FURTHER MATHEMATICS

Paper 9795/01
Further Pure Mathematics

Key Messages

Candidates are best served by a thoughtful approach to each question that considers – in particular – how the question is structured; many candidates frequently ignore the very careful signposts offered within the question. In a similar vein, candidates must be aware of how appropriate the working they are writing down is to the demand of that question or question-part; producing up to a page-and-a-half of working for a result that has been assigned just one or two marks is clearly inappropriate, and candidates should allow themselves to be guided by the number of marks.

Another key point is that many candidates seem unwilling to deploy their 'single maths' skills in the further maths setting and they should be aware that understanding and fluency in GCSE-level and Maths-9794-level techniques are taken as background/assumed knowledge within the further maths papers.

General Comments

Overall, the quality of the candidates' work was very impressive, with a good proportion gaining 100 or more marks. These candidates are to be commended, especially considering the number of places in the paper which required considerable care and due diligence in order to avoid oversights of the small details that many candidates found all too easy to miss.

In **Questions 6** and **12**, in particular, the clear guidance as to how best to proceed was overlooked by many candidates, who seemed to prefer lengthy and unhelpful approaches when a moment's consideration of the questions' wording and structure would have led to a more concise or useful approach being adopted. The importance of remembering 'single maths' skills when taking further maths papers was particularly true in **Question 4** with the integration of $\sin^2 x$, that first required the very standard 'single maths' use of a double-angle identity; the straightforward trigonometric equations that arose at the end of **Question 5**; and the use of either or both of *Pythagoras' Theorem* and some elementary trigonometry work within suitably constructed right-angled triangles in **Question 7** in order to find the argument of a specific point in the complex plane.

Those questions that candidates found most difficult were **Questions 2**, **7** and **12**, while **Questions 1**, **5**, **8** and **9** were handled most comfortably and confidently.

Centres are reminded that there are some changes to the syllabus for 2016 and there are also revised specimen papers on the website: www.cie.org.uk .

Comments on Specific Questions

Question 1

This was a simple starter question, requiring the quoting of a formula and then its use. Almost all candidates managed to attend to these matters both quickly and accurately.

Answer: 3

Question 2

The sigma-notation did not appear to be understood by a number of candidates. In most cases, candidates seemed unsure of what to do with the power of (x – 1) in any chosen term, so they ended up with an answer of zero upon substituting x = 1. It had been expected that candidates would do this question by little more than quoting the result and then dealing with the detail.

Answer: $-\frac{64}{35}$

Question 3

This was a relatively straightforward induction question. However, it was often undertaken in a poor or incomplete way. Amongst the better candidates, the most frequently lost mark was the one at the end for explaining why what they have done up to this point actually means that the result has indeed been proven. Candidates should be reminded of the need for this 'induction round-up'.

Question 4

Candidates should be aware that a sketch does not require the use of graph paper and plotting points. A clear sketch in their answer booklet is all that is required. On this occasion, a closed curve through the pole (O), with a cusp-like shape and some indication of scale was all that was required. The area required the use of a standard, quote-able formula and some very routine integration; however, many candidates made very heavy going of what had been intended to be a quick and easy question.

Answer: **(ii)** $\frac{1}{2}\pi$

Question 5

Once again, the curve-sketching question, on the topic of *Rational Functions*, was by far the most popular and highest-scoring question on the paper, yielding very high marks for almost all candidates. Once again, for the 'sketch', it is important to stress that candidates should not use graph paper, as this has the unfortunate effect of forcing candidates into a particular scale, and also then into the ensuing 'wobbly' curves that occur when trying to accommodate it unreasonably. It is also important to note the demand, stated clearly within the syllabus, that key points – especially those points where the curve meets or crosses the coordinate axes – should be noted somewhere (on or alongside the sketch). On this occasion, the y-intercept was not required because it had little bearing on the validity of the sketch; but the x-intercepts were, and several candidates lost marks for not supplying them.

Answers: **(i)** $x = 3$, $y = 2x + 11$ **(ii)** (1, 9), (5, 25)

Question 6

This was a routine complex numbers question, but it led to several unnecessarily lengthy sets of working from candidates. The allocation of just the one mark to part **(i)** should have given a clear indication that very little was being required of candidates – certainly, those who produced many lines of working should have realised they were doing more than was required. In part **(ii)**, it was often the case that candidates struggled in the use of *de Moivre's theorem* this way round, preferring instead to express $\cos(n\theta)$ as the real part of $(\cos\theta + \text{i}\sin\theta)^n$ in each of three cases. This is a much lengthier approach and most who adopted it made little progress. Part **(iii)** was a relatively straightforward piece of single maths trigonometry. Nonetheless, there were many candidates who were unsuccessful. To begin with, a number of candidates did not notice the difference between the identity in part **(ii)** and the equation in part **(iii)**, thereby attempting to solve $16\cos^5\theta = 0$ (or 1). Many candidates also missed either or both of the $\cos\theta = 0$ and $\cos\theta = -\frac{1}{2}$ cases.

Answers: **(iii)** $\frac{1}{2}\pi$, $\frac{3}{2}\pi$; $\frac{1}{3}\pi$, $\frac{5}{3}\pi$; $\frac{2}{3}\pi$, $\frac{4}{3}\pi$

Question 7

In part **(i)**, the required locus is a circle and its interior. Whilst most candidates identified these details correctly, their circle was often to be found in either the first or the second quadrants. Even for those candidates whose circle was in approximately the right place, the position of z_1 (the point with least argument within this region) was almost never identified correctly. The candidates' work showed, in many cases, a lack of suitable basic trigonometry and *Pythagoras* being used to find the appropriate angle(s).

Answers: **(i)** circle, centre 20 – 15i, radius 7 **(ii)** –0.927 (or 5.356)

Question 8

This was a relatively simple question on this topic. Nevertheless, it was very easy to overlook some of the subgroups – there are seven of order 2 and seven of order 4 – and thus very few candidates gained full marks on the question.

Answers: **(ii)** {*e*, *a*}, {*e*, *b*}, {*e*, *c*}, {*e*, *ab*}, {*e*, *bc*}, {*e*, *ca*}, {*e*, *abc*}, {*e*, *a*, *b*, *ab*}, {*e*, *b*, *c*, *bc*}, {*e*, *c*, *a*, *ca*}, {*e*, *a*, *bc*, *abc*}, {*e*, *b*, *ca*, *abc*}, {*e*, *c*, *ab*, *abc*}, {*e*, *ab*, *bc*, *ca*}

Question 9

This was a relatively gentle question on the *Differential Equations* topic, and proved very popular amongst candidates. A few candidates preferred to differentiate $y=\frac{u}{x}$ in part **(ii)**, rather than the easier $u = xy$ given, frequently confusing themselves as the working is more complicated this way round. Also, despite its relatively straightforward nature, many candidates felt obliged to demonstrate how the auxiliary equation and complementary function arose, rather than quoting and then solving a quadratic, followed by a statement of the relevant CF; this was entirely unnecessary and wasted time, and often led to complex coefficients and complicated expressions.

Answers: **(i)** $u = A\cos 2x + B\sin 2x + 2x + \frac{1}{4}$ **(ii)** $y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2 + \frac{1}{4x}$

Question 10

This was a popular and usually well-attempted question. Algebraic or vector methods apply equally well in part **(i)**, although the simplest method must be that of finding two points on the line and proceeding from there. Once obtained, this line equation can be substituted straight into the third equation in part **(ii)**, to yield both the value of *k* and the evidence for the inconsistency. However, the value of *k* was usually found by evaluating the determinant of the coefficient matrix and then an algebraic elimination process was used to give the required inconsistency, often even from incorrect arithmetic. Many candidates lost one mark unnecessarily by omitting to have an '**r** = ...' (or equivalent) when writing the vector equation of a line.

Answers: **(i)** $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ **(ii)** $k = 5$

Question 11

In general, candidates did not answer this question well. Part of the problem lay in the candidates' misinterpretations of what was being asked of them. In part **(a)**, candidates were asked – in addition to finding the new equation with roots related to those of the first – to show **why** the substitution $y=\frac{4}{x}$ yielded the new equation and not just to verify that it did. In part **(b)**, parts **(i)** and **(ii)** were found to be very routine. Thereafter, in part **(iii)**, the problem lay principally in the justification of the degree of accuracy involved in the given numerical answers. It had been expected that candidates would use the *Change-of-Sign Rule*, but very few did so and thus lost at least two of the four marks available here. Most attempts merely substituted 2.70 into the left hand side of the equation and showed that the outcome was a number close to zero.

Answers: **(a)** $y^3 - 3y^2 - 8y - 16 = 0$ **(b)(i)** complex conjugates

Question 12

There is no doubt that the last question or two on the 9795 paper 1 are intended to differentiate between the most able candidates; it has often been the case – as it was here – that the penultimate question is long and technically demanding, while the last question is relatively short but requires an input of insight in order to untangle what is required. Nonetheless, candidates often seem to ignore the structure supplied. It is also the case that a few moments of careful consideration would show that, despite its appearance, all the parts of this question are essentially free-standing. Thus, an inability to complete one part does not prevent a candidate from continuing with the question. Indeed, part **(i)(a)** is actually a single-maths question, as it involves either integration by 'recognition' or by the most obvious of substitutions. The principal reason for its inclusion becomes abundantly clear when one views the reduction formula in part **(i)(b)**, a process that almost invariably requires the use of *integration by parts* and one of these parts has now been clearly flagged. Candidates who realised this had relatively easy access to the first 9 marks of the question. Those who didn't just seemed to abandon the question, often almost in its entirety, at this point.

For part **(i)(c)**, many unsuccessful candidates often made three or four attempts using unlikely substitutions and this wasted time. A few candidates chose to use $x\sqrt{2} = \tan\theta$ instead of the intended hyperbolic function substitution; this was sufficiently robust to allow them 6 of the 8 marks available, although they did run into difficulties when changing limits. It should then have been clear that the purpose in giving the answer here in part **(i)(c)** was to enable access to part **(ii)**, which now required only the 'surface area of revolution' formula and the preceding results. However, this formula was often incorrectly cited, with the '*y*' part missing.

Answers: **(i) (a)** $\frac{13}{3}$ **(ii)** $\frac{1}{8}\left(51\sqrt{2} - \ln\left(1+\sqrt{2}\right)\right)\pi$

Question 13

Once again, this question was placed last in order to avoid the scenario in which candidates wasted valuable time on something quite tough unless they had attempted all previous questions to the best of their ability. In fact, again, the structure was intended to be helpful and, on this occasion, very specific instructions were given to help guide candidates through the process. It had been expected that candidates would justify in part **(ii)** that the final possible compound angle would actually lie in the interval $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$. However, very few candidates thought to be that thorough. In part **(iii)**, it had been expected that candidates would realise that using the substituted form in the writing of the first term of the given series was invalid, deploy the result of part **(i)** for the sum of the n = 1 and n = 2 cases to obtain $\frac{1}{2}\pi$, and *then* use the 'method of differences' for the cancelling of terms in the remaining infinite series from n = 3 onwards. Again, almost no-one did this, as the option of noting that '$\tan^{-1}\left(\frac{1}{0}\right) = \frac{1}{2}\pi$' was just too inviting. Candidates were not penalised for taking this shortcut, even though this meant that one or other of the earlier parts of the question became redundant as a result.

Answer: **(ii)** $\frac{a \pm b}{1 \mp ab}$.

FURTHER MATHEMATICS

Paper 9795/02
Further Applications of Mathematics

Key Messages

There is a lot of material on this syllabus and the biggest challenge for candidates is selection of the correct method. There is a natural temptation to answer questions in a way similar to those that candidates have recently practiced, but differences that may seem to be fairly small at a casual reading make a big difference in method. Typical examples are confusion of normal and t distributions (**Question 2)**, between the variance of kX and the variance of ($X_1 + X_2 + \ldots + X_k$) (**Question 1**), or between conical pendulums and motion in a vertical circle (**Question 7**).

General Comments

Many candidates found much of the paper challenging, with the mechanics section being found harder than the statistics. Most candidates made a good attempt at the first few statistics questions. The mechanics question on which most candidates scored most of their marks was **Question 11** on relative velocity. However, there were some high scoring scripts, with several candidates scoring full marks.

Diagrams were often drawn well but it is advisable to indicate acceleration on force diagrams, if only to focus attention on which direction is positive.

Centres are reminded that there are some changes to the syllabus for 2016 and there are also revised specimen papers on the website: www.cie.org.uk .

Comments on Specific Questions

Question 1

This question was well answered, with almost all candidates scoring at least 5 out of 8. The mistakes were predictable, with the equation for the variance often written as $22 + 5\sigma^2 = 144$ or $44 + 5\sigma^2 = 144$ instead of the correct $44 + 25\sigma^2 = 144$.

Answers: **(i)** $\mu = 25$, $\sigma^2 = 4$ **(ii)** 0.902

Question 2

About half the candidates obtained the correct confidence interval. The most common mistake was to use a z value (2.326) instead of t_{19} (2.5395); others did not divide 2.382 by both 19 and 20. Almost all candidates realised that 5.5 was or was not a reasonable value for the population mean according to whether it was or was not in the confidence interval. However, some candidates over-interpreted the confidence interval with a statement such as “there is a 98% chance that the value of 5.5 is correct”.

Answer: **(i)** (5.499, 5.901)

Question 3

Most knew that they had to find $G'(1)$ and $G''(1)$, then $G''(1) - G'(1) + (G'(1))^2$, but there were frequent mistakes in working out $G''(t)$. Those who first expanded the expression were more likely to avoid errors. Many could not see what to do in part **(ii)**. Many attempted to use the mean and variance that they had just calculated, but the question required them to show that the distribution was $B\left(4, \frac{1}{3}\right)$ and the main point was to prove that the distribution was actually binomial and not something else.

Answers: **(i)** $E(X) = -\frac{4}{3}$, $\operatorname{Var}(X) = \frac{32}{9}$ **(ii)** $n = 4$, $p = \frac{1}{3}$

Question 4

In part **(i)** very few candidates successfully obtained the correct moment generating function, even though it is in the book of formulae. Some tried to integrate; some wrote out some terms of the series but were unable to take out the factor $\exp(e^{\lambda t})$. Some ended with the PGF. As the questions said “Derive”, a solution starting from the PGF and replacing t by $e^{\lambda t}$ was not acceptable. Most candidates then knew that they had to multiply together two MGFs.

Part **(ii)** was generally very well done. Almost all candidates got $k = 1.1(01)$ and the only errors after that tended to be from muddling k and $k + 2$, or subtracting the wrong tabular value in part **(c)**.

Answers: **(ii)(a)** 1.10 **(b)** 0.0740 **(c)** 0.721

Question 5

This question was done very well by many candidates and almost all knew what to do. The usual errors were with the continuity corrections, though there were a few errors of the type $\Phi^{-1}(1 - p) = 1 - \Phi^{-1}(p)$. Most candidates went straight in with np and $\sqrt{np(1-p)}$; some were then unable to handle the resulting algebra, but many correct answers were seen. Almost all candidates correctly gave the value of n as an integer.

Answers: $n = 192$, $p = 0.25$

Question 6

The first part contained a difficult detail that only careful candidates avoided. Most knew that they had to make U the subject of the formula and almost everyone attempted to go via F_V to F_U and then f_u. However, some candidates simply substituted their formula for U into F_V. It is necessary to work through the inequalities carefully as they lead to $U > \frac{40V}{V-40}$ and hence candidates need to consider $F_U = 1 - F_V(U)$. Most missed this and were surprised to get an extra negative sign in their answer when they differentiated the result.

Most, but not all, sensibly used the given answer rather than their own version in the subsequent working. Many found the median correctly, although not all included the lower limit of 60. Any of parts, partial fractions or substitution worked in evaluating $\int_{60}^{80} \frac{40V}{(v-40)^2} dv$; there were frequent errors, but pleasingly many correct answers were seen.

Answers: **(ii)(a)** $\frac{200}{3}$ **(b)** $40\ln 2 + 40$

***Section B*: Mechanics**

Question 7

As in previous years a question of this sort was found hard by candidates. There is often confusion between motion in a vertical circle and a conical pendulum, while this particular question was not *just* a conical pendulum as there was an extra horizontal force. Many of the candidates were unable to answer the opening piece of Pythagoras. In part **(ii)** many candidates resolved along the string and many others ignored the horizontal part of the string altogether.

It would be beneficial to candidates to stress to them that they should resolve towards the centre of the circle, i.e.

- Simple pendulum (motion in a vertical circle): resolve along the string.
- Conical pendulum (motion in a horizontal circle): resolve horizontally.

Answer: **(ii)** $v = \sqrt{\frac{7ag}{6}}$.

Question 8

To prove SHM, it is necessary to take a general displacement x, but many considered the force only at the extremity. It was also necessary for candidates to state that their equation showed SHM, and not just to produce the equation. In part **(ii)** many did not use the correct displacement/time formula $x = 0.2\cos(10t)$; most split the motion into two parts but then often used the wrong formula for the shorter part.

Answers: **(i)** $\frac{1}{5}\pi$ seconds **(ii)** $\frac{1}{15}\pi$ seconds

Question 9

Those who calculated the change of KE and subtracted the PE made part **(i)** look easy, but attempts to use the acceleration usually went wrong; there was confusion in both parts between the overall acceleration and that part of the acceleration that was due to the driving force. Some candidates produced a fictitious "acceleration from the driving force" although, on this occasion, if they got the right answer they could gain all the marks.

In part **(ii)** there were frequent wrong assumptions, using for instance half the time or half the velocity rather than working out the velocity at half the distance, and then many used the total force down the slope instead of the driving force.

Answers: **(i)** 98 322 J **(ii)** 4.68 kW.

Question 10

Some candidates seemed unfamiliar with this most standard example of motion under a variable force. A large number attempted to use $s = ut + \frac{1}{2}at^2$. Many candidates omitted the negative sign in the equation of motion; those who took downwards as positive generally used U rather than the correct $-U$ as the limit. Those who got as far as the correct integration in part **(i)** were often unable to handle the constants, with $\sqrt{k}$ often in the wrong place or omitted altogether. Part **(ii)** was better done in this respect.

Answers: **(i)** $\frac{1}{\sqrt{gk}}\tan^{-1}\sqrt{\frac{k}{g}}u$ **(ii)** $\frac{1}{2k}\ln\left(\frac{g+ku^2}{g}\right)$.

Question 11

By contrast, this relative velocity question was well done; indeed for quite a few candidates it was their principal source of marks in ***Section B***. Those who used diagrams made it appear straightforward. Those who used column vectors in terms of t had a lot more work to do.

Answers: **(i)** 0.273 km **(ii)(a)** 008.99° **(b)** 13.0 minutes.

Question 12

Many candidates could make only a little progress with this question and it was clear that they could not see the strategy. Those who attempted to take horizontal and vertical axes made life very hard for themselves. Some made unwarranted assumptions, such as assuming that the perpendicular component of velocity was the same at impact as at projection; assumptions such as this should not be quoted for a sloping plane without justification, even by those candidates who rotated the whole system so that the plane was horizontal.

Most could get one equation in part **(i)**, usually the $y = 0$ equation, but then many tried to use the x equation. Those who correctly used the equation $v_x = 0$ and substituted for t got the required answer very easily.

The rest of the question required elimination of θ, and this required some careful algebra; as the answers were given, steps in the working that left too large a gap to be filled by the Examiner were penalised. In fact most candidates had little idea of how to proceed. Those candidates who successfully completed the whole of this question have good reason to feel proud.