

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9795 FURTHER MATHEMATICS

9795/01

Paper 1 (Further Pure Mathematics),
maximum raw mark 120

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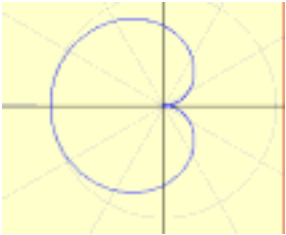
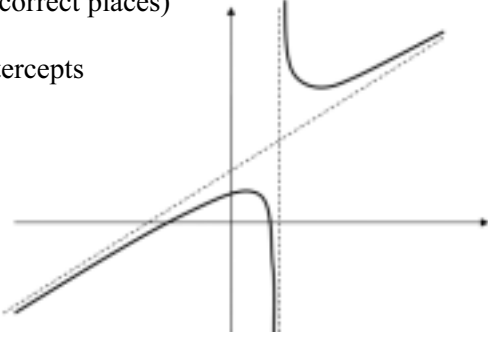
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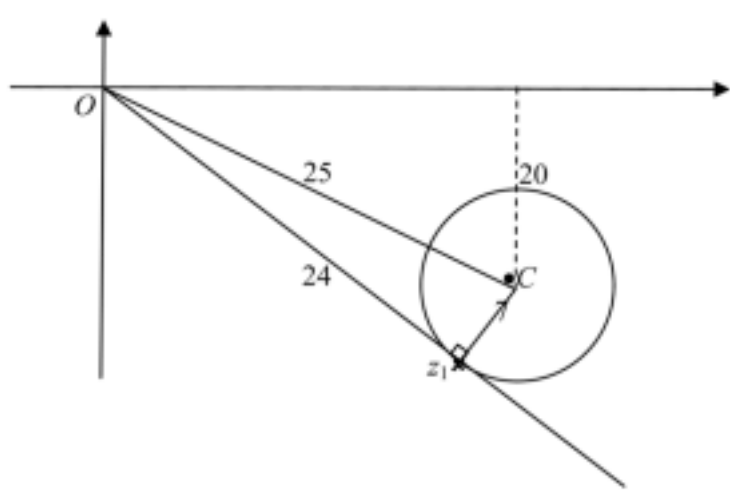
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1	<p>use of formula $V = \frac{1}{6} \mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ or equivalent NB $\mathbf{b} \times \mathbf{c} = -4\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$</p> <p>attempt at relevant scalar triple product = $\begin{vmatrix} 2 & 3 & -2 \\ 2 & 0 & 4 \\ 6 & 1 & 7 \end{vmatrix} = 18$</p> <p>(or a scalar and a vector product)</p> <p>$V = 3$ cso</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>
2	<p>$a_4 = \frac{y^{(4)}(1)}{4!} = \frac{(-2)^3}{1.3.5.7}$ from the Taylor series expansion</p> <p>$\Rightarrow y^{(4)}(1) = \frac{(-2)^3}{1.3.5.7} \times 4! = -\frac{64}{35}$</p> <p>ALTERNATIVE</p> <p>Diff^{te}. four times to get $\frac{d^4 y}{dx^4} = \sum_{n=4}^{\infty} \frac{(-2)^{n-1} n(n-1)(n-2)(n-3)(x-1)^{n-4}}{1.3.5\dots(2n-1)}$</p> <p>When $x = 1$, this is $\frac{(-2)^3 \times 4(3)(2)(1)}{1.3.5.7} + 0 = -\frac{64}{35}$</p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>[3]</p>
3	<p>$n = 1, \mathbf{M} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 2(1)+1 \\ 2(1)^2 + 2(1) \\ 2(1)^2 + 2(1)+1 \end{pmatrix} \Rightarrow$ result true for $n = 1$ (both sides shown)</p> <p>induction hypothesis that $\mathbf{M}^k \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k+1 \end{pmatrix}$</p> <p>attempt at $\mathbf{M}^{k+1} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M} \begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k+1 \end{pmatrix} = \begin{pmatrix} 2k+1-4k^2-4k+4k^2+4k+2 \\ 4k+2-2k^2-2k+4k^2+4k+2 \\ 4k+2-4k^2-4k+6k^2+6k+3 \end{pmatrix}$</p> <p>$= \begin{pmatrix} 2k+3 \\ 2k^2+6k+4 \\ 2k^2+6k+5 \end{pmatrix}$</p> <p>$= \begin{pmatrix} 2(k+1)+1 \\ 2(k+1)^2+2(k+1) \\ 2(k+1)^2+2(k+1)+1 \end{pmatrix}$</p> <p>This is the result with k replaced by $(k+1)$. Hence IF the result is true for $n = k$ THEN it also true for $n = k+1$. Since the result is true for $n = 1$, it follows that it is true for all positive integers n.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>E1</p> <p>[6]</p>

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<p>4 (i)</p> <p>(ii)</p>	<p>Closed curve containing the pole, O Cusp at the pole Essentially all correct, including at least $(r, \theta) = (1, \pi)$</p>  <p>$A = \left(\frac{1}{2}\right) \int_0^{2\pi} \sin^2 \frac{1}{2} \theta \, d\theta$ Condone missing $\frac{1}{2}$ until final A1</p> <p>use of double-angle identity $A = \frac{1}{2} \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{2} \cos \theta\right) d\theta$</p> <p>correct integration. $= \frac{1}{4} [\theta - \sin \theta]$ $= \frac{1}{2} \pi$</p>	<p>B1 B1 B1</p> <p>[3]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>
<p>5 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>VA $x = 3$</p> <p>$y = \frac{2x^2 + 5x - 25}{x - 3} = \frac{2x(x - 3) + 11(x - 3)}{x - 3}$ [+8]</p> <p>$y = 2x + 11$ oblique asymptote</p> <p>for $\frac{dy}{dx} = \frac{(x - 3)(4x + 5) - (2x^2 + 5x - 25)}{(x - 3)^2}$ (good attempt at the <i>Quotient Rule</i>)</p> <p>correct unsimplified OR $\frac{d}{dx} \left(2x + 11 + \frac{8}{x - 3} \right) = 2 - \frac{8}{(x - 3)^2}$</p> <p>solving quadratic equation $4x^2 - 7x - 15 = 2x^2 + 5x - 25$ i.e. $2x^2 - 12x + 10 = 0$ OR $(x - 3)^2 = 4$</p> <p>(1, 9) and (5, 25) (Give one A1 for both x's correct without either y)</p> <p>General shape (with asymptotes and turning points in approximately correct places)</p> <p>$2x^2 + 5x - 25 = 0$ solved to find x-intercepts $x = -5$ or $2\frac{1}{2}$</p> 	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 A1</p> <p>[5]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>

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<p>6 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>$z^n = \cos n\theta + i \sin n\theta$ and $z^{-n} = \cos n\theta - i \sin n\theta \Rightarrow z^n + \frac{1}{z^n} \equiv 2 \cos n\theta$</p> <p>$\left(z + \frac{1}{z}\right)^5 \equiv \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$</p> <p>Repeated use of result $\Rightarrow 32 \cos^5 \theta \equiv 2 \cos 5\theta + 10 \cos 3\theta + 20 \cos \theta$</p> <p>$\Rightarrow 16 \cos^5 \theta \equiv \cos 5\theta + 5 \cos 3\theta + 10 \cos \theta$ Answer Given</p> <p>$16 \cos^5 \theta = \pm \cos \theta$ Condone sign error here to allow for first A1</p> <p>$\cos \theta = 0 \Rightarrow \theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ (both)</p> <p>$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$ or $\frac{5}{3}\pi$ (both)</p> <p>$\cos \theta = -\frac{1}{2} \Rightarrow \theta = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$ (both)</p> <p>but allow one A1 for $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$ if both 2nd answers missing</p>	<p>B1 [1]</p> <p>M1 A1</p> <p>M1</p> <p>A1 [4]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 [4]</p>
<p>7 (i)</p> <p>(ii)</p>	<p>Circle with centre (20, -15)</p> <p>Circle of radius = 7 stated or deducible from diagram</p> <p>Allow B1 for a circle entirely in the 4th Quad.</p> <p>Interior of circle shaded (Ignore boundary in/out)</p>  <p>for z_1 in correct place</p> <p>for (sufficient) distances</p> <p>for $\arg(z_1) = -\left(\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{7}{24}\right)$ or equivalent, using other inverse trig. functions</p> <p>[NB – this is $\left(-\tan^{-1} \frac{4}{3}\right)$ using a result in Q13]</p> <p>for -0.927 (or $2\pi - 0.927 = 5.356$)</p>	<p>B1</p> <p>B1</p> <p>B1 [3]</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1 [4]</p>

8	(i)	<table border="1"> <tr> <td></td> <td><i>e</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>c</i></td> <td><i>ab</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>abc</i></td> </tr> <tr> <td><i>e</i></td> <td><i>e</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>c</i></td> <td><i>ab</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>abc</i></td> </tr> <tr> <td><i>a</i></td> <td><i>a</i></td> <td><i>e</i></td> <td><i>ab</i></td> <td><i>ca</i></td> <td><i>b</i></td> <td><i>abc</i></td> <td><i>c</i></td> <td><i>bc</i></td> </tr> <tr> <td><i>b</i></td> <td><i>b</i></td> <td><i>ab</i></td> <td><i>e</i></td> <td><i>bc</i></td> <td><i>a</i></td> <td><i>c</i></td> <td><i>abc</i></td> <td><i>ca</i></td> </tr> <tr> <td><i>c</i></td> <td><i>c</i></td> <td><i>ca</i></td> <td><i>bc</i></td> <td><i>e</i></td> <td><i>abc</i></td> <td><i>b</i></td> <td><i>a</i></td> <td><i>ab</i></td> </tr> <tr> <td><i>ab</i></td> <td><i>ab</i></td> <td><i>b</i></td> <td><i>a</i></td> <td><i>abc</i></td> <td><i>e</i></td> <td><i>ca</i></td> <td><i>bc</i></td> <td><i>c</i></td> </tr> <tr> <td><i>bc</i></td> <td><i>bc</i></td> <td><i>abc</i></td> <td><i>c</i></td> <td><i>b</i></td> <td><i>ca</i></td> <td><i>e</i></td> <td><i>ab</i></td> <td><i>a</i></td> </tr> <tr> <td><i>ca</i></td> <td><i>ca</i></td> <td><i>c</i></td> <td><i>abc</i></td> <td><i>a</i></td> <td><i>bc</i></td> <td><i>ab</i></td> <td><i>e</i></td> <td><i>b</i></td> </tr> <tr> <td><i>abc</i></td> <td><i>abc</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>ab</i></td> <td><i>c</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>e</i></td> </tr> </table>		<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>	<i>e</i>	<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>	<i>a</i>	<i>a</i>	<i>e</i>	<i>ab</i>	<i>ca</i>	<i>b</i>	<i>abc</i>	<i>c</i>	<i>bc</i>	<i>b</i>	<i>b</i>	<i>ab</i>	<i>e</i>	<i>bc</i>	<i>a</i>	<i>c</i>	<i>abc</i>	<i>ca</i>	<i>c</i>	<i>c</i>	<i>ca</i>	<i>bc</i>	<i>e</i>	<i>abc</i>	<i>b</i>	<i>a</i>	<i>ab</i>	<i>ab</i>	<i>ab</i>	<i>b</i>	<i>a</i>	<i>abc</i>	<i>e</i>	<i>ca</i>	<i>bc</i>	<i>c</i>	<i>bc</i>	<i>bc</i>	<i>abc</i>	<i>c</i>	<i>b</i>	<i>ca</i>	<i>e</i>	<i>ab</i>	<i>a</i>	<i>ca</i>	<i>ca</i>	<i>c</i>	<i>abc</i>	<i>a</i>	<i>bc</i>	<i>ab</i>	<i>e</i>	<i>b</i>	<i>abc</i>	<i>abc</i>	<i>bc</i>	<i>ca</i>	<i>ab</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>e</i>	B4, 3, 2, 1, 0
		<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>																																																																											
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	(ii)	<p>Subgroups of order 2: $\{e, a\}, \{e, b\}, \{e, c\}, \{e, ab\}, \{e, bc\}, \{e, ca\}, \{e, abc\}$ 6 distinct given all 7, no repeats</p> <p>Subgroups of order 4: $\{e, a, b, ab\}, \{e, b, c, bc\}, \{e, c, a, ca\}$ $\{e, a, bc, abc\}, \{e, b, ca, abc\}, \{e, c, ab, abc\}$ $\{e, ab, bc, ca\}$</p> <p>Either (all 3 of 1st type) + B1 (all 3 of 2nd type) + B1 (last one) Or (5 distinct) + B1 (6th) + B1 (all 7 and no repeats)</p>	B1 B1 B2 [5]																																																																																	
9	(i)	<p>Comp. Fn. is $u = A \cos 2x + B \sin 2x$ Allow $Ae^{2ix} + Be^{-2ix}$ here For Part. Int. try $u = ax + b$ ($u' = a$ and $u'' = 0$) and substd. into given d.e. $a = 2, b = \frac{1}{4}$</p> <p>Gen. Soln. is $u = A \cos 2x + B \sin 2x + 2x + \frac{1}{4}$ ft</p> <p>Must have two arbitrary constants Condone apparently complex coefficients here Don't allow $Ae^{2ix} + Be^{-2ix}$ here</p>	M1 A1 M1 A1 B1 [5]																																																																																	
	(ii)	<p>$xy = u \Rightarrow x \frac{dy}{dx} + y = \frac{du}{dx}$ and $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = \frac{d^2u}{dx^2}$</p> <p>$\Rightarrow (*)$ becomes $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 4xy = 8x + 1$</p> <p>$\Rightarrow y = \frac{A \cos 2x}{x} + \frac{B \sin 2x}{x} + 2 + \frac{1}{4x}$ ft</p> <p>Accept $xy = \dots$ Must have two real arbitrary constants</p>	M1 A1 A1 B1 [4]																																																																																	

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<p>10 (i)</p>	<p>d.v. = $\begin{pmatrix} 1 \\ 7 \\ -6 \end{pmatrix} \times \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix} = \begin{pmatrix} 26 \\ -26 \\ -26 \end{pmatrix} \equiv \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$</p> <p>finding one point on line: e.g. (0, 8, 11), (8, 0, 3), (11, -3, 0)</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots, t = \dots$)</p> <p>ALTERNATIVE 1</p> <p>finding two points on line: e.g. (0, 8, 11), (1, 7, 10), (8, 0, 3), (11, -3, 0), ...</p> <p>ft d.v. $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (e.g.)</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots$)</p> <p>ALTERNATIVE 2</p> <p>Eliminating x (say): $z = y + 3$ Setting $y = \lambda$ (or equivalent) and finding z and x in terms of the parameter $z = \lambda + 3$ and $x = 8 - \lambda$</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots$)</p>	<p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p>[5]</p> <p>M1 A1 A1</p> <p>B1</p> <p>B1</p> <p>[5]</p> <p>M1 A1 M1 A1</p> <p>B1</p> <p>[5]</p>
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<p>(ii)</p>	$\begin{vmatrix} 1 & 7 & -6 \\ 3 & -5 & 8 \\ k & 2 & 3 \end{vmatrix} = 26k - 130$ <p>= 0 when $k = 5$ ft from a linear eqn.</p> $x + 7y - 6z = -10$ <p>e.g. $3x - 5y + 8z = 48 \rightarrow -33y + 33z = 66 \rightarrow -y + z = 2$ $5x + 2y + 3z = 16 \rightarrow -26y + 26z = 78 \rightarrow -y + z = 3$</p> <p>eliminating one variable (twice); correctly Inconsistency noted/explained (allow valid ft)</p> <p>ALTERNATIVE</p> <p>Substituting $x = 8 - \lambda$, $y = \lambda$ and $z = 3 + \lambda$ into $kx + 2y + 3z = 16$ $\Rightarrow \lambda(5 - k) + (11k - 22) = 0$ λ terms; other terms $k = 5$ gives non-unique soln. Then $33 = 0$ ft and system inconsistent (allow valid ft)</p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1 A1 [5]</p> <p>M1 A1 A1 A1 A1 B1 [5]</p>
<p>11 (a)</p>	<p>Roots $\{qr, rp, pq\} = \left\{ \frac{pqr}{p}, \frac{pqr}{q}, \frac{pqr}{r} \right\}$</p> $= \left\{ \frac{4}{p}, \frac{4}{q}, \frac{4}{r} \right\}$ from $pqr = 4$, so subst. $y = \frac{4}{x}$ <p>Setting $x = \frac{4}{y} \Rightarrow \frac{64}{y^3} + 2 \cdot \frac{16}{y^2} + 3 \cdot \frac{4}{y} - 4 = 0$</p> $\Rightarrow 64 + 32y + 12y^2 - 4y^3 = 0$ $\Rightarrow y^3 - 3y^2 - 8y - 16 = 0$ <p>ALTERNATIVE (for last four-mark section)</p> $\sum \alpha' = qr + rp + pq = 3$ $\sum \alpha' \beta' = pqr(p + q + r) = 4 \times (-2) = -8$ $\sum \alpha' \beta' \gamma' = (pqr)^2 = 16$ <p>New eqn. is $x^3 - 3x^2 - 8x - 16 = 0$ (must have “= 0”)</p> <p>(b) (i) β and γ are complex conjugates (since the coefficients are real)</p> <p>(ii) $\alpha\beta\gamma = 4 \Rightarrow \beta\gamma = \frac{4}{\alpha}$ and $\beta = \gamma$ since conjugates $\Rightarrow \beta\gamma = \beta \cdot \gamma = \beta ^2 = \frac{4}{\alpha}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1 A1</p> <p>A1 [6]</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1 [6]</p> <p>B1 [1]</p> <p>M1</p>

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(iii)	and $ \beta = \frac{2}{\sqrt{\alpha}}$ (since $\alpha > 0$ given) Answer Given	A1	[2]
	$p(2.695) = -0.003... < 0$ and $p(2.705) = 0.049... > 0$ so $2.695 < \alpha < 2.705$ (by the “ <i>Change-of-Sign Rule</i> ”) $\Rightarrow \alpha = 2.70$ to 3s.f. (Allow numerical methods, e.g. <i>Newton-Raphson</i> – to find $\alpha = 2.70$ to 3s.f.)	B1	
	$\alpha + \beta + \gamma = 4 \Rightarrow \beta + \gamma = 4 - \alpha$ Then $2.695 < \alpha < 2.705 \Rightarrow 1.295 < 4 - \alpha < 1.305$	M1	
	Now, if $\beta = u + iv$, then $\gamma = u - iv$ and $\beta + \gamma = 2u$, i.e. $2 \operatorname{Re}(\beta)$ Thus $1.295 < 2 \operatorname{Re}(\beta) < 1.305 \Rightarrow 0.6475 < \operatorname{Re}(\beta) < 0.6525$	M1	
and $\operatorname{Re}(\beta) = 0.65$ to 2 s.f. (A0 for failure to justify 2s.f. accuracy properly)	A1	[4]	

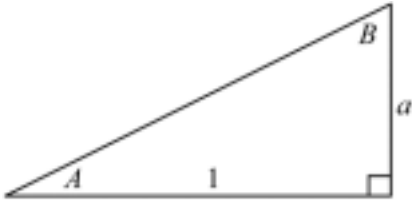
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12 (i)	<p>(a) $I_1 = \int_0^2 x\sqrt{1+2x^2} \, dx = \left[\frac{1}{6}(1+2x^2)^{\frac{3}{2}} \right]_0^2 = \frac{13}{3}$</p> <p>(b) $I_n = \int_0^2 x^{n-1} \cdot x\sqrt{1+2x^2} \, dx$ Correct splitting and use of parts</p> $= \left[x^{n-1} \cdot \frac{1}{6}(1+2x^2)^{\frac{3}{2}} \right]_0^2 - \int_0^2 (n-1)x^{n-2} \cdot \frac{1}{6}(1+2x^2)^{\frac{3}{2}} \, dx$ $= 2^{n-1} \cdot \frac{27}{6} - 0 - \frac{1}{6}(n-1) \int_0^2 x^{n-2} \cdot (1+2x^2)\sqrt{1+2x^2} \, dx$ $= 2^{n-1} \cdot \frac{27}{6} - \frac{1}{6}(n-1)\{2I_n + I_{n-2}\}$ $\Rightarrow 6I_n = 27 \times 2^{n-1} - 2(n-1)I_n - (n-1)I_{n-2}$ $\Rightarrow (2n+4)I_n = 27 \times 2^{n-1} - (n-1)I_{n-2} \quad \text{Answer Given}$ <p>(c) $I_0 = \int_0^2 \sqrt{1+2x^2} \, dx$</p> <p>Let $x\sqrt{2} = \sinh \theta$ ($\sqrt{2} \, dx = \cosh \theta \, d\theta$, $\sqrt{1+2x^2} = \cosh \theta$)</p> <p>full substitution $I_0 = \frac{1}{\sqrt{2}} \int \cosh^2 \theta \, d\theta$ (Ignore limits for now)</p> <p>trig. identity $= \frac{1}{2\sqrt{2}} \int (1 + \cosh 2\theta) \, d\theta$</p> $= \frac{1}{2\sqrt{2}} \left[\theta + \frac{1}{2} \sinh 2\theta \right] \text{ft ln. of } a + b \cosh 2\theta \text{ only}$ <p>$(0, 2) \rightarrow (0, \sinh^{-1} 2\sqrt{2})$ Limits properly dealt with</p> $= \frac{1}{2\sqrt{2}} \left[\sinh^{-1} 2\sqrt{2} + 6\sqrt{2} \right]$ <p>$\sinh^{-1} 2\sqrt{2} = \ln(3+2\sqrt{2})$ to at least here $= \ln(1+\sqrt{2})^2 = 2 \ln(1+\sqrt{2})$</p> <p>$I_0 = 3 + \frac{1}{\sqrt{2}} \ln(1+\sqrt{2})$ legitimately Answer Given</p>	<p>M1 A1 A1 [3]</p> <p>M1 A1 A1 M1 A1 A1 [6]</p> <p>M1 M1 A1 M1 A1 M1 M1 A1 A1 [8]</p>
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	<p>ALTERNATIVE (partial)</p> <p>Let $x\sqrt{2} = \tan \theta$ ($\sqrt{2}dx = \sec^2 \theta d\theta$)</p> <p>full substitution $I_0 = \frac{1}{\sqrt{2}} \int \sec^3 \theta d\theta$ (Ignore limits for now)</p> <p>use of \int by parts on $\int \sec \theta \sec^2 \theta d\theta$ and use of $\tan^2 \theta = \sec^2 \theta - 1$</p> $2\sqrt{2}I_0 = \sec \theta \tan \theta + \ln \sec \theta + \tan \theta $ <p>Further progress (limits, etc.) essentially impossible</p> <p>ALTERNATIVE</p> $I_0 = \int_0^2 \sqrt{1+2x^2} \times 1 dx = x\sqrt{1+2x^2} - \int \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \cdot 4x \cdot x dx \quad \int \text{n. by parts}$ $= x\sqrt{1+2x^2} - \int \frac{(2x^2+1)-1}{\sqrt{1+2x^2}} dx$ $= x\sqrt{1+2x^2} - I_0 + \int \frac{1}{\sqrt{1+2x^2}} dx$ $\Rightarrow 2I_0 = x\sqrt{1+2x^2} + \int \frac{1}{\sqrt{1+2x^2}} dx$ $\Rightarrow I_0 = \frac{1}{2} \left(x\sqrt{1+2x^2} + \frac{1}{\sqrt{2}} \sinh^{-1}(x\sqrt{2}) \right) \quad (\text{from MF20})$ <p>Use of limits (0, 2) $\Rightarrow I_0 = 3 + \frac{1}{2\sqrt{2}} \sinh^{-1}(2\sqrt{2})$</p> $\sinh^{-1} 2\sqrt{2} = \ln(3+2\sqrt{2}) = \ln(1+\sqrt{2})^2 = 2 \ln(1+\sqrt{2})$ $I_0 = 3 + \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \quad \text{legitimately Answer Given}$	M1 M1 A1 M1 A1
(ii)	$y = \frac{1}{\sqrt{2}}x^2 \Rightarrow \frac{dy}{dx} = x\sqrt{2} \quad \text{and } S = 2\pi \int_0^2 \frac{x^2}{\sqrt{2}} \cdot \sqrt{1+2x^2} dx$ $= \pi\sqrt{2} (I_2)$ <p>use of R.F. for $n = 2$: $8 I_2 = 54 - I_0$</p> <p>use of <i>their</i> (i) result $= 51 - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2})$</p> $\Rightarrow S = \frac{\pi\sqrt{2}}{8} \left(51 - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \right) \quad \text{or } \frac{\pi}{8} (51\sqrt{2} - \ln(1+\sqrt{2}))$	M1 A1 M1 M1 A1
		[8] [5]

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<p>13 (i)</p>	 $\tan A = \frac{a}{1}, \tan B = \frac{1}{a}$ $\Rightarrow \tan^{-1} a + \tan^{-1} \frac{1}{a} = A + B = \frac{\pi}{2}$	<p>B1</p> <p>[1]</p>
<p>(ii)</p>	$\tan(\tan^{-1} a \pm \tan^{-1} b) = \frac{\tan(\tan^{-1} a) \pm \tan(\tan^{-1} b)}{1 \mp \tan(\tan^{-1} a) \tan(\tan^{-1} b)} = \frac{a \pm b}{1 \mp ab}$	<p>M1 A1</p> <p>[2]</p>
<p>(iii)</p>	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right) \text{ noted at any stage}$ $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}(2) + \tan^{-1}\left(\frac{1}{2}\right) + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) \text{ Splitting off 1st 2 terms}$ $= \frac{\pi}{2} + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) \text{ using (i)'s result}$ <p>use of difference method (finite or infinite series)</p> $\Rightarrow \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{\pi}{2} +$ $\left(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{7} \dots\right)$ $-\left(\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{7} + \dots\right)$ <p>Cancelling of terms made clear</p> $= \frac{\pi}{2} + \left(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}\right)$ <p>and $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$ using (ii)'s result</p> <p>leading to ... Answer Given</p>	<p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[7]</p>

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	<p>ALTERNATIVE</p> <p>$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage</p> <p>Use of difference method (finite or infinite series)</p> $\sum_{n=1}^N \tan^{-1}\left(\frac{2}{n^2}\right) = \left(\tan^{-1}\frac{1}{0} + \tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \dots\right)$ $-\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \dots\right)$ <p>Cancelling of remaining terms made clear</p> $\tan^{-1}\infty + \tan^{-1}1 = \frac{\pi}{2} + \frac{\pi}{4}$ $= \frac{3\pi}{4} \quad \text{Given Answer}$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>
		[7]