\right]_0^{0.41} = 0.506\ldots$	Correct integral	M1
	$\Rightarrow 0.41 <$ median < 0.42 **[AG]**	One correct output	A1
	Or: $\frac{4}{\pi}\left[\tan^{-1}x\right]_0^{m} = 0.5$; $m = \frac{\pi}{8} = 0.414\ldots$	Conclusion fully justified (need some comment if 0.41 and 0.42 used)	A1 [3]
(ii)	$\mathrm{E}(X) = \frac{4}{\pi}\int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x$	$\int x\,\mathrm{f}(x)\,\mathrm{d}x$ attempted	M1
	$= \frac{2}{\pi}\ln\left[1+x^2\right]_0^1 = \frac{2}{\pi}\ln 2$ **[AG]**		A1 [2]
(iii)	$\mathrm{E}(X^2) = \frac{4}{\pi}\int_0^1 \frac{x^2}{1+x^2}\,\mathrm{d}x\ \ \frac{4}{\pi}\int_0^1 1-\frac{1}{1+x^2}\,\mathrm{d}x$	$\int x^2\,\mathrm{f}(x)\,\mathrm{d}x$ seen Method for integration	M1 M1
	$= \frac{4}{\pi}\left[x-\tan^{-1}x\right]_0^1 = \frac{4}{\pi}\left(1-\frac{\pi}{4}\right) = \frac{4}{\pi}-1$	$\left(\frac{4}{\pi}-1\right)$ or 0.2732	A1 M1
	$\mathrm{Var}(X) = \left(\frac{4}{\pi}-1\right) - \left(\frac{2}{\pi}\ln 2\right)^2 = 0.0785$	Answers from calculator can get full marks	A1 [5]
(iv)	$\frac{4}{\pi}\left[\tan^{-1}x\right]_{\sqrt{2}-1}^{1} = 1-\frac{1}{2} = \frac{1}{2}$	Quotient of relevant probabilities	M1
	$\frac{4}{\pi}\left[\tan^{-1}x\right]_{\sqrt{3/3}}^{1} = 1-\frac{2}{3} = \frac{1}{3}$	One correct	A1
	$\mathrm{P}(X > 1/3\sqrt{3} \mid X > \sqrt{2}-1) = \frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$ or $\dfrac{\frac{\pi}{4}-\frac{\pi}{6}}{\frac{\pi}{4}-\frac{\pi}{8}} = \frac{2}{3}$		A1 [3]
7	Recognising (3, 4, 5) triangle Resolving vertically for P: $T = 0.5\mathrm{g}$ Resolving vertically for Q: $\frac{3}{5}T = mg$ Resolving horizontally for Q: $\frac{4}{5}T = 4m\omega^2$ $\Rightarrow$ $m = 0.3$ $\omega = \sqrt{\frac{g}{3}}$ or **1.83**	[can award B1 if answer correctly obtained without use of $T = 5$] Needs resolving with trig (if resolve ∥ QR must have cos or sin × $mr\omega^2$)	B1 B1 M1 A1 M1 A1 A1 A1 [8]

8	Components of speed ‖ to AB are 12 and 3 ms^{-1}. CLM: $2 \times 12 - 3 \times 3 = 2v_A + 3v_B$ NEL: $-\frac{2}{3}(12+3) = v_A - v_B$ $v_A = -3$, $v_B = 7$ Components of velocity ⊥ to AB are 5 and 4 ms^{-1}. Speed $A = \sqrt{3^2+5^2} = \sqrt{34}$ or 5.83 ms^{-1}, Speed $B = \sqrt{7^2+4^2} = \sqrt{65}$ or 8.06 ms^{-1} (OE)	 *Any* signs on RHS for A1 *Consistent* signs on RHS for A1 (both √ on x-components) Both either Both answers, correct to 3SF if necessary	B1 M1A1√ M1A1√ A1 B1 M1 A1 [9]
9 **(i)**	$750 = \frac{1}{2}(5+15)t \Rightarrow t = 75$	If a [$=\frac{2}{15}$] found, need second equation for M1 Or: Energy: $750F = \frac{1}{2}m(15^2 - 5^2)$ $15 = 5 + \frac{F}{m}t \Rightarrow t = \frac{10m}{F} = 75$	M1 A1 (B1) (B1) [2]
(ii)	Let P denote the constant power of the engine. Newton II: $\frac{P}{v} = m\frac{dv}{dt} \Rightarrow \frac{P}{m}\int_0^t dt = \int_5^{15} v\,dv$ $\Rightarrow \frac{Pt}{m} = \left[\frac{v^2}{2}\right]_5^{15} (=100)$ Also $\frac{P}{v} = mv\frac{dv}{dx} \Rightarrow \frac{P}{m}\int_0^{750} dx = \int_5^{15} v^2\,dv$ $\Rightarrow \frac{750P}{m} = \left[\frac{v^3}{3}\right]_5^{15} \left(=\frac{3250}{3}\right)$ $\Rightarrow \frac{P}{m} = \frac{13}{9}t$ and $t = \frac{900}{13}$ or **69.2 seconds**	For $F = \frac{P}{v}$ seen If ΔKE used, must equate to Pt	B1 M1 A1 A1 A1 M1 A1 A1 A1 [9]

10 (i)	Hooke's Law: $\frac{0.05\lambda}{0.5} = 2.5g \Rightarrow \lambda = 250$ N **[AG]**		M1A1 [2]
(ii)	Let e m be the extension in the new equilibrium position. $4g = \frac{250e}{0.5} \Rightarrow e = 0.08$ m Newton II: $4\ddot{x} = 4g - \frac{250(0.08+x)}{0.5} \Rightarrow \ddot{x} = -125x$	Find new e Needs 0.08√ SR: $\ddot{x} = 10 - 125x$ and then $x = y - 0.08$: B2	M1 A1 M1 A1 [4]
(iii)	**(a)** Amplitude of new motion is 0.08 – 0.05 = 0.03 m	cwo	B1 [1]
	(b) Maximum speed = $\sqrt{125} \times 0.03 = 0.335$ ms^{-1}.	√ on a or ω	B1√ [1]
	(c) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{125}} = 0.562$ seconds.	√ on ω	B1√ [1]
	(d) $\frac{1}{\sqrt{125}}\left(\frac{\pi}{2} + \sin^{-1}\frac{2}{3}\right) = \mathbf{0.206}$ seconds	$\sin^{-1}$("$\frac{2}{3}$")/ω or $\cos^{-1}$("$\frac{2}{3}$")/ω Deal with quadrants	M1 M1 A1 [3]
11 (i)	$gx^2 \tan^2\alpha - 2V^2x \tan\alpha + (gx^2 + 2V^2y) = 0$ or equiv For boundary of accessible points $B^2 - 4AC = 0$ $\Rightarrow 4V^4x^2 - 4gx^2(gx^2 + 2V^2y) = 0$ (*) $\Rightarrow 2gV^2y = V^4 - g^2x^2 \Rightarrow y = \frac{1}{2gV^2}\left(V^4 - g^2x^2\right)$ **[AG]**	y must be part of quadratic Allow $\Delta \geq 0$	B1 M1 A1 A1 [4]
	Or: $\frac{dy}{d\alpha} = x\sec^2\alpha - \frac{gx^2}{2V^2}\tan\alpha\sec^2\alpha$ $= 0 \Rightarrow x = \frac{V^2}{g\tan\alpha} \Rightarrow y = \frac{1}{2gV^2}\left(V^4 - g^2x^2\right)$	"Quadratic equation" required in question, so B0	B0M1 A1 A1 [max 3]
(ii) (a)	Putting $y = -h$ and $V^2 = gh \Rightarrow -h = \frac{1}{2g^2h}\left(g^2h^2 - g^2x^2\right)$ $\Rightarrow x = \sqrt{3h}$	$y = h$: M1A0	M1 A1 A1 [3]
(b)	Substituting for x in (*) gives $3gh^2 \tan^2\alpha - 2\sqrt{3}\,gh^2 \tan\alpha + 3gh^2 - 2gh^2 = 0$ $\Rightarrow 3\tan^2\alpha - 2\sqrt{3}\tan\alpha + 1 = 0$ $\Rightarrow \tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \boldsymbol{\alpha = 30°}$ or $\boldsymbol{\frac{\pi}{6}}$	Or quote $\tan\alpha = \frac{-B}{2A}$	M1 A1 A1 [3]

	Notation: Speed of wind relative to: ground, w; cyclist at $15\,\text{ms}^{-1}$, u; cyclist at $10\,\text{ms}^{-1}$, v Angle of wind velocity with due west, θ (see diagram in method α)		
12	**Method α:**  $\angle ABC = 45°$ $\angle BAC = 120°$ $\angle BCA = 15°$	Diagram, with all three	B1
	$\dfrac{v}{\sin 120°} = \dfrac{5}{\sin 15°} \Rightarrow v = \dfrac{5\sin 120°}{\sin 15°} = 16.73$	Or : $\dfrac{u}{\sin 135°} = \dfrac{5}{\sin 15°}$	M1A1 A1
	$w^2 = 10^2 + 16.73^2 - 2 \times 10 \times 16.73 \times \cos 135°$ $\Rightarrow w = 24.8(3)$	$\Rightarrow u = \dfrac{5\sin 135°}{\sin 15°} = 13.66$ or $5 + 5\sqrt{\ }$	M1A1 A1
	$\dfrac{\sin\theta}{16.73} = \dfrac{\sin 135°}{24.83}$ $\Rightarrow \theta = 28.45°$. $\Rightarrow$ Wind blows from bearing 118°.		M1 A1 A1 [10]
	Method β: $u\begin{pmatrix} -\sin 30° \\ \cos 30° \end{pmatrix} + \begin{pmatrix} -15 \\ 0 \end{pmatrix} = w\begin{pmatrix} -\sin 45° \\ \cos 45° \end{pmatrix} + \begin{pmatrix} -10 \\ 0 \end{pmatrix}$		M1A1 M1A1
	$\dfrac{u}{2} + 15 = \dfrac{w}{\sqrt{2}} + 10$		M1
	$\dfrac{\sqrt{3}u}{2} = \dfrac{w}{\sqrt{2}}$		M1
	$u = 5 + 5\sqrt{3} = 13.66$ or $v = \dfrac{5}{2}\sqrt{2}\left(3+\sqrt{3}\right) = 9.66$		A1
	Either side = $\begin{pmatrix} -\frac{5}{2}\left(7+\sqrt{3}\right) \\ \frac{5}{2}\left(3+\sqrt{3}\right) \end{pmatrix} = \begin{pmatrix} -21.83 \\ 11.83 \end{pmatrix}$	(N.B. Accept this vector answer from any method)	M1A1 A1 [10]
	Method γ: $15 + \dfrac{w\sin\theta}{\sqrt{3}} = w\cos\theta$	[Note: If a and b for $w\sin\theta$ and $w\cos\theta$ in the above equations, each equation gets M1A1, followed by M1A1A1 for v and M1A1A1 for θ.]	M1A1A1
	$10 + w\sin\theta = w\cos\theta$ $\tan\theta = \dfrac{9+2\sqrt{3}}{23} \Rightarrow \theta = 28.45°$.		M1A1A1
	Wind blows from bearing 118°. $v = 24.8(3)$. Speed of wind is 24.8 km h^{-1}.		M1A1 M1A1[10]

	Method δ: $\frac{w}{\sin 120°}=\frac{15}{\sin(60°-\theta)} \Rightarrow w=\frac{15\sin 120°}{\sin 60°\cos\theta-\cos 60°\sin\theta}$	*Two* equations for θ and w	M1A1
	$\Rightarrow w=\frac{15\sqrt{3}}{\sqrt{3}\cos\theta-\sin\theta}$	Compound angle formulae used	M1A1
	$\frac{w}{\sin 135°}=\frac{10}{\sin(45°-\theta)} \Rightarrow w=\frac{10\sin 135°}{\sin 45°\cos\theta-\cos 45°\sin\theta}$	Correct values of sin and cos	M1A1
	$\Rightarrow w\frac{10}{\cos\theta-\sin\theta}$ Equate: $15\sqrt{3}\ \cos\theta - 15\sqrt{3}\ \sin\theta = 10\sqrt{3}\ \cos\theta - 10\sin\theta$ $\Rightarrow \tan\theta=\frac{5\sqrt{3}}{15\sqrt{3}-10}=\frac{9+2\sqrt{3}}{23}$ [= 0.5419]		
	$\Rightarrow \theta = 28.45 \Rightarrow$ Wind blows from bearing 118° and $w = 24.83$	Solve Correct θ, correct bearing Correct w	M1 A1A1 A1 [10]
	Method ε: Components:		
	$w_x = u_x + 15 = v_x + 10$	Any two	M1
	$w_y = u_y = v_y$	All four	A1
	$\frac{w_y}{w_x-15}=\tan(60°)=\sqrt{3},\ \frac{w_y}{w_x-10}=\tan(45°)=1$		M1 A1 A1
	$w_x - 10 = \sqrt{3}\,(w_x - 15) \Rightarrow w_x=\frac{5}{2}(7+\sqrt{3})$,		M1A1
	$w_y=\frac{5}{2}(3+\sqrt{3})$		
	$\Rightarrow$ speed = 24.8(3) ms^{-1}		A1
	Angle is $\tan^{-1}$ (0.5419) = 28.45°		M1
	$\Rightarrow$ wind blows from bearing 118°.		A1 [10]
	Summary of answers: $\boldsymbol{w}=\begin{pmatrix}-\frac{5}{2}(7+\sqrt{3})\\ \frac{5}{2}(3+\sqrt{3})\end{pmatrix}=\begin{pmatrix}-21.83\\ 11.83\end{pmatrix}, w=2\sqrt{16+5\sqrt{3}}=24.83$ $\theta=\tan^{-1}\left(\frac{3+\sqrt{3}}{7+\sqrt{3}}\right)$ or $\tan^{-1}\left(\frac{18+4\sqrt{3}}{46}\right)=28.45° \Rightarrow$ wind blows from bearing 118(.45)° $\boldsymbol{u}=\begin{pmatrix}-\frac{5}{2}(1+\sqrt{3})\\ \frac{5}{2}(3+\sqrt{3})\end{pmatrix}=\begin{pmatrix}-6.83\\ 11.83\end{pmatrix}, u=13.66$ $\quad \boldsymbol{v}=\begin{pmatrix}-\frac{5}{2}(1+\sqrt{3})\\ \frac{5}{2}(3+\sqrt{3})\end{pmatrix}=\begin{pmatrix}-11.83\\ 11.83\end{pmatrix}, v=16.73$		