Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2022**

3 hours

* 4 0 6 9 4 3 8 9 8 7 *

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document has **8** pages. Any blank pages are indicated.

Section A: Probability (60 marks)

1 The number Y of notifications of problems received by a helpline in one month has the distribution $\mathrm{Po}(\lambda)$.

(a) Suppose that $\mathrm{P}(Y = 16) = \mathrm{P}(Y = 18)$. Find the value of λ. [4]

(b) Suppose instead that the probability that exactly 8 notifications of problems are received in a period of two consecutive months is 4 times the probability that $Y = 8$. Find the exact value of λ. [4]

2 **(a)** The random variable U has the distribution $\mathrm{Po}(\lambda)$. Show that the probability generating function of U is $\mathrm{e}^{\lambda(t-1)}$. [4]

(b) The random variable V has probability generating function $\mathrm{G}(t)$. It is given that $\mathrm{E}(V) = 3$ and $\mathrm{Var}(V) = 6.75$.

(i) Find the values of $\mathrm{G}(1)$, $\mathrm{G}'(1)$ and $\mathrm{G}''(1)$. [4]

(ii) Find the mean and variance of the random variable with probability generating function $[\mathrm{G}(t)]^4$. [2]

3 It is given that, for a random variable X with the distribution $\mathrm{N}(0,\ \sigma^2)$, the moment generating function $\mathrm{M}(t)$ is $\mathrm{e}^{\frac{1}{2}\sigma^2 t^2}$.

The random variable Z has the distribution $\mathrm{N}(0,\ 1)$.

(a) By consideration of the variance of kZ, where k is a constant, find the moment generating function of kZ. [2]

(b) $Z_1, Z_2, \ldots, Z_n$ form a random sample of observations of Z. Show that the moment generating function of the sample mean $\overline{Z} = \dfrac{1}{n}\left(Z_1 + Z_2 + \ldots + Z_n\right)$ is $\mathrm{e}^{\frac{t^2}{2n}}$. [4]

4 In the town of Ayford, 8% of residents are retired.

(a) A researcher randomly selects 60 residents in Ayford. Use a suitable approximation to find the probability that at least 6 of those selected are retired. Justify the approximation used. [5]

In the town of Beebury, 45% of residents are of working age.

(b) A second researcher randomly selects 200 residents in Ayford and 200 residents in Beebury. Use suitable approximations to find the probability that the number of residents selected from Beebury who are of working age is more than 6 times the number of residents selected from Ayford who are retired. [6]

5 Sheila wishes to obtain information about the times taken to travel to work by residents in her town.

(a) Give a reason why Sheila might choose to obtain data

(i) from a sample rather than from the whole working population of the town, [1]

(ii) from a random sample rather than from a non-random sample. [1]

(b) Sheila selects a random sample of 20 people in the town. The times, t minutes, that these 20 people take to travel to work are summarised by

$$n = 20, \qquad \Sigma t = 940, \qquad \Sigma t^2 = 48\,360.$$

(i) Calculate an unbiased estimate of the population variance. [3]

Sheila calculates a 95% confidence interval for the mean time taken to travel to work.

(ii) Determine what answer Sheila should obtain, giving the end-points of the interval correct to 4 significant figures. [4]

(iii) State an assumption needed for Sheila's calculation to be valid, and give a reason why this assumption might not in practice be valid. [2]

6 A function $\mathrm{F}(x)$ is defined for $x \in \mathbb{R}$ by

$$\mathrm{F}(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right).$$

(a) Show that $\mathrm{F}(x)$ has the following properties.

- $\mathrm{F}(x) \to 0$ as $x \to -\infty$
- $\mathrm{F}(x) \to 1$ as $x \to \infty$
- $\mathrm{F}(x)$ is an increasing function for all x. [4]

$\mathrm{F}(x)$ is the cumulative distribution function of the continuous random variable X.

(b) Find $\mathrm{P}(X > 1)$. [2]

(c) Show that $\mathrm{E}(X^2)$ does not have a finite value. [5]

(d) The random variable Y is defined by $Y = 1 - X$. Find the cumulative distribution function of Y. [3]

Section B: Mechanics (60 marks)

7 The engine of a car of mass 800 kg has constant power output 96 kW. The car is subject to a resistive force R N, modelled by $R = kv^2$, where $v\,\text{m}\,\text{s}^{-1}$ is the speed and k is a constant. When travelling on a horizontal road its steady speed is $40\,\text{m}\,\text{s}^{-1}$.

(a) Find the value of k. [2]

(b) The car ascends a hill which makes an angle of $\sin^{-1} 0.1$ to the horizontal. Find the acceleration of the car at the instant when $v = 20$. [3]

(c) The car is accelerating at $0.5\,\text{m}\,\text{s}^{-2}$ on a horizontal road.

(i) Show that v satisfies an equation of the form $v^3 + av = b$, where a and b are constants whose values are to be stated. [3]

(ii) Verify that $v = 37.8$, correct to 3 significant figures. [2]

8 A ball is dropped from a height 2.45 m vertically above a point O on a smooth plane that slopes at an angle of 30° to the horizontal. The coefficient of restitution between the ball and the plane is 0.8.

(a) Find the components, parallel and perpendicular to the plane, of the velocity of the ball immediately after it hits the plane at O. [4]

(b) After bouncing at O the ball next hits the plane at the point A. Find the distance OA. [4]

9 A particle of mass m is fixed to one end of a light inextensible string of length l. The other end of the string is fixed to the ceiling. Initially the particle is held with the string taut so that it makes an angle of 60° with the downward vertical, and it is then released so that it moves in part of a vertical circle. Find the angle between the string and the downward vertical at an instant when the acceleration of the particle has no vertical component. [8]

10 A charged particle E of mass 5×10^{-5} kg can move in a horizontal straight line OA of length 0.12 m. An electric field exerts a force on E of $0.0024(0.12 - 3x)$ N in the direction OA, where x m is the displacement of E from O. Initially E is released from rest at the midpoint of OA.

(a) Show that E performs simple harmonic motion about a point which is to be defined, and state the period of the motion. [6]

(b) Find the time taken for E to reach the point where $x = 0.035$ for the first time. [4]

11 A particle P of weight 0.1 N is fixed to two light elastic strings AP and BP. The natural lengths of AP and BP are 0.4 m and 0.6 m respectively. The modulus of elasticity of each string is 0.15 N. The ends A and B of the strings are fixed to two points 1.6 m apart, with A vertically above B. P is projected vertically downwards with kinetic energy 0.02375 J from a position 0.7 m below A.

(a) Find the distance of P below A when the speed of P is greatest. [4]

(b) Find the greatest distance below A that P reaches, given that PB becomes slack. [7]

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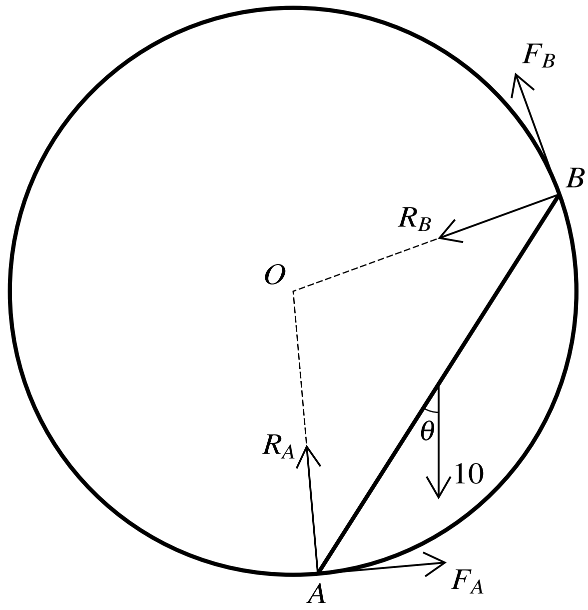

A uniform rod of weight 10 N and length 0.8 m rests in limiting equilibrium on the rough inside surface of a hollow sphere of radius 0.5 m. The plane containing the rod and the centre of the sphere is vertical. The rod makes an angle θ with the vertical. The magnitudes of the normal and frictional forces acting on the rod at A are denoted by R_A and F_A and those at B are denoted by R_B and F_B (see diagram). The coefficient of friction between the inside of the sphere and each end of the rod is 0.4.

(a) By taking moments about the centre of the rod, show that the ratio $R_A : R_B$ is $23 : 7$. [5]

(b) By resolving both parallel to the rod and perpendicular to the rod, find the value of θ. [8]

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